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Frederico Pinheiro, Onofre Alves Simões

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1249-078 Lisboa,
Portugal

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REM – Research in Economics and Mathematics

Rua Miguel Lupi, 20
1249-078 LISBOA
Portugal

Telephone: +351 - 213 925 912
E-mail: rem@iseg.ulisboa.pt

https://rem.rc.iseg.ulisboa.pt/

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An 'Eiopean' Tool to Project Post Retirement Income in Portuguese Defined Contribution Pension Schemes

Frederico Pinheiro[1]

Onofre Alves Simões[2],[3]

Abstract

Ageing of the populations is leading to reforms in Social Security systems with a negative impact on post retirement income. One way to minimize this is to reinforce the role of complementary pension schemes, and pension projections can be an important tool to assist workers in making their decisions on saving for retirement. The topic has been discussed by the European Union (EU) and the European Insurance and Occupational Pensions Authority (EIOPA).

This work focuses on a tool for making pension projections in the scope of occupational defined contribution pension schemes, based on EIOPA's guidance. We aim to study the potential performance of different investment strategies using an Economic Scenario Generator framework and evaluate the impact on the retirement income that such investment strategies produce, under different assumptions. The model underlying the tool takes in three main risk factors: the financial market risk, which includes uncertainty over returns on investments, inflation and interest rates; the labor risk, originated from uncertainty over real wage growth paths; the demographic risk, as a result of the increasing life expectancy.

Keywords: Retirement income, Pension projection, Economic scenario generator, Life tables, Real-world valuation, EIOPA

1 Introduction

The pension landscape across countries is very diverse in the European Union, but in general pension providers can be divided into three pillars: the public system; the occupational pension schemes; the personal pension schemes. In Portugal, the main source of retirement income is public Social Security, which is a pay-as-you-go system.

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[1] ISEG, frederico.mpinheiro@aln.iseg.ulisboa.pt

[2] ISEG-UL, onofre@iseg.ulisboa.pt

[3] REM, CEMAPRE

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As in other European countries, the ageing of the population, due to an increase in life expectancy and lower birth rates, is placing Portuguese Social Security under financial pressure, leading to regular reforms with negative implications on the levels of the provided retirement income. According to projections presented in the 2021 Ageing Report of the European Commission[4], it is expected that the replacement rate from public pensions will decrease in Portugal from 74% in 2019 to 41,4% in 2070. One way to minimize the impact of such a reduction is to strengthen the role of complementary pension schemes.
There are different measures that a country can implement to encourage participation and increase the coverage of complementary pension schemes, as well as to ensure the adequacy of retirement outcomes from these schemes. For instance, raising individuals' awareness to the importance of planning for retirement and promoting individuals' active engagement with their pensions. In this context, pension projections of the foreseeable levels of future retirement benefits can be an important tool to support people in their decisions about saving for retirement. This has been emphasized by the EU.
From a regulatory perspective, one of the main actions was the publication of the Directive 2016/2341, commonly known as the IORP II Directive [13], which provides an updated legislative framework regarding institutions for occupational retirement provision (IORPs). Among other goals, this Directive aims to ensure that IORPs provide clear and adequate information to pension schemes' members and beneficiaries, including regular information on projected levels of retirement benefits, via the Pension Benefit Statement (PBS), which IORPs should make available to all members on an annual basis – cf. also [15].
Regarding personal schemes, the Regulation 2019/1238 on a Pan-European Personal Pension Product (PEPP) [14], created a legislative framework for a new individual product, aiming to all EU citizens, with a harmonized set of key features, offering savers more choice and more competitive products. The PEPP may be offered by financial institutions from different sectors and, similar to the IORP II Directive, the regulation requires the provision of standardized information, namely an annual statement on PEPP benefits for savers, including information on pension benefit projections.
The European Commission further asked EIOPA for technical advice on the development of best practices for setting up national pension tracking systems, see [16] [5], which would consist of a tool to give individuals projections of their future retirement income, including entitlements from all pension schemes in which they participate. While pension projections are made for all types of schemes, uncertainty tends to be higher in Defined Contribution (DC) plans, as in Defined Benefit (DB) plans the retirement income is usually based on a pre-defined function of the years of service and past salaries. In DC plans, the sponsor and/or individuals make regular contributions to an

[4] 2021 Ageing Report of the European Commission
[5] Call for advice to EIOPA on pension tools

account to fund the retirement income and the accumulated value (to be converted into this income) is unknown.

