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# A coordination game approach to higher education growth 

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April 2023


#### Abstract

This paper examines the evolution of higher education in Portugal under the light of an n -person (Stag Hunt) coordination game. Such a game exhibits two strict Nash equilibrium points, namely $\bar{\alpha}$ when all youngsters decide to work immediately, and $\bar{\beta}$ when they all decide to join a university. Harsanyi and Selten (1988)'s risk dominance concept is used to select the $\bar{\beta}$ Nash equilibrium. We consider two alternative coordination requirements in the n - person Stag Hunt, namely unanimity and the $k$-coordination requirement, that allows the university to break even. Even though the unanimity game is formally noncooperative, it represents in fact the result of a cooperative agreement as was emphasized by John $\operatorname{Nash}(1950,1953)$. By contrast, the $k-$ coordination game is purely noncooperative and it is driven by efficiency considerations. By applying these concepts to higher education spread across Portuguese regions between 2001 and 2021, we could reach two main conclusions. First, the distribution of higher education across regions seems to be mainly affected by a $k$-coordination constraint, i.e., the share of tertiary-educated people appears to be higher in densely populated regions where the high fixed costs of setting up a college are more easily covered. Second, public policy appears to be oriented to achieve unanimity in the youngsters' decisions to join a university by stimulating college attendance in sparsely populated regions. Such a policy purpose might make the college system less effective and limit its expansion in the future.


Keywords: Education, Regional Development, Coordination Games, Risk Dominance.
JEL Classification: C72, I20, O12, R11

[^0]
## 1. Introduction

Compulsory education levels in Portugal progressed fast since the establishment of a democratic regime in 1974. At that time, it involved only six schooling years. In the aftermath, it increased to nine years from 1985 on and it was eventually set at twelve years after 2008.

Tertiary schooling (ISCED 5-8) stands now for post-compulsory education, and it is the core signal of progress in overall education. Table 1 shows growth rates in percentage of tertiary-educated people and in real per head GDP during the periods 1981-2001 and 2001-2021.

Table 1

| Time period | $(1)$ | $(2)$ | $(1)-(2)$ |
| :---: | :---: | :---: | :---: |
| $1981-2001$ | 6.4 | 3.0 | 3.4 |
| $2001-2021$ | 4.8 | 0.4 | 4.4 |

$(1) \equiv$ average annual growth rate in the share of people older than 15 with a complete higher education degree according to the Censuses.
(2) $\equiv$ average annual growth rate of real per head GDP.

Source: PORDATA. INE

While there was a steady progress in college attendance rates, the positive correlation between higher education spread and economic growth, which was clear in the first period, vanished during the last twenty years.

The evolution in Portugal matches the global one. As Mankiw, Romer and Weil (1992) showed, while global convergence trends are present both in schooling rates and in per head income, the former evolution appears to be much faster than the latter one. By restricting to Europe and to higher education, Pontes and Buhse (2019) computed speeds of $\beta$ - convergence in schooling rates and per head GDP with contrasting values, namely $4.0 \%$ and $2.3 \%$, respectively. Hence, economic incentives may not fully account for the differential development of college education across countries.

This paper tries to assess the driving forces behind the expansion of universities to measure the level of efficiency in this process. Such an analysis might enable us to
explain why higher education spread and aggregate productivity growth became apparently unrelated in the more recent past.

Since tertiary education is non-compulsory, the decisions to enrol in a university and the student's performance have an individual character, which is mostly related with the socioeconomic background of the family. For the university, Spiess and Wrolich (2010) single out factors such as the education level of parents, the per head income of the household and the distance separating the parental home and the nearest university. The impact of distance interacts with the family socioeconomic background as it seems stronger for students coming from less favoured families (see, among others, Dickerson and McIntosh, 2013, and Frenette, 2006).

