Explaining higher education progress through risk dominance in an n-person coordination (Stag Hunt) game

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Explaining higher education progress through risk dominance in an n-person coordination (Stag Hunt) game

By

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Abstract: In this paper, we use HARSANYI and SELTEN (1988)'s risk dominance concept to explain the growth in the Portuguese higher education system during two time periods: 1998 - 2005 and 2005-2018.

During the first time period, the high annual growth rate in tertiary schooling (8.2%) can be accounted for by a n – person, k – coordination Stag Hunt game framework. Hence, the progress in university education can be described as the outcome of a noncooperative game, where youngsters and their families can take decisions without needing to communicate previously.

By contrast, during 2005-2018, the former coordination game seems inadequate to rationalize the continued progress in college schooling at an annual rate of 5%, since the wage premium of tertiary education fell drastically (more than 20%) during the same interval. Hence, we switch to an “unanimity” game as framework of analysis. Within such a game, the widespread tertiary enrolment can be accounted for a diminishing “unanimity” requirement, derived from a shrinking demography and the sheer cumulative effect of past spread of college education.

We apply here NASH (1950, 1953)'s intuition that the selection of an equilibrium point within an unanimity game is a tool for modelling the outcome of a game, where the players discuss in order to reach an agreement. Hence, we can describe the rise in college education in Portugal in the more recent time period as the outcome of a cooperative process, leading to a wide policy consensus.

Keywords: Education; Regional Development; Coordination Games; Risk Dominance

JEL classification: C72, I20, O12, R11

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Introduction

Since MANKIW, ROMER and WEIL (1992), it is well known that global convergence trends are at present both in schooling rates and in per head income, but the former evolution appears to be much faster than the latter one. By restricting to Europe and to higher education, PONTES and BUHSE (2019) compute speeds of $\beta$ convergence in schooling rates and of real per head GDP with contrasting values, namely 4.0% and 2.3%, respectively. Hence, economic incentives cannot fully account for the differential development of college education across countries.

The growth rates of tertiary schooling rates and economic growth are still more at odds with each other in the case of a medium income European country such as Portugal. In this country for the period 1998 – 2005, while the tertiary schooling rate grew at an average yearly rate of 8.2%, the growth rate in per head real GDP was only 1.2%. The gap between either trend continued during the period 2005 -2018, the respective growth rates being 5.0% and 0.7%.\(^2\)

It can be objected that the private incentives to complete tertiary education are related with the \textit{wage premium} of college attendance, i.e. the relative difference between the skilled labour reward and the unskilled one, rather than with the average income. In terms of such a wage advantage, Portugal fared well in comparison to other European countries. MISIKOVA and VECERNIK (2019) find a value of 0.374 for the wage premium of college education in Portugal during the time period 2004 – 2013, which far exceeds the median (0.183) and the first \(\frac{1}{3}\) quantile (0.211) within a set of 27 European countries.

However, it is widely acknowledged that the wage advantage yielded by higher education in Portugal has been eroding during this time period. The \textit{OECD Employment Outlook 2019} found that such a decrease in the wage premium has been quite sharp. Portugal is deemed to be the OECD country where this fall was more sudden and severe, reaching -22.8% for the period 2006 – 2016., while the average decrease was only 3.3%. This fact matches with the surge in the emigration of skilled Portuguese youngsters from 2013 on.

Nevertheless, during this time period the progress in the tertiary schooling rate in Portugal continued at high pace, although private incentives diminished sharply. This paper attempts to provide a reasonable explanation for this apparent contradiction.

Higher education (i.e. \textit{ISCED 5-8}) is non-compulsory, so that the decision to enrol is an individual one. However, several authors have stressed that the accumulation of human capital is by its nature a “group process”, on three different grounds.

Firstly, learning only takes place in groups of people who freely discuss face-to-face during extended periods (LUCAS, 1988; MARSHALL, 1949). Hence, local externalities in human capital are crucial. BENABOU (1993) finds that the effort cost for a youngster to complete a college degree strongly decreases strongly with the share of residents with complete higher education living in the same neighbourhood.