During the accumulation phase, two sets of risk factors, dependent on the economic and financial conditions, can be identified: the financial market conditions, that have an impact on the savings accumulated in the DC account, and the labour market conditions, which include the employment prospect and the real wage growth path, in case contributions are based on salaries. During the decumulation phase, there is uncertainty from mortality and interest rates.

Given the EU concerns, the main objective of our work is to construct a tool for making pension projections in the scope of Portuguese occupational DC pension schemes, whose importance shows an increasing trend. There are several reasons that explain this, one of them being employers' difficulties to bear the financial costs of providing DB plans, due to financial market conditions (low interest rates) and demographic changes (higher life expectancy).

The model underlying the tool takes in three main risk factors: the financial market risk, which includes uncertainty over returns on investments, inflation, and interest rates; the labour risk, originated from uncertainty over real wage growth paths; the demographic risk, as a result of the increasing life expectancy. Based on the stochastic models presented in [17] for the assessment of the risk and performance of PEPP products, we use an economic scenario generator to study the potential performance of different investment strategies and evaluate the impact on the retirement income that such investment strategies produce, considering also different assumptions with regard to mortality and interest rates.

The relevance of our paper is to provide the actuarial community with a case study that shows how to thoroughly implement "EIOPA's stochastic model for a holistic assessment of the risk profile and potential performance", in the specific context of defined contribution pension schemes. We give evidence that it is a well-designed and powerful tool to address the important problem of how to finance the ageing of the population in Portugal (or in any country).

This work demonstrates that the implementation of the model is quite complex and needs to be adapted to the specific environment of each pension plan at the national level. Overall, the "Eiopean" tool addresses a question of high priority.

The progression of the paper is as follows: Section 2 outlines the principles and good practices when making pension projections. Sections 3 and 4 provide the models for both the accumulation and decumulation phases. Section 5 is the application. Section 6 concludes.

2 Principles and Good Practices in Pension Projections

In what concerns the Portuguese pension funds sector, a law was published in 2020 (Law no. 27/2020), transposing the IORP II Directive into the national legal framework. It approves a new legal regime for the operation of pension funds and management entities and, among other aspects, it gives special attention to the projections of pension benefits. Members of the schemes must have an overview of their current situation, the accrued entitlements or accumulated capital, and an estimation of the level of benefits received at retirement, so that they can make informed decisions to achieve the expected retirement income and, where possible, to take pro-active actions to change contributions or the investment profile.

Although the Portuguese Insurance and Pension Funds Supervisory Authority (ASF) has the power to issue further requirements on information provision, the applicable legal framework is flexible on how pension projections should be made. It establishes that the management entities should provide information about the benefit projections based on the expected retirement age, income level and contribution period, and should include a warning that the final value of benefits can be different from projections. It is important that the uncertainty surrounding results is clearly communicated to the schemes' members, either by using appropriate disclaimers or by showing a range of possible scenarios.

The disclosure of the methodology and assumptions can also contribute to improve communication. In this regard, the Law establishes that savers should be told where and how to obtain additional information, when applicable, on the assumptions used for expressing amounts in annuities, namely the interest rate and the mortality table. About the scenarios to be used, it requires that, if projections use economic scenarios, the information should include a best estimate scenario and an unfavorable scenario, considering the nature of the pension scheme. It leaves the choice on whether to use a deterministic or stochastic approach to managing entities. The use of a stochastic approach is more complex but allows the simulation of a large variety of outcomes and to attach probabilities to results. It also allows the calculation of performance indicators that can be used to assess whether the investment strategies' risk-reward profile is in line with the members' retirement goals and risk tolerance.