However, the individual decision to join a university is strongly affected by region-level economic factors such as the wage premium of tertiary-educated workers in relation to unskilled ones. Since in this paper we will assume that labour is perfectly mobile, the wage premium is not considered a possible source of discrimination across regions. Instead, we focus here the determinants associated with the "group process" nature of education, which is based on three different grounds.

First, as Lucas (1988) emphasized, human capital is effectively a "social capital", so that individuals who engage in a training process "learn with each other" within a group of neighbours. As Benabou (1993) emphasized, the effort cost for a candidate to join a university decreases steadily with the share of people with higher education living in the same local area. Second, the operation of a college is feasible only if a minimum number of students "share" a set of fixed inputs, such as "buildings", "professors", "laboratories", "libraries" and so on. Finally, tertiary education is specialized by nature. Diamond (1982) argued that a graduate may use his training period profitably only if he is matched with complementary specialists within a work organization. In the same line of reasoning, Helsley and Strange (1990) said that a minimum population density is required to allow a sufficiently dense network of colleges to break even, thus ensuring that each student's residence lies within an acceptable travel distance to the closest university.

In this paper, we model the tertiary schooling process as a simultaneous coordination Stag Hunt - game, where a set of players (i.e., youngsters) decide whether to enrol in the university or to engage in work immediately. The latter option guarantees the lower wage of unskilled labour. The former choice gives the higher wage of skilled labour, as long as a "critical mass" of $k$ youngsters decide to engage in college. Otherwise, the student becomes unemployed at the end of the graduation period and thus receives a zero reward.

The existence of a $k$-coordination requirement models add a "group process" nature to learning in a university. But the "critical mass" in the coordination game may be described in two different ways. Either the unanimity of players is required for each student to obtain the wage premium, or only a strict subset of individuals is necessary. In the latter case, the size of the students' "critical mass" is determined to allow the group effects of learning to take place effectively. We will realize ahead that these alternative specifications of the coordination requirement have opposite meanings.

# 2. Modelling decisions to join a university by means of a $n$ person Stag Hunt game. 

### 2.1. Assumptions

We feature an economy along two consecutive periods. In each period $t=0,1$, the economy is composed by $n_{t}$ families.

Let $s_{t}$ be the share of youngsters in period $t$ who complete a college degree. We assume that in the next period $t+1$ these youngsters become parents and they will predetermine their children to enrol in the university. Consequently, in each period $t$, only $n_{t}-s_{t-1} n_{t}$ youngsters are free to decide whether to engage in higher education.

We presuppose that that the values $n_{0}$ and $s_{0}$ in period 0 , and $n_{1}$ in period 1 are exogenously determined. Then we try to explain the value of $s_{1}$. For that purpose, we assume that in period 1 each youngster either enters immediately the labour market (pure strategy $\alpha$ ) or enrols in college (pure strategy $\beta$ ) thus postponing one period his participation in the labour market. These decisions are made simultaneously by all players.

The payoffs of the youngsters' pure strategies are as follows. If a youngster plays $\alpha$, i.e., he decides to find a job immediately, he obtains the wage of unskilled labour as a certain payoff. Otherwise, if he plays $\beta$, i.e., he decides to join a university, he might obtain one of two possible rewards.

If at least $k$ of the $n_{1}$ youngsters decide to join a university, then each student obtains the payoff $\frac{w_{S}}{1+r}$, the discounted value of the wage of skilled labour with $w_{S}>w_{U}$. For simplicity, we assume that the discount rate $r$ is close to zero, so that the payoff of higher education under $k$-coordination among candidates might be approximated by $w_{S}$, the wage of skilled labour. The $k$-coordination requirement stems from the group nature of higher education as we stressed above in the introduction.

Otherwise, if the coordination requirement is not satisfied, then the graduate is assumed to become unemployed and his payoff is zero.