Secondly, to complete higher education leads to the acquisition of specialized skills, which become profitable only if each graduate is matched with complementary specialists within the

\(^2\) In this paper, all data whose source is not otherwise mentioned were drawn from the Portuguese database \textit{PORDATA}, published by the \textit{Fundação Francisco Manuel dos Santos.}
same geographical area (WYDICK, 2008, DIAMOND, 1982). Hence, tertiary education skills are not supposed to be “self-consumed” by the graduate, but rather “traded” in the local labour market with complementary graduates.

Lastly, college students must share a fixed infrastructure (i.e. professors, libraries, labs) in order to be able to learn. Hence, the size of students group should meet a minimum requirement in order that the teaching institution breaks even.

In this paper, we will use the framework of a n-person coordination game (a Stag Hunt game) in order to rationalize the decision by a group of students to enrol in the university, in spite of the fact that private returns to higher education are small and fast decreasing. We will try to answer the question: can these decisions be modelled as the equilibrium outcome of a non-cooperative game or in alternative, should they be regarded as the result of an explicit agreement involving the students and their families?

The coordination game approach, with multiple Nash equilibria, has already been applied to the educational field, namely to the issue of “child labour”, in order to model the parental decision to either send a youngster to school or to assign to work outside the home (see BASU and VAN, 1998, and BASU, 1999).

A Stag Hunt game to model decisions to join higher education

Assumptions

We feature an economy along three consecutive time periods. In each period $t = 0, 1, 2$, the economy is composed by $n_t$ families. Each family is composed by an adult and by a youngster who has the proper age to enrol in the university.

Let $s_t$ be the share of families in period $t$, whose adult holds a college degree. We assume that such an adult predetermines his child to enrol in the university. Consequently, in each period $t$, only $n_t - s_{t-1}n_{t-1}$ students are free to decide whether to engage in higher education.

We presuppose that in period 0, the values of $n_0$ and $s_0$ are exogenously determined. Then, in each time $t = 1, 2$, each one of the $n_t - s_{t-1}n_{t-1}$ students with a low background decides whether to enter immediately in the labour market (strategy $\alpha$) or to enrol in college (strategy $\beta$), thus postponing one period his participation in the labour market. Such decisions are made simultaneously.

The payoffs of the pure strategies are as follows. If the youngster decides to find a job immediately, he obtains the certain payoff $w_U$, the wage for unskilled labour. Otherwise, if he decides to enrol in college, he may obtain one of two rewards. If at least $k$ of the $n_t - s_{t-1}n_{t-1}$ students decide to join higher education, then he obtains the payoff $\frac{w_S}{1 + r}$, the discounted value of the wage of unskilled labour, with $w_S > w_U$. For
simplicity, we will assume that the discount rate \( r \approx 0 \), so that the payoff of higher education with \( k \)-coordination among candidates is \( w_s \), the wage of skilled labour. The minimum requirement \( k \) stems from the group nature of higher education, whose foundations were discussed above in the Introduction.

Otherwise, if the minimum coordination requirement \( k \) is not satisfied, then the youngster’s payoff is zero.

The set of Nash equilibria of the n-person Stag Hunt game

The n-person Stag Hunt game has been well treated in CARLSSON and van DAMME (1993-b) and van DAMME (2002). They prove that this game has two strict Nash equilibria, namely \( \alpha \) (all players select pure strategy \( \alpha \)) and \( \beta \) (all players choose the pure strategy \( \beta \)), which is rather intuitive.

This game involves the selection of a Nash equilibrium, which amounts to the specification for each player of beliefs about the opponents’ behaviour thus enabling each player to deal with the situation of strategic uncertainty where he finds himself from the start.

