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# Education spread and economic development- a coordination game approach

José Pedro Pontes[1]

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**Abstract**: Even though education exhibits strong positive externalities, there exists abundant evidence that the positive correlation between the spread of higher education and aggregate productivity tends to gradually disappear as a larger number of students attend college.
To rationalize this apparent contradiction, we model college attendance through a coordination (*Stag Hunt*) game, where a set of youngsters decide whether to enrol in college or immediately enter the labour market. The benefits of higher education are reaped by everyone only if *all* youngsters decide to engage in higher education.
We argue that the reward to college education tends to decline as it becomes widespread on account of two factors. First, the wage premium of skilled labour falls due to an increased supply. Then, new colleges tend to be set up in less dense areas, so that the average distance between a youngster's residence and the closest university rises.
The decrease in the reward of tertiary education is multiplied by its impact on the result of a *Stag Hunt* game. If such a reward is high, there will likely be a switch from the "risk dominant" Nash equilibrium, where no youngster decides to engage in tertiary education, to the "payoff dominant" equilibrium with unanimous enrolment in college. By contrast when the net reward of higher education becomes relatively low the outcome of the *Stag Hunt* will be instead a *coordination breakdown*, which might even lead to a fall in the productivity of the agents involved.
The latter result was found in laboratory studies by Straub (1995) and Schmidt et al. (2003) among others and we can explain it in theoretical terms if we allow for incomplete information in the *Stag Hunt* game.

**Keywords**: Education and Economic Development, Coordination Games, Incomplete Information Games

**JEL codes**: C72, I25, 040

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# 1. Introduction

Rising schooling rates have been widely regarded as a crucial factor of economic development. Since in many countries secondary education is now compulsory, there is a consensus that the proportion of youngsters aged 18 or more with college attendance strongly determines the pace of economic growth.

This belief is founded on that the allocation of resources to formal education causes the accumulation of “human capital”, an asset that increases labour productivity. Such a technical progress enhancing mechanism is shared by different views on economic growth, both by Solow (1956)’s type theories (see Mankiw et al., 1992) and by endogenous technical progress models (see Lucas, 1988).

Nevertheless, a positive causality exerted by “human capital” on productivity growth is not straightforward to detect. By restricting to Europe and higher education, Pontes and Buhse (2019) computed speeds of $\beta$ - convergence in higher education attendance rates and real per head GDP with contrasting values, namely 0.04 and 0.023, respectively, between 2004 and 2018.

Using a middle-income country such as Portugal as a benchmark, we write in Table 1 the average annual growth rates of the proportion of people with a complete higher education degree, of real per head GDP and their difference along to two twenty-year time periods.

Table 1

| Time period | (1) | (2) | (1)−(2) |
|---|---|---|---|
| 1981−2001 | 6.4 | 3.0 | 3.4 |
| 2001−2021 | 4.8 | 0.4 | 4.4 |

Legend

(1) ≡ average annual growth in the percentage of people older than 15 with a complete higher education degree according to censuses.

(2) ≡ average annual growth of real per head GDP

Source: PORDATA - INE

Some comments to Table 1 are in order. During a 40-year period, a steady growth in the proportion of people with tertiary education took place, albeit at a decreasing rate. By contrast, the significant economic growth that occurred between 1991 and 2001 ceased afterwards, the economy almost experimenting a stagnation from 2001 onwards.

The increase in the higher education schooling rate always by far exceeded the economic growth rate, but the gap became more severe during the last twenty years. Such a widening gap expresses a structural change. While between 1981 and 2001 both variables grew - albeit at different rates - during the more recent period tertiary schooling continued to rise but aggregate productivity remained almost unchanged.

The aim of this paper is twofold. First, we purport to explain why there is a structural gap between the spread of higher education and the evolution of economic incentives to college attendance. Second, we wish to account for the fact that such a gap became more severe in recent times.