### 2.2. The set of strict Nash equilibria in the n-person Stag Hunt

The n-person Stag Hunt game was well described by Carlsson and van Damme (1993-b) and van Damme (2002). They prove that this game has two Nash equilibria in pure strategies, namely $\bar{\alpha} \equiv$ all players select the pure strategy $\alpha$, and $\bar{\beta} \equiv$ all players select the pure strategy $\beta$, a result that is quite intuitive.

This game involves the selection of a Nash equilibrium, which amounts to the specification for each player of beliefs about the opponents' behaviour. Such beliefs enable everyone to deal with the situation of strategic uncertainty where he finds himself from the start.

In the experimental economics literature of repeated coordination games (see Van Huyck et al., 1990, 1991; Schmidt et al. 2003), the selection of a Nash equilibrium point has used two kinds of considerations, namely inductive reasons related with historical precedent, and deductive arguments founded on the mathematical structure of the game. In this paper, inductive considerations are accounted by the fact that the share of individuals who decide to join a college in period $t$ predetermines the decisions made by their offspring in the subsequent period $t+1$.

According to Kim (1996), three strands of literature deal with the selection from the deductive perspective. Harsanyi and Selten (1988) handle this problem for static games of complete information by defining the concept of risk dominance which we treat in more detail further ahead.

Carlsson and van Damme (1993-a) redefine risk dominance for $2 \times 2$ games of incomplete information, the so-called global games, where players observe their payoffs imperfectly, with a small amount of "noise", albeit their observations are correlated. The selection of a Nash equilibrium follows from the observation of a sequence of perturbed games, as the amount of "noise" converges to zero. The Nash equilibrium that is the limit of this sequence is regarded as being coincident with Harsanyi and Selten (1988)'s risk dominance. Then Carlsson and van Damme (1993-b) extend the concept of a global game to a symmetric Stag Hunt with $n$ players.

The issue of Nash equilibrium selection may also be dealt with by means of explicitly dynamic and evolutionary processes in line with Kandori et al. (1995) and Young (1993).

### 2.3. How to select a Nash equilibrium in the symmetric n-person Stag Hunt game?

It is well known that Harsanyi and Selten (1988) define two criteria for ranking multiple Nash equilibria, namely payoff dominance and risk dominance. While the former concept is related with collective rationality, the latter expresses the individual attitude of each player while dealing with the strategic uncertainty about the opponent's behaviour.

In theoretical terms, Harsanyi and Selten (1988) contend that, when the two criteria conflict each other, payoff dominance should prevail over risk dominance. This is so because the criterion of payoff dominance is focal. Even though the players are not allowed to bargain before the game, there is common knowledge that, if they might, then they would settle in the Pareto dominant equilibrium. Consequently, they will tacitly coordinate in the payoff dominant equilibrium.

This assertion has been widely debated in experimental economics. Earlier results with coordination games, such as those by Cooper et al. (1990) and Van Huyck et a. (1990,
1991) stressed that Pareto dominant outcomes failed to be observed in many instances. Most recent laboratory studies with two-person Stag Hunt such as Straub (1995) and Schmidt et al. (2003) emphasize risk dominance considerations in relation to those Pareto dominance. In situations where the two criteria choose different equilibrium points, Schmidt et al. (2003) say that, even though the payoff dominant strategy is selected more often than not, players appear to be quite responsive to changes in risk dominance levels, whereas they seem not ready to react to variations in payoff dominance levels.

The importance of the risk dominance criterion derives not only from that it is purely based on individual rationality in a situation of strategic uncertainty, but also from that it takes into account more information about the payoff functions than payoff dominance on two different grounds. First, while payoff dominance in $2 \times 2$ Stag Hunt games is based only on Nash equilibrium payoffs, risk dominance takes additionally into account payoffs related with out-of-equilibrium outcomes. Second, while payoff dominance in $n$ - person Stag Hunt only compares individual payoffs, risk dominance is also influenced by the size of the group of players.