In the experimental economics literature of repeated coordination games (Van HUYCK, BATTALIO and BEIL, 1990, 1991; SCHMIDT et Al. 2003), the selection of a Nash equilibrium point has used two kind of reasons, namely “inductive” arguments related with historical precedents; and “deductive” arguments, which are founded on the mathematical structure of the game situation. In this paper, “inductive” considerations are accounted by the fact that the number of individuals who decide to enrol in college, \( n_s \), predetermine the decisions made by their children in the subsequent time period \( t + 1 \).

According to KIM (1996), three strands of literature deal with the selection of a Nash equilibrium from the “deductive” perspective. HARSANYI and SELTEN (1988) handle this problem for static games of complete information by defining the concept of “risk dominance”, which we will treat further beyond.

CARLSONN and van DAMME (1993-a) redefine “risk dominance” for \( 2 \times 2 \) games of incomplete information, the so called “global games” where the players observe their payoffs imperfectly, with a small amount of “noise”. The selection of a Nash equilibrium follows from the observation of a sequence of perturbed games, as the amount of “noise” converges to zero. The Nash equilibrium that is the limit of this sequence is regarded to be coincident with HARSANYI and SELTEN (1988)’s “risk dominance”. CARLSSON and van DAMME (1993-b) extend the concept of “global game” to a symmetric Stag Hunt with \( n \) players.

The issue of Nash equilibrium selection can also be dealt with by means of explicitly dynamic and evolutionary processes, as in KANDORI, MAILATH and ROB (1995), among others.

In this paper, we will adopt the HARSANYI and SELTEN (1988)’s “risk dominance” as a comparative statics device, which we will use to consider successive time periods 1998-2005 and 2005-2018, in order to explain reasonably the evolution of higher education in Portugal.
How to select a Nash equilibrium in the symmetric n-person *Stag Hunt* game?

It is well known that HARSANYI and SELTEN (1988) define two criteria for ranking multiple Nash equilibria, namely *payoff dominance* and *risk dominance*. While the former concept is related with collective rationality, the latter expresses the individual attitude of each player while dealing with the strategic uncertainty he holds about his opponents’ behaviour.

In theoretical terms, HARSANYI and SELTEN (1988) contend that, when the two criteria conflict, the payoff dominance should prevail over risk dominance. This is so because the criterium of payoff dominance is “focal”. In this case, even if the players are not allowed to bargain before the game, there is common knowledge that, *if they might*, then they would settle in the Pareto dominant equilibrium. Thus, they will tacitly coordinate in the payoff dominant equilibrium.

This assertion has been widely debated in the experimental economics. Earlier results with coordination games, such as those by COOPER et Al. (1990), Van HUYCK, BATTALIO and BEIL (1990, 1991) stressed that Pareto dominant outcomes were often not observed in equilibrium. More recent laboratory studies with two-person *Stag Hunt* such as STRAUB (1995) and SCHMIDT et Al. (2003) stress the importance of “risk dominance” considerations in relation to those concerning Pareto dominance. In situations where the two criteria single out different Nash equilibrium points, SCHMIDT et Al. (2003) say that, even if the payoff dominant strategy is selected *more often than not*, players tend to respond to changes in the risk dominance levels, while they seem not be ready to react to variations in payoff dominance levels.

The importance of the risk dominance criterium derives not only from that it is purely based on individual rationality in a situation of strategic uncertainty, but also from the fact that it takes into account more information about the players’ payoff functions than the payoff dominance criterium. Hence, in $2 \times 2$ *Stag Hunt* games, while the latter criterium is based only upon equilibrium payoffs, the former one depends also from payoffs in outcomes that are out of equilibrium. Furthermore, while payoff dominance in n-person *Stag Hunt* games only compares individual payoffs, risk dominance is also influenced by the size of the group of players.
Checking risk dominance in the n-person *Stag Hunt* educational game

Since the game is symmetric, we presuppose that each player holds the same belief $p \in (0,1)$ on the likelihood of each opponent selecting the pure strategy $\beta$. Then, the expected payoff of a player choosing to enrol in college in time period $t$ is

$$\sum_{x=k_t-1}^{n_t-n_{t-1}s_{t-1}-1} \binom{n_t-n_{t-1}s_{t-1}-1}{x} p^x (1-p)^{(n_t-n_{t-1}s_{t-1}-1)-x} W_s$$