To rationalize this structural gap, we base ourselves on the premise that the decisions to complete college education are not only driven by private economic incentives, but they generate strong positive externalities that make them complementary across agents. In addition to raising individual private productivity and revenue, higher education works as a *collective input* that directly impacts the aggregate productivity of the economy, an effect that was put forward by endogenous growth theories (see, among others, Frankel, 1962, and Lucas, 1988).

In this paper, we use the framework of $2 \times 2$ coordination (*Stag Hunt*) games to explain why individuals, who are driven by complementarities, engage in higher education even though the revenue enhancing of such decisions might be limited, even non-existent. Our use of coordination *Stag Hunt* games is based on the idea that the accumulation of "human capital" is essentially a *group process*, on three different grounds.

First, learning implies that groups of people meet and discuss face-to-face during extended periods of time (Lucas, 1988; Marshall, 1949). Consequently, positive externalities are especially strong at a local level. Following this line, Benabou (1993) presupposed that the effort cost for a youngster to obtain a college degree strongly decreases with the share of residents with higher education living in the same neighbourhood.

Second, college students share a set of fixed inputs such as "professors", "laboratories", "classrooms" and so on, which might break even only if they are jointly used by a minimum number of youngsters.

Finally, higher education provides the student a specialized skill that may be made profitable only if each graduate is matched with other complementary specialists within a professional organization. Hence, the college skill that is developed through college attendance is not destined to be "self-consumed" by the graduate, but instead "traded" with complementary graduates in a local labour market (Wydick, 2008; Diamond, 1982). Consequently, the returns of training a specialist are quite variable and depend on the quality of this *match* to other complementary specialists.

In what follows, we feature an economy with $n$ individuals who interact along two stages. In the first stage, each agent decides whether to enter the labour market immediately – strategy labelled as "Work" – or to postpone this entry to enrol in a college – a strategy named as "School".

In the second stage, the participants are allocated to the production of a composite consumer good. In this production stage, a college educated worker receives a higher wage than an unskilled one, provided that a "critical mass" of highly educated individuals is attained. Otherwise, the "School" strategy entails a loss in relation to the decision to participate immediately in the labour market.

It is intuitive that this coordination *Stag Hunt* game has two symmetric Nash equilibrium points – either all youngsters decide to enter college, or none does. For simplicity, we assume that the "critical mass" requirement on the number of students is equivalent to "unanimity"", i.e., the benefits of higher education may be reaped only if *all* youngsters decide to enrol in college. This assumption allows us to model this situation more simply by means of a two-person game.

The framework of a complete information game with two equilibrium points in basic alternatives of action is of limited help to explain reasonably the apparent lack of correlation between higher education levels and aggregate productivity. Basically, the switch from a situation where none completes college to an outcome of unanimous enrolment always implies a rise - however small - in output per worker. Nevertheless, we must still account for an utter absence of correlation between higher education levels and productivity.

Consequently, we change the coordination game to allow *incomplete* information by at least a subset of players. As Harsanyi (1973) argued, the change in this assumption makes possible that, in the context of a (Bayesian) Nash equilibrium in type-contingent pure strategies, a player might base his strategic decision on the belief that his opponent uses an “induced” mixed strategy. Thus, a coordination breakdown might emerge with the result that the game terminates in an out-of-equilibrium outcome.

Hence, we may depict the evolution of the economy as follows. During the first period, both individuals decide join a university, so that both the college schooling rate and per head income rise. Then, in the second period, they fail to coordinate themselves, so that a youngster chooses to work immediately, while the other one joins college. Along this sequence of events, the college schooling rate increases, while the average revenue falls, as the educated player is unable to find a suitable job after completing college.

In section 2, we provide some evidence about the share of college-educated people and population density across the Portuguese (NUTS2) regions. In section 3, we model the decisions to enrol in higher education by means of a coordination (*Stag Hunt*) game. In section 4, we apply this theoretical framework to the evolution in period 1981-2001, when both higher education spread out and economic growth took place. In section 5, we allow for incomplete information in the schooling game to explain why the above correlation stopped existing in the period 2001-2021. Section 6 draws the main conclusions.