A key element of pension projections, especially for DC schemes, is the set of the underlying assumptions, which according to the Law should be chosen in the most realistic way possible and reviewed regularly. For the accumulation phase, the main economic and/or financial assumptions used are typically related to investments' return, volatility and correlations of assets classes. When benefits depend on inflation, assumptions on the inflation rate are also required. For the decumulation phase, if annuities are calculated, at least assumptions on the interest rate and the mortality table are needed. When setting these assumptions, especially for younger members, one has to consider that the annuity rates used as reference at the time projections are performed may not be an appropriate estimator for the technical basis that will be used to price annuities in 30 or 40 years. Therefore, the inclusion of the evolution of life expectancy, e.g., by using dynamic life tables, might provide a more realistic view. Similarly, different scenarios should be considered also for the interest rate. Further detail on Communication and explanation of results is in Section 5.4.6.

3 Generating the Economic Scenario

An economic scenario generator (ESG) is a computer-based model used to produce simulations of the joint behaviour of financial and economic variables. The primary goal of ESG is to generate

future economic scenarios in order to evaluate the potential outcomes and their likelihood, giving an extremely useful insight into future risks [26]. The design and components of an ESG model can vary significantly with the goals of the specific application. For instance, pension providers can use ESG to evaluate different funding strategies and investment performance.
The calibration of real-world ESG models is a forward-looking procedure, that requires a view of the future economic development and expert judgement to determine the accuracy of the scenarios that result from the parameterization process. Following [26] and [17], the ESG model for this work, see Figure 1, comprises the nominal interest rate model, the equity index model, the inflation model and the real wage growth model, briefly described next. Although we sometimes discuss on the different models included in this holistic approach, we do not contest the choices made by EIOPA, because we are clear that they will not lead to misinformation of savers or to reduce risk awareness, which would obviously contradict the essence of the methodology.

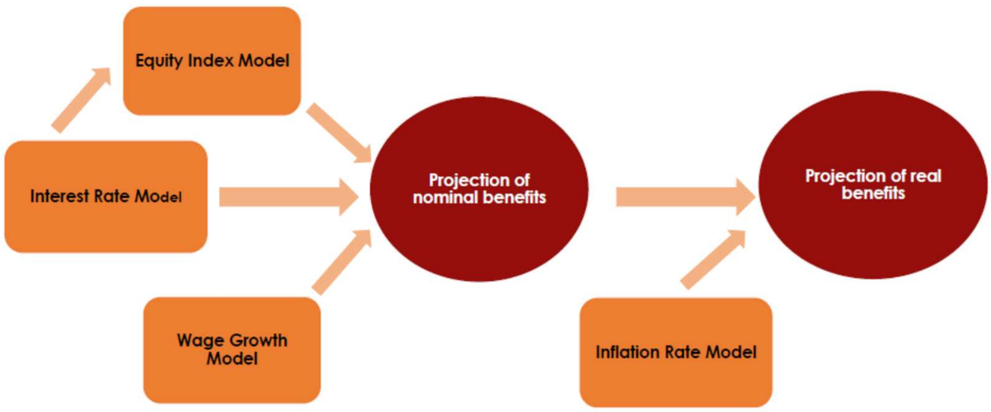

Figure 1: Economic Scenario Generator

3.1 Nominal Interest Rate – model, estimation, simulation and discussion

The interest rate model is a key component of most ESG models. It is used to generate the price of risk-free bonds and therefore to calculate the bond investment return.
Following for instance [2], the price of a zero-coupon bond and the yield to maturity are related. The price per unit at time t of a zero-coupon bond with maturity at time T, and assuming continuous compounding, is equal to

$$P(t,T) = e^{-y(t,T)(T-t)}, \tag{1}$$

where $y(t,T)$ is the continuous yield to maturity from t to T. Then

$$y(t,T) = -\frac{\log P(t,T)}{(T-t)}. \tag{2}$$

The instantaneous forward rate is

$$f(t,T) = -\frac{\partial}{\partial T}\log P(t,T) \tag{3}$$

and the instantaneous short rate is

$$r(t) = f(t,t) = -\lim_{T\to t}\frac{\log P(t,T)}{T-t} = -\frac{\partial}{\partial T}\log P(t,T)\,|_{T\to t}\,. \tag{4}$$