(1)

In (1), $k_t$ is the requirement of coordination in the game, which expresses the fact that learning takes place in groups. We assume here this requirement follows from the fact that students share a fixed learning infrastructure, hence a minimum number of students in necessary for a college to break even. We can write this condition as the inequality

$$(y_t + n_{t-1}s_{t-1})c \geq E_t \cdot F$$

(2)

Where

$y_t$ is the number of youngsters that take freely the decision whether to enrol in higher education in time period $t$.

c is the cost per student of higher education.

$E_t$ is the number of higher education institutions in time period $t$.

$F$ is the fixed cost of a higher education institution.

If we solve inequality (2) in relation to $y_t$, we obtain

$$y_t \geq E_t \left( \frac{F}{c} \right) - n_{t-1}s_{t-1} = k_t$$

(3)

Since the cost structure of colleges tends to be invariant, we can set

$$\frac{F}{c} = \gamma$$

a positive constant, and the coordination requirement in time period $t$ becomes

$$k_t = \gamma E_t - n_{t-1}s_{t-1}$$

(4)

The breakeven point of the higher education system varies directly with the number of universities and it is negatively influenced by the overall number of college graduates in the previous time period.
HARSANYI and SELTEN (1988) derive the symmetric expectation each player holds about every other participant selecting a pure strategy in a quite intricate way, which is also described in CARLSSON and van DAMME (1993-b), van DAMME (2002). Their result is as follows. Let us define the “wage premium” of higher education as

$$\bar{w} = \frac{w_2 - w_U}{w_U}$$  \hspace{1cm} (5)

Then HARSANYI and SELTEN (1988) derive the following belief about every other player choosing to join college.

$$p = \bar{w}$$  \hspace{1cm} (6)

This result is quite intuitive since a high “wage premium” of college education will lead each player to believe that the other candidates will choose to join the university with a high probability. However, specification (6) is not the unique that is consistent with the “risk dominance criterium. By contrast, Güth and Kalkofen (1989) proposed

$$p = \frac{1}{2}$$  \hspace{1cm} (7)

which is also intuitive, since an individual faced with uncertainty tends to assign the same probability to each alternative.

In what follows, we will follow the former specification of $p$. Bearing in mind expressions (4), (5) and (6), the expected payoff of playing the pure strategy $\beta$ in (1) becomes

$$\sum_{x=\gamma E_t-n_{t-1}S_{t-1}}^{n_t-n_{t-1}S_{t-1}-1} \left( \frac{n_t - n_{t-1}S_{t-1} - 1}{x} \right) \bar{w}^x \left( 1 - \bar{w} \right)^{(n_t-n_{t-1}S_{t-1})-x} W_S$$  \hspace{1cm} (8)

Then, the condition that pure strategy $\beta$ has an higher expected payoff than pure strategy $\alpha$ is simply

$$\sum_{x=\gamma E_t-n_{t-1}S_{t-1}}^{n_t-n_{t-1}S_{t-1}-1} \left( \frac{n_t - n_{t-1}S_{t-1} - 1}{x} \right) \bar{w}^x \left( 1 - \bar{w} \right)^{(n_t-n_{t-1}S_{t-1})-x} W_S > W_U$$

Bearing in mind (5), this inequality can be manipulated to give

$$\sum_{x=\gamma E_t-n_{t-1}S_{t-1}}^{n_t-n_{t-1}S_{t-1}-1} \left( \frac{n_t - n_{t-1}S_{t-1} - 1}{x} \right) \bar{w}^x \left( 1 - \bar{w} \right)^{(n_t-n_{t-1}S_{t-1})-x} + \bar{w} > 1$$  \hspace{1cm} (9)

Let us define $Bin(n, k, p)$ the cumulative binomial (or Bernoulli) distribution function.