## 2. Empirical motivation

In Table 2, we gather data of population density and share of tertiary-educated people in 2001, and the relative variation in the latter indicator between 2001 and 2021 for the five Portuguese mainland NUTS2 regions (North, Centre, Lisbon Metropolitan Area, Alentejo and Algarve) and the NUTS1 insular regions of Azores and Madeira.

Table 2

| Regions | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| North | 173.2 | 6.2 | 17.8 | 1.87 |
| Centre | 83.3 | 6.1 | 17.4 | 1.85 |
| Lisbon M.A. | 899.6 | 12.0 | 26.6 | 1.22 |
| Alentejo | 24.6 | 5.0 | 14.7 | 1.94 |
| Algarve | 79.5 | 6.5 | 17.3 | 1.66 |
| Azores | 104.2 | 5.2 | 14.7 | 1.83 |
| Madeira | 313.6 | 5.6 | 16.5 | 1.95 |

where,

$(1) \equiv$ Population density (people per $Km^2$) in 2001. Source: PORDATA with data compiled from DGT/MAAC - MCT, INE

$(2) \equiv$ Share of resident population older than 15 with a complete tertiary education degree according to the 2001 Census. Source: PORDATA - INE

$(3) \equiv$ Share of resident population older than 15 with a complete tertiary education degree according to the 2021 Census. Source: PORDATA - INE

$(4) \equiv$ Relative variation in the share of tertiary-educated people between 2001 and 2021, i.e.,

$$(4) = \frac{(3)-(2)}{(2)}$$

Three stylized facts can be found in Table 2.

First, in 2001 areas with a low share of tertiary-educated people are either mainland regions with low population density or insular regions, which are geographically isolated.

Second, a strong convergence in the share of tertiary-educated people took place across regions between 2001 and 2021. Regions with a low initial share of college-educated people increased higher education attendance at a much faster pace.

Finally – and this follows direct from the two previous facts – the growth in higher education was relatively stronger either in sparsely populated or in geographically isolated regions.

## 3. Modelling the correlation of higher education and labour productivity through a *Stag Hunt* game under complete information

We said above that learning through college attendance has the nature of a "group process", where students share a set of fixed inputs (i.e., professors, buildings, libraries and so on) and develop specialized skills which are combined to each other to generate increasing amounts of output.

We will use here an approach inspired by Helsley and Strange (1990) to model the workings of the "fixed input sharing" and "skill matching" mechanisms, which form the economic foundations of a higher education process.

We feature in Figure 1 a linear region plotted as the interval $\left[0, L\right]$.

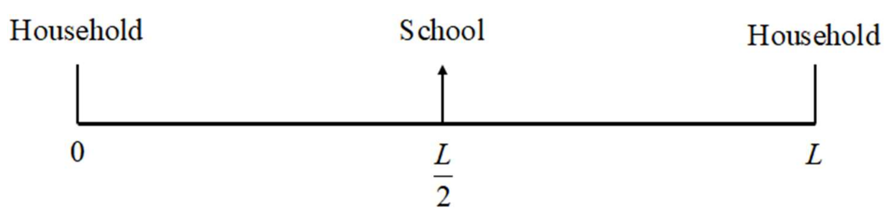


Figure 1: Linear region $\left[0, L\right]$

A different household lives in each extreme point $r = 0$ or $r = L$ and has an offspring who must be educated. Each youngster can be trained in two alternative ways. Either he is "self-educated" within his family, or he attends a school located at the midpoint $r = \frac{L}{2}$. In the former way, he gets the wage $w_u$ of unskilled labour and must support a fixed cost $f < w_u$. This fixed cost consists in the outlay of hiring a "coach" or "professor" who instructs him at his residence. The payoff of this learning method is thus $w_u - f$.