To model $r(t)$, which is not directly observed in the market, we use the G2++ model, see [3]. Being a 2-factor model, it captures more accurately the shape of the interest rate curve. In addition, it has the advantage to allow for negative interest rates. Our setting will be a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathcal{M})$, where $\mathcal{M}$ is either the risk-neutral measure $\mathbb{Q}$ or the real-world measure $\mathbb{P}$, as appropriate.

The dynamic of the short rate under a risk-neutral measure $\mathbb{Q}$ is

$$r(t) = x(t) + y(t) + \varphi(t)\,, \quad r(0) = r_0\,. \tag{5}$$

The 2-factor stochastic differential equation on $\{x(t): t \geq 0\}$ and $\{y(t): t \geq 0\}$ can be written

$$dx(t) = -ax(t)dt + \sigma dW_1^{\mathbb{Q}}(t)\,, \quad x(0) = 0\,, \tag{6}$$

$$dy(t) = -by(t)dt + \eta dW_2^{\mathbb{Q}}(t)\,, \quad y(0) = 0\,, \tag{7}$$

where a, b, σ, η are positive parameters and $r_0 = \varphi(0)$; (W_1^Q, W_2^Q) are correlated Wiener processes under the risk-neutral measure $\mathbb{Q}$, the instantaneous correlation parameter ρ being defined by $\rho dt = dW_1^Q(t)\, dW_2^Q(t)$, $-1 \leq \rho \leq 1$; $\varphi(t)$ is a deterministic function that makes the model to fit the initial market term structure.

We can define $r(t)$ conditional to the information up to time $s < t$, contained in a sigma-field $\mathcal{F}_s$. Following [3] and rewriting the price of the zero-coupon bond in the framework of affine term structure models, as [12], the result is

$$P(t,T) = \mathcal{A}(t,T)\exp\left[-\mathcal{B}_x(t,T)x(t) - \mathcal{B}_y(t,T)y(t)\right], \tag{8}$$

where

$$\mathcal{A}(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\, exp\left\{\frac{1}{2}[V(t,T) - V(0,T) + V(0,t)]\right\}, \tag{9}$$

$$P^M(0,t) = exp\left[-\int_0^t \varphi(u)du + \frac{1}{2}V(0,t)\right], \tag{10}$$

$$\begin{aligned} V(t,T) = {} & \frac{\sigma^2}{a^2}\left[(T-t) + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right] + \\ & + \frac{\eta^2}{b^2}\left[(T-t) + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right] + \\ & + \\ & + 2\rho\frac{\sigma\eta}{ab}\left[(T-t) + \frac{e^{-a(T-.t)}-1}{a} + \frac{e^{-b(T-.t)}-1}{b} - \frac{e^{-(a+b)(T-.t)}-1}{a+b}\right]. \end{aligned} \tag{11}$$

$$\mathcal{B}_x(t,T) = \frac{1-e^{-a(T-t)}}{a}, \qquad \mathcal{B}_y(t,T) = \frac{1-e^{-b(T-t)}}{b}\,. \tag{12}$$

To perform real-world scenario projections, G2++ must be regarded under the real-world measure $\mathbb{P}$. According to [17], this is done by using a constant independent market price of risk, to preserve the risk neutral structure with an additional constant drift term. The change of measure using Girsanov's theorem is operated through

$$dW_i^{\mathbb{P}} = -\lambda_i \, dt + dW_i^{\mathbb{Q}}, \quad i = 1,2, \tag{13}$$

where λ_i is the market price of risk.