$$Bin(n, k, p) = \sum_{x=0}^{k} \binom{n}{x} p^x (1 - p)^{n-x}$$
Then, inequality (9) can be written simply as
\[
\left[1 - \text{Bin}(n_t - n_{t-1}, s_{t-1} - 1, \gamma E_t - n_{t-1} s_{t-1} - 2, \tilde{w})\right] + \tilde{w} > 1
\] (10)

Although we have modelled the choice of the pure strategy $\beta$ as best reply against a subjective belief rather than against the actual equilibrium behaviour of the opponents, it is easy to realize that the fulfilment of inequality (10) means that the equilibrium point $\bar{\beta}$ is indeed the risk dominant equilibrium (see HARSANYI and SELTEN, 1988, lemma 4.17.7). This happens because the game is symmetric and $\alpha$ and $\bar{\beta}$ are the only strict Nash equilibria in this game.

A direct observation of (10) allows us to state the factors that either ease or make difficult that the equilibrium point $\bar{\beta}$ arises as “risk dominant” (see HEINEMANN, NAGEL and OCKENFELS, 2009).

**Proposition 1:** The educational game outcome where all candidates decide to enrol in the university (equilibrium $\bar{\beta}$) will become less risky in strategic terms if

1. The total population in the same time period is high.
2. The “wage premium” in the same period is high.
3. The number of universities is low, i.e. the higher education system is concentrated.

Applying the k – coordination game to the evolution of higher education schooling in Portugal

The time period 1998 -2005

During this time period, the share of population with age 30/34 with a complete university degree increased at a high annual growth rate of 8.2%. Resident population within the same age interval increased at an average rate of 1.2%, while the “wage premium” of college education remained relatively stable and high in comparison to other European countries. Some concentration of the college network took place as some private universities ceased to exist. The number of students enrolled in private universities per hundred students enrolled in the public system fell at the annual rate of -6%.

We can observe that the causal factors described in Proposition 1 provide a reasonable explanation for the evolution of the higher education system during this time period.
The time period 2005 – 2018

During this period, the schooling rate of higher education continued to increase at a fast pace, with an annual growth rate of 5.0%, and it reached a level of 34% in 2018, not far from the 39% mean of the EU countries.

However, it is difficult to rationalize the continuation of higher education growth in connection with the causal factors that were listed in Proposition 1. Resident population aged between 30/34 fell at a -2.7% average annual rate. As we mentioned, the “wage premium” decreased almost 23% between 2006 and 2016, the highest fall recorded in OECD countries. The only positive factor was the steady decrease in the number of private universities and the associated concentration of the university system. In whole, it seems that Proposition 1 fails to rationalize the evolution along the time period 2005-2018.

Hence, we rewrite the n-person Stag Hunt as an “unanimity game”, where a candidate who decides to join the university obtains the positive payoff \( w_s \) only if every free youngster makes the same choice. This amounts to presuppose that the total number of candidates \( n_t \) is just equal to the minimum scale \( \gamma E_t \) for the college system to operate profitably. With this change, condition (9) that strategy \( \beta \) is risk dominant for each player becomes

\[
\tilde{w}^{(n_t-n_{t-1}s_{t-1})} + \tilde{w} > 1
\]

which is equivalent to

\[
n_t - n_{t-1}s_{t-1} < 1 + \frac{\ln(1-\tilde{w})}{\ln \tilde{w}}
\]

(12)

It can be easily checked that the right hand side of inequality (12) is an increasing function of \( \tilde{w} \). As before with the k-coordination game, the strategic riskiness of strategy \( \beta \) is now directly associated with the wage premium yielded by higher education.