In alternative, the youngster may attend the school sited at the midpoint $r=\frac{L}{2}$. By doing so, he receives the higher wage $w_s > w_u$ of skilled labour and additionally he saves one half of the fixed cost $f$, since he now shares the “professor” with the student living in the opposite extreme of the region. However, he must bear a travel cost over the distance $\frac{L}{2}$ that separates his residence from the school. We assume that the travel cost per unit of distance is expressed by $t$. Hence, the payoff of the strategy of attending the school is $w_s-\frac{f}{2}-\frac{tL}{2}$.

We should consider a third possibility, namely that the youngster decides to attend school, but he is the only one to behave so. Hence, he must bear the full cost $f$ of hiring a professor. In this case, the student’s payoff is $w_s-f-\frac{tL}{2}$.

Since the game is symmetric, we only need to write the payoff matrix of a single player, i.e., “Row” without loss of generality.

$$\begin{array}{llcc} & & \text{Column} & \\ & & \text{School} & \text{Work} \\ \text{Row} & \text{School} & a_{11}=w_s-\frac{f}{2}-\frac{tL}{2} & a_{12}=w_s-f-\frac{tL}{2} \\ & \text{Work} & a_{21}=w_u-f & a_{22}=w_u-f \end{array} \quad (1)$$

In matrix (1), we label the strategy of self-teaching the youngster within the household as “Work”. Hence, we implicitly presuppose that the time length of education within the family is shorter than the duration of college schooling. The self-taught youngster is thus ready to enter the labour market *before* the formal student does.

In payoff matrix (1), we make two kinds of assumptions. First, we presuppose that each payoff number is positive, i.e., $a_{ij}>0, i, j=1,2$. Second, the game has two strict Nash equilibria $(\text{School, School})$ and $(\text{Work, Work})$, i.e., this a coordination (*Stag Hunt*) game, where the following inequalities are satisfied,

$$a_{11}>a_{21} \text{ and } a_{22}>a_{12} \quad (2)$$

Since $a_{21}=a_{22}$, the inequality $a_{11}>a_{22}$ also holds, i.e., the equilibrium $(\text{School, School})$ always dominates in payoffs the $(\text{Work, Work})$ Nash equilibrium.

Let us define $\Delta w \equiv w_s - w_u$ as the wage premium of completing higher education. Then inequality $a_{11}>a_{21}$ in (2) can be shown to mean that,

$$\Delta w > \frac{1}{2}(tL-f)$$

The other inequality $a_{22} > a_{12}$ in (2) may be shown to mean that,

$$\Delta w < \frac{tL}{2}$$

Consequently, the conditions for a coordination game may be summarized by setting lower and upper bounds on the wage premium of higher education $\Delta w$.

$$\frac{1}{2}\left(tL - f\right) < \Delta w < \frac{tL}{2} \qquad (3)$$

It is useful to convert the payoff matrix (1) into a form that singles out the net reward of complete university education by applying to positive linear transformations that preserve the set of Nash equilibria of the game.

The first transformation consists in subtracting $w_s - f - \frac{tL}{2}$ to each payoff number in (1), so that the payoff matrix becomes,

| | | Column | |
|---|---|---|---|
| | | School | Work |
| Row | School | $a_{11} = \frac{f}{2}$ | $a_{12} = 0$ |
| | Work | $a_{21} = \frac{tL}{2} - \Delta w$ | $a_{22} = \frac{tL}{2} - \Delta w$ |

The second positive linear transformation consists in multiplying each payoff number in the matrix by $\frac{1}{\frac{tL}{2} - \Delta w} = \frac{2}{tL - 2\Delta w}$ - which is positive by (3) – to obtain the equivalent matrix,

| | | Column | |
|---|---|---|---|
| | | School | Work |
| Row | School | $a_{11} = x \equiv \frac{f}{tL - 2\Delta w}$ | $a_{12} = 0$ |
| | Work | $a_{21} = 1$ | $a_{22} = 1$ |

(4)

In payoff matrix (4), $x > 1$ stands for the net reward of university attendance, which is influenced by the basic parameters of the game as follows.