The estimation procedure is based on minimizing the negative loglikelihood function by means of the differential evolution algorithm (current-to-p-best), as presented in [17]. The differential evolution is an algorithm of global optimization [29] that belongs to the family of evolutionary computing algorithms and starts with an initial population of candidate solutions. Resorting to iterations, these candidates are improved by minimizing an adequate objective function. In this work, the candidate solutions are defined by the lower and upper bounds of the parameters and the objective function is the negative likelihood given by the Kalman filter [10]. It is a method that consists of consecutive cycles of predicting the state of an observed variable, comparing the prediction with the historical observed data, and updating the parameters to reach the optimal predictive. In our study, using equations (2), (8), (9), (12), and (13), it is possible to set a relationship between the observable variables (yields) and the non-observable $x(t)$ and $y(t)$, see Figure 2 for an overview of the method.

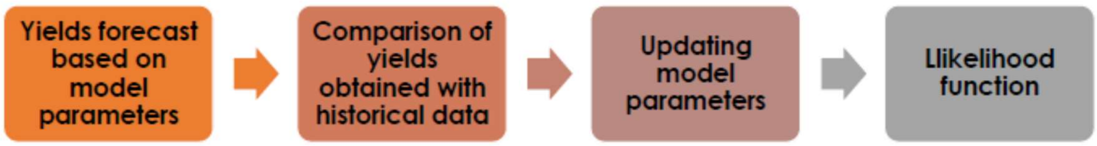

Figure 2: Kalman filter method

For simulation purposes, Cholesky decomposition [20] allows to discretize the interest rate model factors as shown in equations (14) and (15),

$$x(t+dt) = x(t)e^{-adt} + \frac{\lambda_1 \sigma}{a}(1 - e^{-adt}) + \sqrt{\frac{\sigma^2}{2a}(1 - e^{-2adt})}\, Z_x \tag{14}$$

$$y(t+dt) = y(t)e^{-adt} + \frac{\lambda_2 \eta}{b}(1 - e^{-bdt}) + \sqrt{\frac{\eta^2}{2b}(1 - e^{-2bdt})}\, \left(\rho Z_x + \sqrt{1-\rho^2} Z_y\right), \tag{15}$$

where Z_x and Z_y follow the standard normal distribution.

Adding the two factors to the deterministic function, calculated using Nelson-Siegel-Svensson parameters [25], the short rate is simulated. Next, the calculation of the price of the zero-coupon bonds follows, applying (8). To complete the process, the bonds investment returns are computed, using a rolling down strategy.

The G2++ model has been selected because it allows projecting negative interest rates, as is the case in our study. Other options do not allow this, for instance, the Black-Karasinski model, which is based on the short-rate but has a log-normal structure and cannot project negative interest rates.

An alternative also found in practice is to use the CIR3+ model; although it is based on a CIR model that does not allow for negative interest rates, the addition of a deterministic function overcomes this limitation, but the model becomes unnecessarily complex.
Currently, market models (LMM - libor model market; SABR – stochastic alpha, beta, rho; LMM+; SABR-LMM) are perhaps more common than the G2++ model, but they are modeled directly from asset prices in the market (swaption prices) and are calibrated by adjusting the curve of model volatility to the volatility curve observed in the market, which is not the case in our work. As we do a real-world valuation we use historical data and these models become less appropriate. In fact, for a real-world valuation, it is more common to use models from the "affine" family, which are based on the short-rate and therefore do not have a direct link to market prices, being calibrated through approximate formulas for risk neutral valuations. Since we are calibrating according to the spot rates relative to a historical period, it makes more sense to use a model from the "affine" family, based on the short-rate. In addition, the calibration using the Kalman Filter allows a direct connection to the model formulas making the whole process easier. In practice, solutions can be found that provide interest rate models based on the short rate using the Kalman Filter to calibrate, converting afterwards the real-world valuation into risk-neutral (required in Solvency II), based on deflators.