However, the left hand side of the inequality (12) tells an utterly different story. For a youngster, the degree of safety while joining a college is inversely related with the number of youngsters who freely make the same decision, since “unanimity” is easier to achieve the smaller the number of concerned agents is. The equilibrium point \( \bar{\beta} \) becomes more easily selected on account of negative demographic trends (i.e. a low value of \( n_t - n_{t-1} \)) and by the sheer cumulative effect of former spread of higher education (i.e. a high value of \( s_{t-1} \)). This is what seemingly explains the increase in tertiary schooling between 2005 and 2018, when the computed value of \( n_t - n_{t-1}s_{t-1} \) fell sharply at an annual growth rate of -4.2%.

The results of the unanimity game can be summarized by Proposition 2.

**Proposition 2:** In the context of the unanimity game, strategy \( \beta \) is risk dominant for each player if

1. The “wage premium” of higher education is high.
2. Overall population tends to decrease.
3. Past level of tertiary schooling is high.
Hence, the fast increase of tertiary schooling in Portugal during the period 2005-2018 can be rationalized by negative demographic trends and by a circular, cumulative process associated with the spread of higher education. A continued development of university education took place, although the “wage premium” of college attendance fell sharply during the time period.

The “unanimity” game requires that the decline in population is matched by a proportional decrease in the breakeven point of the higher education system, made feasible through its concentration. Such a process seems to have taken place in the time period 2005-2018 through the closure of private universities. The number of students attending private universities per a hundred of students enrolled in the public system fell at the annual rate of -4% during this period.

The k-coordination game expressed by inequality (10) and the unanimity game represented by condition (12) have a qualitatively different meaning. The former may be viewed as a purely non cooperative game, where the players are not allowed to communicate before they move. By contrast, as John NASH (1950, 1953) contended, the selection of an equilibrium point in a noncooperative “unanimity” game is nothing but a different way of representing a situation where the players may discuss explicitly in order to reach an agreement. Hence, the fast growth of university education in Portugal between 2005 and 2018, which happened although the college “wage premium” fell sharply, mainly reflects a wide social consensus which was established through the political system in the country.

Concluding remarks

In this paper, we used HARSANYI and SELTEN (1988)’s risk dominance concept to explain the growth in the Portuguese higher education system during two time periods: 1998 -2005 and 2005-2018.

During the first time period 1998-2005, the high annual growth rate in tertiary schooling (8.2%) can be accounted for within a n – person, k – coordination Stag Hunt game framework. The fast growing tertiary schooling rate appears to be the outcome of several factors, namely a high and relatively stable “wage premium” of college education, a relatively stable overall population and a concentration trend of the higher education system through the closing of several private universities. Hence, the progress in university education can be described as a noncooperative game, where youngsters and their families take decisions without any kind of previous communication. In this process, coordination around an equilibrium point seems to stem only from common knowledge of rationality and game rules.

By contrast, during the most recent time period 2005-2018, the former game seems inadequate to rationalize the high 5% annual growth rate of tertiary schooling. Hence, we switch to the framework of an “unanimity” game. Although the sharp fall in “wage premium” of higher education remained an obstacle to tertiary schooling, the widespread decision to join the university by youngsters can be reasonable explained by the decrease in the “unanimity” requirement, derived from a shrinking demography and the sheer cumulative effect of past spread of college education.
We apply here NASH (1950, 1953)’s insight that the selection of an equilibrium point within a noncooperative game is but a tool for modelling the outcome of a game where the players settle in an agreement by means of previous discussion. Hence, we can describe the rise in college education in Portugal in the more recent time period as the outcome of a cooperative process, leading to a wide consensus that was established through the operation of the political system.

If the recent progress in university education in Portugal is best regarded as a cooperative process, why not to use “payoff dominance” rather than “risk dominance” as the framework for examining college schooling trends? We believe that adopting the “risk dominance” perspective is still preferable on two grounds. Firstly, the “level of payoff dominance” has fallen sharply, although joining the university by all candidates continues to be a Pareto optimum. Secondly, the evolution of the university system can be understood only the variation in the number of the enrolled individuals is taken in account. Group size considerations are included within the “risk dominance” framework, but they are remarkably absent from Pareto comparisons between outcomes of higher education development.
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