- $x$ increases with $\Delta w$, an intuitive fact.
- $x$ decreases with $L$, as $\frac{L}{2}$ is the average commuting distance between a student's residence and the nearest university.
- $x$ decreases with $t$ the commuting cost per unit of distance. Students with a low socioeconomic background usually have a high value of $t$, so that they get a lower payoff from attending a distant college.
- $x$ increases with $f$. The efficiency of higher education derives from the fact that students share fixed inputs, so that it increases with the value of these assets.

In the ensuing sections, we will use the coordination game (4) as a framework to rationalize the evolution in the college schooling rate and per head real GDP during periods 1981-2001 and 2001-2021, respectively.

## 4. The evolution in the period 1981-2001: growth in the share of tertiary-educated people and in aggregate productivity

From Table 1, we are aware that between 1981 and 2001 a steady growth took place both in the college schooling rate and in real per head GDP.

During this period, the relative net reward of higher education $x \equiv \frac{f}{tL - 2\Delta w}$ in (4) was quite high on account of several factors.

First, the wage premium of skilled labour $\Delta w \equiv w_s - w_u$ was high as compared with other European countries, a fact that was well documented by Teixeira et al. (2014). In addition to a wage advantage, completing college also entailed an unemployment rate that was far below the average one. For the period 2004-2013, Mysikova and Vecernik (2019) found a value 0.374 for the wage premium of higher education in Portugal, which far exceeded the median (0.183) and even the first $\frac{1}{3}$ quantile (0.211) across a set of 27 European countries.

We can explain the relatively high premium of higher education in Portugal on the grounds that there is a statistically significant impact of the share of tertiary-educated people on the returns of higher education across the European countries, especially the countries of Western Europe. We should bear in mind that in 2002 Portugal had only a 9.4 percent share of population aged 25-64 with complete tertiary education whereas this share for the EU27 was about 18.8 per cent.

In addition to the relative wage of skilled labour, other factors contributed to a high net reward of higher education. Table 2 shows that in 2001 college attendance was concentrated in densely populated regions in mainland Portugal. This implies that the size of $L$ in Figure 1 was quite low, so that the average distance $\frac{L}{2}$ between a student's residence and the closest college small thus implying a positive impact on the net reward $x$. Spiess and Wrolich (2010) singled out the student's travel cost to the nearest university as an important hindrance to enrolment.

Furthermore, the spread of higher education tended to involve firstly youngsters from a relatively high socioeconomic background with students from less favoured families being admitted in college at a later stage. The impact of distance on college attendance seems to be stronger for lower class students, so that it appears to be less influent during early stages of higher education spread. We can model this effect on the net reward of higher education $x$ by arguing that the unit travel cost $t$ is smaller in early stages of higher education.

Finally, it is likely that in the early stages of higher education spread the quality provided was quite high, thus entailing a significant level of fixed costs $f$. For instance, in 1981 in Portugal, undergraduate training lasted for five years, a length of time that was reduced in the aftermath to three years.

To select a Nash equilibrium in the game with payoff matrix (4), we compute Harsanyi and Selten (1988)'s *risk dominant* equilibrium. Individual losses of deviating from a strict equilibrium point are,

- $a_1 = x - 1$ for (School, School), and
- $a_2 = 1$ for (Work, Work)

Since we assumed that, in the first period 1981-2001, $x$ was quite high, it is reasonable to suppose that (School, School) was then risk dominant, i.e., we had $a_1 > a_2$ or $x > 2$.
Since (School, School) is also payoff dominant, it appears to be the equilibrium point prevailing in this period, which was thus characterized both by a steep rise in college schooling rates and aggregate productivity.