3.2 Equity Index – model, estimation, simulation and discussion

The Geometric Brownian motion (GBm) was selected to model the development of the equity index. As it is widely known, see for instance [2], the model can be described by two parameters, the volatility σ and the equity risk premium λ_{eq}. The risk-free rate used $r(t)$ is the one calculated by the nominal interest rate model. The stochastic differential equation of the price of the index, S_t, is given by

$$dS_t = \left(r(t) + \lambda_{eq} \right) S_t \, dt + \sigma \, S_t dW \tag{16}$$

and the solution is

$$S_t = S_0 \exp\left[\left(r(t) + \lambda_{eq} - \frac{\sigma^2}{2}\right)t + \sigma \, dW_t\right] . \tag{17}$$

Knowing the prices of the equity index, we can compute the annual equity return,

$$Ret_t = \frac{S_t - S_{t-1}}{S_{t-1}} . \tag{18}$$

The equity risk premium is estimated following Damodaran method [9]. Since the rate of return expected by investors is

$$E[R_m] = R_f + \lambda_{eq} , \tag{19}$$

where R_f is the risk-free rate, to estimate the equity risk premium we estimate the implied premium based on the market rates related to current prices. [9] proposes an expansion of the classic Dividend Discounted Model that inputs the potential dividends. Adding stock buybacks to aggregate dividend paid gives a better measure of total cash flow to equity. The general formula of the value of equity (present value of the index) can then be written as

$$Value\ of\ Equity = \sum_{t=1}^{N} \frac{E[FCF_t]}{(1+E[R_m])^t} + \frac{E[FCF_{N+1}]}{(E[R_m]-g_N)(1+E[R_{m]})^N}\ , \tag{20}$$

where N is the number of years of high growth, $E[FCF_t]$ is the Expected Free Cash Flow to equity (potential dividends) in year t, and g_N is the stable growth (after year N).

Following the assumptions considered in the reference document from [17], the Expected Free Cash Flow is computed using the long-term growth EPS (Earnings Per Share) forecast (g), the sum of the dividend yield and the buyback yield (γ), and the price of the index (P_0), at time $t = 0$. We consider a constant growth rate for five years followed by a perpetuity with growth rate equal to risk-free rate (R_f). Then (20) can be rewritten as

$$\begin{aligned} Value\ of\ Equity = &\frac{\gamma P_0}{(1+E[R_m])} + \frac{\gamma(1+g)P_0}{(1+E[R_m])^2} + \frac{\gamma(1+g)^2 P_0}{(1+E[R_m])^3} + \frac{\gamma(1+g)^3 P_0}{(1+E[R_m])^4} \\ &+ \frac{\gamma(1+g)^4 P_0}{(1+E[R_m])^5} + \frac{\gamma(1+g)^4(1+R_f)P_0}{(E[R_m]-R_f)(1+E[R_m])^5}. \end{aligned} \tag{21}$$

To compute the equity risk premium, we calculate the expected return, $E[R_m]$, by imposing

$$P_0 = Value\ of\ Equity. \tag{22}$$

Inserting the short rate from the interest rate simulated model, the simulation of the equity price follows the discretization

$$S(t+dt) = S(t)\ exp\left[\ \left(r(t) + \lambda_{eq} - \frac{1}{2}\sigma^2\right) dt + \sigma\sqrt{dt}\ Z_w\right], \tag{23}$$

where Z_w is a standard normal random variable. The equity return is computed using (18).

The option for a Geometric Brownian Motion (that converges to infinity almost surely for a positive drift) is arguable, but there are three possible reasons why EIOPA (and SOA) suggest its use, instead of, for instance, the SVJ model: (i) the inclusion of the ECB's 10-year spot rate and the application of the Damodaran method (which also depends on market conditions, in particular what the market expects to happen with buybacks and dividends) ultimately adjust the model; (ii) projections must be reviewed periodically, at least every year - in practice, with a shorter periodicity; (iii) "By their nature, jumps are difficult to measure, and the empirical finance literature is not settled on the matter", can be read on p 164 [26], followed by a discussion on omitting/including the jumps component.

3.3 Real Wage Growth – model, estimation and simulation

Labour market risk, in particular employment and wages, have an impact on the value of the contributions and consequently on the asset accumulation and retirement income. Contributions to DC plans depend, among other elements, on the length of employment and the wage growth path. We assume an uninterrupted career path, so only the real wage growth needs to be modelled.