### 2.4. Checking risk dominance in the $n$-person Stag Hunt educational game.

In the $n$ - person Stag Hunt game applied to simultaneous decisions to join a college, Harsanyi and Selten (1988)'s risk dominance can be checked in two steps. First, we need to specify for each player a belief about another player deciding to join an university, i.e., that he selects his pure strategy $\beta$. Since the game is fully symmetric, this belief should be the same across players and might be expressed by the probability $p \in(0,1)$.

Second, we need to compute each player's best reply against his belief $p$. As we will realize ahead, the profile of players' best replies is necessarily is necessarily one of the two pure strategy Nash equilibria of the coordination game, i.e., it is either $\bar{\alpha}$ (all
individuals decide to work immediately), or $\bar{\beta}$ (all youngsters decide to join a university). Hence, no further adjustment of beliefs and strategies is required. The emerging profile of best replies to beliefs might be considered to be the risk dominant Nash equilibrium of the game.

While in the $2 \times 2$ Stag Hunt the specification of players' beliefs compatible with the determination of the risk dominant equilibrium is unique, Carlsson and van Damme (1993-b) showed that in the n-person version of the game there might be several different forms of specifying them.

The basic assumption underlying the specification of $p \in(0,1)$ is the so called principle of insufficient reason. If a player is uncertain about the alternative of action that an opponent might take and he has no additional information allowing him to discriminate, then he should assign the same probability to each one of the other player's pure strategies. In this context, each player should expect an opponent to select pure strategy $\beta$ with probability $p=\frac{1}{2}$, an idea that was put forward by Güth and Kalkofen (1989).

In a different line of reasoning, Harsanyi and Selten (1988) propose for the n-person educational game $p=\tilde{w}$, where $\tilde{w}$ stands for the "wage premium" of higher education, i.e., $\tilde{w} \equiv \frac{w_{S}-w_{U}}{w_{S}} \in(0,1)$. The latter specification of $p$ is intuitive since the expectation that an opponent joins a university should be directly proportional to the wage premium he might earn by behaving in this way. Hence, we adopt here the specification $p=\tilde{w}$.

There is more than one form to model the n -person Stag Hunt educational game, depending on the specification of the $k$ - coordination requirement. We examine two ways of stating this requirement. Under the first alternative statement (see van Damme, 2002), a student may reap the benefits of completing higher education only if the $n$ neighbouring youngsters decide unanimously to join the university. Under the second specification (see Heinemann et al., 2009), we only require that $k<n$
candidates decide to enrol in college. This subset of $k$ individuals is considered a "critical mass", i.e., the minimum number of students that allow complementarities and "group effects" to take place across individuals.

In what follows, we will realize these two ways of stating the coordination requirement give contrasting meanings to the educational game.

### 2.4.1. The unanimity game

Following van Damme (2002), the decision by participants in this game to attend higher education is profitable only if it is taken unanimously by the candidates. Since $n_{t} s_{t-1}$ youngsters are constrained to join the university by parental orientation, the required unanimity concerns in fact only $n_{t}\left(1-s_{t-1}\right)$ players.

As van Damme (2002) argued, action $\beta$ will be a risk dominant equilibrium strategy if

$$
\begin{equation*}
\tilde{w}^{\left[n_{t}\left(1-s_{t-1}\right)-1\right]} w_{S}>w_{U} \tag{1}
\end{equation*}
$$

In inequality (1), $\tilde{w}$ is the probability that a youngster expects another player to enrol in college. Since each individual faces other $n_{t}\left(1-s_{t-1}\right)-1$ freely deciding youngsters and he expects these individuals to take independent choices, the chance that the coordination requirement is met is $\tilde{w}^{\left[n_{t}\left(1-s_{t-1}\right)^{-1}\right]}$. Consequently, the expected payoff of pure strategy $\beta$ is just the left-hand side of (1).