# 5. Evolution in period 2001-2021: spread in higher education without growth of aggregate productivity

To explain reasonably the lack of correlation between the rising share of college-educated people and a stagnant aggregate productivity in the most recent period we should realize that the net reward of higher education $x \equiv \frac{f}{tL - 2\Delta w}$ fell significantly after the beginning of the century. Such a decline concerned *all* the determinants of $x$.

First, there is a consensus on that the wage premium of higher education $\Delta w$ decreased steeply. While authors such as Teixeira et al. (2014) acknowledge a "slight decline" in this indicator, the *OECD Employment Outlook 2019* says that the fall was quite sudden and severe reaching 22.8 per cent between 2006 and 2016, whereas the average degree was only 3.3 per cent. This fact is confirmed by the surge in the emigration of skilled Portuguese Youngsters from 2013 onwards.

A steep decline in the returns of tertiary education in Portugal should be expected given the significant negative impact of the share of college-educated people on the returns of higher education found by Mysikova and Vecernik (2019). Indeed, Portugal changed from a share of tertiary-educated people in active age (i.e., 25-64 years) of about one half the average of the EU27[2] (9.4 per cent and 18.8 per cent, respectively) to a situation in 2021 where these shares were virtually identical (33.1 per cent and 33.5 per cent, respectively).

Furthermore, as we noticed in Table 2, there was a strong convergence across the Portuguese regions in the college-educated people between 2001 and 2021, an intended result of the public policy for higher education. Consequently, growth in higher education was concentrated in sparsely populated areas or in peripheral regions. Hence, we should expect that the average distance separating a student's residence from the nearest college, i.e., $\frac{L}{2}$ in Figure 1, was strongly increased. *Otherwise, where public policy tried to limit students' travel costs by means of a proliferation of colleges, a steep rise in fixed costs ensued with a similar negative impact on the net reward of higher education* $x$. This cost increasing effect was made worse by the fact that youngsters from a less favoured socioeconomic background tended to be enrolled by the university system as it spread out. As studies by Dickerson and McIntosh (2013) and Frenette (2006) among others showed, the unit travel cost $t$ of lower-class students is higher when compared to their more affluent fellows. Consequently, the average student's commuting cost $\frac{tL}{2}$ was further raised. *Again, a policy that attempts to contain students' travel costs by increasing the number of colleges and spreading them to many different places merely duplicate fixed costs and has the same detrimental impact on the net reward of higher education.*

Finally, the spread out of higher education went ahead probably together with a decline in quality, with an emphasis on educational programs with comparatively low fixed costs $f$.

[2] 27 EU countries in 2020.

Given the fall in $x$ during the period 2001-2021, we may reasonably suppose that (Work, Work) became risk dominant during this period, i.e., we have $a_1 = x - 1 < 1 = a_2$ which is equivalent to $x < 2$.

The economic implications of this change in risk dominance is not straightforward to detect, as (School, School) is still selected according to payoff dominance, whereas (Work, Work) becomes the most likely result from risk dominance. Hence, some kind of indeterminacy arises here concerning the analytic solution of the game, so that a brief discussion of the methods for selecting a Nash equilibrium is in order now.

According to Harsanyi and Selten (1988), the two methods for selecting a Nash equilibrium point have a quite different nature. While "payoff dominance" is related to a *collective* kind of rationality based on the existence of *trust* between the players, "risk dominance" derives from the *individual* behaviour of a player who seeks "safety" in a situation of strategic uncertainty.

Hence, when the "payoff dominance" and "risk dominance" criteria select different equilibrium points, it is in general impossible to know a priori the true solution of the game. Consequently, we should rely on experimental evidence.

Following this line of reasoning, Schmidt et al. (2003) conducted a laboratory inquiry of the *Stag Hunt* game when the two criteria select a different equilibrium point and they reached two main conclusions. First, experimental subjects tend to select the payoff dominant equilibrium *more often than not*. Second, the selection of the "risk-dominant" equilibrium is influenced not only by the qualitative fact of "risk dominance" but also by its *level*, which they measure as the logarithm of the ratio of unilateral deviation losses from the strict equilibrium points. In the context of the diagonal payoff matrix (4), this measure is,

$$R \equiv \log\left(\frac{1}{x-1}\right) \tag{5}$$

It is clear that $R$ increases with $tL$ and decreases with $\Delta w$ and $f$. Hence, the level of risk dominance of the *Work* strategy has probably experimented a significant rise during the 2001-2021 period. During this period, (Work, Work) became the most likely outcome of the schooling game, an evolution that might account for the gradually fading growth in both college schooling and aggregate productivity as compared with the earlier 1981-2001 period.

Nevertheless, the "risk-dominant" solution (Work, Work) would mean that *both* the share in college-educated people and the output per worker remain unchanged, while instead we wish to explain the lack of correlation between the two variables. Such absence of correlation might be rationalized if the *Stag Hunt* game ended in a coordination breakdown with each player selecting a different pure strategy. Starting with a $(\text{Work, Work})$ situation, the transition to a $(\text{Work, School})$ or $(\text{School, Work})$ outcome would imply *both* an *increase* in the college schooling rate and a *decrease* in the output per worker from 1 to $\frac{1+0}{2} = \frac{1}{2}$.

Such an outcome takes indeed place in experimental studies. An example is due to Straub (1995), who found that a $2 \times 2$ symmetric coordination game where the payoff dominance and risk dominance criteria select different equilibrium points might end in an out-of-equilibrium outcome. This kind of result becomes likelier the higher the level of "risk dominance" is, i.e., the smaller and closer to 1 the net reward $x$ of higher education is.

It remains to encompass the possibility of an out-of-equilibrium result in theoretical terms. To reach this purpose, we follow Harsanyi (1973) to include incomplete information in the framework expressed by matrix (6), so that one player – Row, without loss of generality – knows more about his own payoff function than the opponent does. The payoff matrix for both players is now,

| | | Column | |
|---|---|---|---|
| | | School | Work |
| Row | School | $x-\theta, x$ | $0,1$ |
| | Work | $1,0$ | $1,1$ |

(6)

where $\theta$ is a random variable uniformly distributed in $[0,1]$.This is a game with asymmetric information. While Row knows the realization of $\theta$, Column's knowledge is limited to the distribution of $\theta$. Hence, $\theta$ is a *type* of Row's payoff function.

The coordination game depicted by (6) still has a Nash equilibrium (Work, Work) in unconditional pure strategies.

To detect a second Nash equilibrium, we need to consider type-contingent pure strategies in line with Harsanyi (1973). Hence, let us consider a Row's pure strategy such that,

- Row chooses *School* if $\theta < s$
- Row selects *Work* if $\theta > s$

where $0 \leq s \leq 1$. For Column, Row's type-contingent pure strategy is viewed as if he were using an "induced" mixed strategy that assigns probabilities $s, 1-s$ to the pure strategies *School* and *Work*, respectively. We assume that *Row* sets the value of the threshold $s$ so that the "induced" mixed strategy $s, 1-s$ equalizes the expected payoffs of Column's pure strategies, which are,

- Expected payoff of *School* = $xs + 0 \cdot (1-s) = xs$
- Expected payoff of *Work* = $1 \cdot s + 1 \cdot (1-s) = 1$

Hence, the value of $s$ is determined by the equality,

$$xs = 1 \leftrightarrow s = \frac{1}{x} \tag{7}$$

The Column player has two pure best replies to Row's type-contingent pure strategy with threshold $s$ as given by (7). If he chooses to reply with *Work*, Row's reply in turn is not his type-contingent pure strategy, but instead the (unconditional) pure strategy *Work*.