Inequality (1) may be written as,

$$
\begin{equation*}
\tilde{w}^{\left[n_{t}\left(1-s_{t-1}\right)-1\right]}+\tilde{w}>1 \tag{2}
\end{equation*}
$$

Inequality (2) may be solved for $n_{t}$ to give,

$$
\begin{equation*}
n_{t}<\left(\frac{1}{1-s_{t-1}}\right)\left[\frac{\ln (1-\tilde{w})}{\ln \tilde{w}}+1\right] \tag{3}
\end{equation*}
$$

Since the ratio $\frac{\ln (1-\tilde{w})}{\ln \tilde{w}}$ is a strictly increasing function of $\tilde{w}$, the $\bar{\beta}$ equilibrium point will be likelier if $\tilde{w}$ and $s_{t-1}$ are high.

Furthermore, a low $n_{t}$ makes the inequality easier to be satisfied. In other words, the unanimity constraint becomes less binding when the number of players is reduced.

To understand this, we should realize in line with John $\operatorname{Nash}(1950,1953)$ that, even though the selection of an equilibrium point in a formally noncooperative unanimity game, it is just in fact an implicit way of representing a cooperative situation where players discuss to reach a binding agreement. It is not surprising that a cooperative agreement becomes harder to achieve when the number of participants in the bargaining is high.

### 2.4.2. The k - coordination game

While there exist different kinds of complementarity among youngsters deciding whether to join a university, we assume here that the "critical mass" is determined only by the constraint that the number of students should sufficient for a college to break even. This means that the group interaction follows from the fact that students must share fixed inputs, such as "professors", "buildings", "libraries", "laboratories" and so on, thereby benefiting from economies of scale.

We assume that there is a university in each region, whose cost $F$ is utterly fixed. This university is fully financed by a tuition fee $c$, which is contributed by each student.

Even though there exist $n_{t}$ students in period $t$, a subset of $n_{t} s_{t-1}$ youngsters have their enrolment decision determined by parents, so that they are not effectively players in the coordination game. Let $y_{t}$ be the number of youngsters who decide freely to join a university in period $t$. Then, the $k$-coordination requirement may be expressed by the inequality,

$$
\begin{equation*}
\left(y_{t}+n_{t} s_{t-1}\right) c \geq F \tag{4}
\end{equation*}
$$

By solving (4) in relation to $y_{t}$, we obtain the $k$-coordination requirement in period $t$, i.e., $k_{t}$.

$$
\begin{equation*}
y_{t} \geq \frac{F}{c}-n_{t} s_{t-1} \equiv k_{t} \tag{5}
\end{equation*}
$$

For a given tuition fee, the breakeven point of higher education increases with the college fixed cost and decreases with the number of students in period $t$ and with the share of tertiary-educated people in the previous period $t-1$.

Following Heinemann et al. (2009), we can use a Bernoulli (or binomial) distribution to write the probability that the $k_{t}$-coordination requirement in (5) is satisfied as,

$$
\begin{align*}
& \sum_{x=k_{t}-1}^{n_{t}-1}\binom{n_{t}-1}{x} \tilde{w}^{x}(1-\tilde{w})^{\left[\left(n_{t}-1\right)-x\right]}  \tag{6}\\
& \text { where } k_{t} \equiv \frac{F}{c}-n_{t} s_{t-1}
\end{align*}
$$

The expression in (6) is just the probability that at least $k_{t}-1$ out of $n_{t}-1$ players select the pure strategy $\beta$. Then, the condition that $\beta$ is a risk dominant Nash equilibrium strategy is just,

$$
\begin{equation*}
\sum_{x=\frac{F}{c}-n_{S} S_{1-1}-1}^{n_{t}-1}\binom{n_{t}-1}{x} \tilde{w}^{x}(1-\tilde{w})^{\left[\left(n_{t}-1\right)-x\right]} w_{S}>w_{U} \tag{7}
\end{equation*}
$$

which may also be written as,

$$
\begin{equation*}
\sum_{x=\frac{F}{c}-n_{t} s_{s-1}-1}^{n_{t}-1}\binom{n_{t}-1}{x} \tilde{w}^{x}(1-\tilde{w})^{\left[\left(n_{t}-1\right)-x\right]}+\tilde{w}>1 \tag{8}
\end{equation*}
$$

Clearly, Nash equilibrium selection corresponds here to a noncooperative situation. Youngsters living in a region will decide to join the university if they are numerous enough to allow college fixed costs to be covered by tuition fees.

Let us define as $\operatorname{Bin}(n, k, p)$ the cumulative Bernoulli (or binomial) distribution function.

$$
\begin{equation*}
\operatorname{Bin}(n, k, p) \equiv \sum_{x=0}^{k}\binom{n}{x} p^{x}(1-p)^{(n-x)} \tag{9}
\end{equation*}
$$

Let $p$ be the probability of success in each trial. Then, $\operatorname{Bin}(n, k, p)$ stands for the probability that that a number of successes equal to or smaller than $k$ arise in $n$ trials.

It is clear that $\operatorname{Bin}(n, k, p)$ decreases with $n$ and $p$, and increases with $k$.

Hence, we may write condition (8) that $\beta$ is a risk dominant equilibrium pure strategy in terms of $\operatorname{Bin}(n, k, p)$ as,

$$
\begin{equation*}
\left[1-\operatorname{Bin}\left(n_{t}-1, \frac{F}{c}-n_{t} s_{t-1}-2, \tilde{w}\right)\right]+\tilde{w}>1 \tag{10}
\end{equation*}
$$

Hence, $\bar{\beta}$ will be the risk dominant Nash equilibrium in period $t$ in case that,

- $n_{t}$, the total number of youngsters in period $t$, is high.
- $s_{t-1}$, the share of college educated people in the former period $t-1$, is high.
- $\tilde{w}$, the wage premium of skilled labour, is high.

If we compare these conditions with those yielded by the game under the unanimity requirement, we can draw two main conclusions. First, variables $s_{t-1}$ and $\tilde{w}$ have the same kind of influence on the decision to enrol in college in both game specifications, i.e., they favour this decision. Second, variable $n_{t}$ has a contrasting influence on tertiary
schooling rate in the two models, i.e., while it hinders the spread of higher education in the context of the unanimity game, it helps it in the $k$-coordination situation.

In the following section, we seek to determine which kind of coordination requirement fits better the evolution of college attendance across the Portuguese regions (NUTS3 in the mainland and NUTS1 insular regions).

## 3. Testing the coordination requirement implicit in higher education spread

We now try to find out which kind of coordination requirement - either unanimity or $k$ - coordination - better explains the expansion of universities in Portugal between 2001 and 2021.

In the Appendix, we gather data on higher education (ISCED 5-8) schooling rates in 2001, $s_{t-1}$, and 2021, $s_{t-1}$, across NUTS3 regions in mainland Portugal and the NUTS1 insular regions of Azores and Madeira. More precisely, $s_{t}$ is the share of population aged over 15 with a complete higher education degree as it is recorded through a population census. We also record the resident population in the region in the same years, $n_{t}$ in 2021 and $n_{t-1}$ in 2001, with the same statistical source.

We first use a cross-section OLS model to assess the kind of coordination requirement that describes more accurately how universities spread out in Portugal. We selected above the causes that might account for the fact that joining a university (pure strategy $\beta$ ) is risk dominant in period $t$, namely population in period $t, n_{t}$, the share of college educated people in the previous period $t-1, s_{t-1}$, and the wage premium of skilled labour, $\tilde{w}$. From these factors, we only keep $n_{t}$ and $s_{t-1}$, since we assumed that workers are freely mobile so that the wage premium $\tilde{w}$ is equalized across regions.