Hence, let Column reply with *School*. If Row uses his type-contingent pure strategy to reply against *School,* then he will obtain,

- $x-\theta$ if $\theta < s$, with $s$ given by (7)
- 1 if $\theta > s$

Let us assume that,

$$x - \theta > 1 \tag{8}$$

Then, Row's type-contingent pure strategy is a best reply against *School* if $\theta < s$. A *sufficient* condition for this to hold is,

$$x - s > 1 \quad (9)$$

By substituting $s$ from (7) in (9), we obtain the condition,

$$x - \frac{1}{x} > 1$$

which can be solved to mean that,

$$x > x^* \equiv \frac{1+\sqrt{5}}{2} \approx 1.618 \quad (10)$$

By contrast, let us assume that,

$$x - \theta < 1 \quad (11)$$

Then, the type-contingent pure strategy by Row is a best reply to *School* by Column if,

$$\theta > s$$

A *sufficient* condition for (11) to hold is,

$$x - s < 1 \quad (12)$$

By substituting $s$ from (7) in (12) we obtain the inequality,

$$x - \frac{1}{x} < 1$$

which may be solved to give,

$$x < x^* \equiv \frac{1+\sqrt{5}}{2} \approx 1.618 \quad (13)$$

We may summarize the findings of the incomplete information game as follows. There exist two pure strategy Nash equilibria, namely the (Work, Work) equilibrium in unconditional pure strategies, and an equilibrium involving the unconditional choice of *School* by the uninformed agent and a type-contingent pure strategy by the informed player. If the reward of higher education is high, then the latter participant will likely choose *School*, otherwise he will probably select *Work*. Hence, in the latter equilibrium point, the participants either will coordinate around the *School* pure strategy, or they will fail to coordinate. Then, the game will end in an out-of-equilibrium outcome where the uninformed player selects *School*, while the informed one chooses *Work*. The coordination breakdown outcome appears to be likelier if the reward of college education $x$ is low.

The asymmetric information game rationalizes the breakdown of coordination noticed by Straub (1995) in the context of an experimental study. Not only an out-of-equilibrium outcome arises in games when the “payoff dominance” and “risk dominance” criteria select different equilibrium points, but also players fail to coordinate more likely in games where the measure $R$ of “risk dominance” in  is high, i.e., when the net reward of college attendance is relatively low.

Consequently, we may use the asymmetric information game (6) to rationalize the evolution in the more recent period 2001-2021. We assume that for the start both players choose the pure strategy *Work*. Then, they play the incomplete information version of the *Stag Hunt* schooling game. Since the net reward of college education is low, players fail to coordinate with the uninformed agent choosing *School* and the informed one selecting *Work*. In conformity with evidence, there is a significant increase in the proportion of youngsters who attend college, but the average output generated by a worker effectively falls as it passes from 1 in the first stage to $\frac{1+0}{2}=\frac{1}{2}$.

## 6. Concluding remarks

Even though education exhibits strong positive externalities, there exists abundant evidence that the positive correlation between the spread of higher education and aggregate productivity tends to gradually disappear as a larger number of students attend college.

To rationalize this apparent contradiction, we model college attendance through a coordination (*Stag Hunt*) game, where a set of youngsters decide whether to enrol in college or immediately enter the labour market. The benefits of higher education are reaped by everyone only if *all* youngsters decide to engage in higher education.

We argue that the reward to college education tends to decline as it becomes widespread on account of two factors. First, the wage premium of skilled labour falls due to an increased supply. Then, new colleges tend to be set up in less dense areas, so that the average distance between a youngster's residence and the closest university rises.

The decrease in the reward of tertiary education is multiplied by its impact on the result of a *Stag Hunt* game. If such a reward is high, there will likely be a switch from the "risk dominant" Nash equilibrium, where no youngster decides to engage in tertiary education, to the "payoff dominant" equilibrium with unanimous enrolment in college. By contrast when the net reward of higher education becomes relatively low the outcome of the *Stag Hunt* will be instead a *coordination breakdown*, which might even lead to a fall in the productivity of the agents involved.

The latter result was found in laboratory studies by Straub (1995) and Schmidt et al. (2003) among others and we can explain it in theoretical terms if we allow for incomplete information in the *Stag Hunt* game.

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