We consider first the unanimity game. Condition (3) of risk dominance of strategy $\beta$ might be written as,

$$
\begin{equation*}
n_{t}\left(1-s_{t-1}\right)<\left[\frac{\ln (1-\tilde{w})}{\ln \tilde{w}}+1\right] \tag{11}
\end{equation*}
$$

The fact that this inequality is satisfied may be checked by estimating the decreasing function,

$$
\begin{equation*}
s_{t}=f\left[n_{t}\left(1-s_{t-1}\right)\right] \tag{12}
\end{equation*}
$$

We turn now to the condition of risk dominance of strategy $\beta$ in the k - coordination game. Basically, for a given wage premium $\tilde{w}$, the left-hand side of (10) increases with the population of youngsters in period $t, n_{t}$, and it decreases with the coordination requirement $k_{t} \equiv \frac{F}{c}-s_{t-1} n_{t}$. Hence, it can be approximately described by the difference $n_{t}-k_{t}$, which is given by,

$$
\begin{align*}
& n_{t}-k_{t}=n_{t}-\left(\frac{F}{c}-s_{t-1} n_{t}\right)  \tag{13}\\
& =n_{t}\left(1+s_{t-1}\right)+\frac{F}{c}
\end{align*}
$$

As the price and cost parameters $F$ and $c$ are supposed to be invariant across regions, the $k$-coordination requirement may be checked by estimating in cross-section the increasing function,

$$
\begin{equation*}
s_{t}=g\left[n_{t}\left(1+s_{t-1}\right)\right] \tag{14}
\end{equation*}
$$

We estimate jointly by OLS the functions $f(\cdot)$ and $g(\cdot)$ in the cross-section model,

$$
\begin{equation*}
s_{t}=\beta_{1}+\beta_{2}\left[n_{t}\left(1-s_{t-1}\right)\right]+\beta_{3}\left[n_{t}\left(1+s_{t-1}\right)\right]+u_{t} \tag{15}
\end{equation*}
$$

where $u_{t}$ is an error term and the coefficients have expected signs $\beta_{2}<0$ and $\beta_{3}>0$.
The estimated structure is (with $p$ - values in parenthesis)

$$
\begin{aligned}
& \hat{\beta}_{1}=0.1462 \\
& \hat{\beta}_{2}=-4.2847 \times 10^{-6} \\
& \hat{\beta}_{3}=4.0903 \times 10^{-5}
\end{aligned}
$$

We have also

$$
\begin{aligned}
& R^{2}=0.57767 \\
& F=15.04629\left(7.62 \times 10^{-5}\right)
\end{aligned}
$$

The overall explanatory power of the model is high and the $\hat{\beta}$ coefficients have the theoretically expected signs. However, while the coefficient $\hat{\beta}_{3}$ associated with the $k$ coordination game is significant at the $1 \%$ confidence level, the coefficient $\hat{\beta}_{2}$ of the unanimity game is not significant. Hence, we may say that higher education attendance tends to be concentrated in densely populated regions, where colleges can break even more easily.

A different issue concerns the orientation of public policy towards higher education. To assess it, we estimate by OLS the cross-section model,

$$
\begin{equation*}
\frac{s_{t}-s_{t-1}}{s_{t-1}}=\beta_{1}+\beta_{2} n_{t-1}+u_{t} \tag{16}
\end{equation*}
$$

The estimated structure is,

$$
\begin{aligned}
& \hat{\beta}_{1}=2.08876 \\
& \hat{\beta}_{2}=-0.00029(0.011)
\end{aligned}
$$

With an $R^{2}=0.248$, the model is significant at the $5 \%$ confidence level. Hence, public policy in the field of higher education seems oriented to an (almost) universal inclusion of youngsters even though they might live in sparsely populated regions. Such a unanimity driven policy might lead in the future to a rise in the breakeven point of colleges, thus diminishing the chances of achieving quality in higher education.

## 4. Concluding remarks

This paper examines the evolution of higher education in Portugal under the light of an n-person (Stag Hunt) coordination game. Such a game exhibits two strict Nash equilibrium points, namely $\bar{\alpha}$ when all youngsters decide to work immediately, and $\bar{\beta}$ when they all decide to join a university. Harsanyi and Selten (1988)'s risk dominance concept is used to select the $\bar{\beta}$ Nash equilibrium.

We consider two alternative coordination requirements in the n - person Stag Hunt, namely unanimity and the $k$-coordination requirement, that allows the university to break even. Even though he unanimity game is formally noncooperative, it represents in fact the result of a cooperative agreement as was emphasized by John Nash (1950, 1953). By contrast, the $k$ - coordination game is purely noncooperative and it is driven by efficiency considerations.

By applying these concepts to higher education spread across the Portuguese regions between 20201 and 2021, we could reach two main conclusions. First, the distribution of higher education across regions seems to be mainly affected by a $k$-coordination constraint, i.e., the share of tertiary-educated people appears to be higher in densely populated regions where the high fixed costs of setting up a college are more easily covered. Second, public policy seems oriented to achieve unanimity in the youngsters' decisions to join a university by stimulating college attendance in sparely populated regions. Such a policy purpose might make the college system less effective and limit its expansion in the future.

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Appendix: Data on higher education schooling rates and resident population across Portuguese regions

Meaning of variables
$s_{t} \equiv$ share of regional population aged over 15
with a complete higher education (ISCED $5-8$ ) degree in time period $t$
$n_{t} \equiv$ resident population in the region in time period $t$ (unit: 1000 people)
Time periods
$t-1 \equiv 2001$
$t \equiv 2021$
Statistical sources: PORDATA. INE. Censuses of 2001 and 2021

| Portuguese Regions | $s_{t}$ | $s_{t-1}$ | $n_{t}$ | $n_{t-1}$ |
| :---: | :---: | :---: | :---: | :---: |
| Alto Minho | 0.147 | 0.047 | 231 | 250 |
| Cávado | 0.199 | 0.062 | 417 | 394 |
| Ave | 0.141 | 0.039 | 418 | 426 |
| Área Metropolitana do Porto | 0.21 | 0.081 | 1736 | 1732 |
| Alto Tâmega | 0.118 | 0.042 | 84 | 104 |
| Tâmega e Sousa | 0.104 | 0.027 | 409 | 434 |
| Douro | 0.148 | 0.051 | 184 | 220 |
| Terras de Trás-os-Montes | 0.166 | 0.056 | 107 | 127 |
| Oeste | 0.155 | 0.049 | 364 | 340 |
| Região de Aveiro | 0.186 | 0.067 | 367 | 365 |
| Região de Coimbra | 0.214 | 0.083 | 437 | 472 |
| Região de Leiria | 0.169 | 0.052 | 287 | 289 |
| Viseu, Dão, Lafões | 0.162 | 0.055 | 253 | 276 |
| Beira Baixa | 0.169 | 0.054 | 80 | 94 |
| Médio Tejo | 0.153 | 0.053 | 228 | 254 |
| Beiras e Serra da Estrela | 0.154 | 0.052 | 211 | 258 |
| Área Metropolitana de Lisboa | 0.266 | 0.12 | 2870 | 2665 |
| Alentejo Litoral | 0.124 | 0.039 | 96 | 100 |
| Baixo Alentejo | 0.139 | 0.046 | 115 | 135 |
| Lezíria do Tejo | 0.151 | 0.053 | 236 | 241 |
| Alto Alentejo | 0.140 | 0.047 | 105 | 127 |
| Alentejo Central | 0.164 | 0.059 | 152 | 173 |
| Algarve | 0.173 | 0.065 | 467 | 397 |
| Região Autónoma dos Açores | 0.147 | 0.052 | 236 | 242 |
| Região Autónoma da Madeira | 0.165 | 0.056 | 251 | 246 |


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