[5] and [1] conclude that there are three main career paths for real wages: (1) paths that reach a plateau at the end of the career (high real-wage gains); (2) paths where the plateau is reached earlier, between ages 45 and 55 (medium real-wage gains), and then real wage path falls; (3) flat real wages paths during the whole career (a minority).
The model recommended in [17] is a quadratic equation with age,

$$wage(t) = a\,(max - age(t))^2 + b\,, \tag{24}$$

where a is related to the range of the wage and follows a uniform distribution between 0.011 and 0.15, max is related to the age when the real wage reaches the plateau and follows a uniform distribution between 52 and 69 and b can be found by solving the equation above, given the initial *wage* and *age*.
Finally, simulations are performed based on (24).

3.4 Inflation – model, estimation, simulation and discussion

In this work, inflation rates follow one factor Vasicek process. The model was proposed by Vasicek [28] and is a particular case of Hull-White model [21] with time dependent drift and diffusion parameters. It is a mean reverting stochastic model which ensures that the interest rates adhere to a long run reference level.
The corresponding stochastic differential equation is

$$di_t = k(\theta - i_t)\,dt + \sigma dW_t\,, \qquad i(0) = i_0\,, \tag{25}$$

where i_t is the inflation rate at time t, k is the speed of mean reversion, θ is the level of mean reversion, σ is the volatility, and W_t is the Wiener process.
Considering the variable change $i_t = z_t e^{-kt}$ and applying Itô formula and Itô isometry it is possible to derive that

$$i_t = \theta + (i_0 - \theta)e^{-k} + \sigma e^{-k}\int_0^t e^{ks}dW_s\,, \tag{26}$$

$$E[i_t] = \theta + (i_0 - \theta)e^{-kt}, \tag{27}$$

$$V[i_t] = \frac{\sigma^2}{2k}(1 - e^{-2kt})\,. \tag{28}$$

When $t \to \infty$, the distribution of i_t converges to $N(\theta, \frac{\sigma^2}{2k})$, and we obtain the stationary distribution.
To estimate the parameters, and following [18], the loglikelihood function is

$$L(\theta) = L(\kappa; \theta; \sigma^2) = -\frac{n}{2}\log\left[\frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa dt})\right] - \frac{n}{2}\log(2\pi)$$
$$-\frac{k}{\sigma^2(1 - e^{-2\kappa dt})}\sum_{i=1}^{n}\left[I_{t_i} - I_{t_{i-1}}e^{-kdt} - \theta(1 - e^{-\kappa dt})\right]^2. \tag{29}$$

and the estimators are

$$\hat{\kappa} = -\frac{1}{dt}\log\left[\frac{n\sum_{i=1}^{n} I_{t_i}I_{t_{i-1}} - \sum_{i=1}^{n} I_{t_i}\sum_{i=1}^{n} I_{t_{i-1}}}{n\sum_{i=1}^{n}\sum_{i=1}^{n} {I_{t_i}}^2 - \left(\sum_{i=1}^{n} I_{t_{i-1}}\right)^2}\right]; \tag{30}$$

$$\hat{\theta} = \frac{1}{n\left(1 - e^{-\hat{k}dt}\right)}\left(\sum_{i=1}^{n} I_{t_i} - e^{\hat{k}dt}\sum_{i=1}^{n} I_{t_{i-1}}\right); \tag{31}$$

$$\hat{\sigma}^2 = \frac{2\hat{k}}{n\left(1 - e^{-2\hat{k}dt}\right)}\sum_{i=1}^{n}\left[I_{t_i} - I_{t_{i-1}}e^{-\hat{k}dt} - \hat{\theta}\left(1 - e^{-\hat{k}dt}\right)\right]^2 . \tag{32}$$

From historical data and the future projection of inflation rates, we estimate the parameters. The inflation rates are simulated using the discretization

$$i(t + dt) = i(t) + \kappa\big(\theta - i(t)\big)dt + \sigma\sqrt{dt}\, Z_s\,, \tag{33}$$

where Z_s is a normal (0,1) random variable.

Some of the world most influential actuarial entities (CAS, EIOPA, SOA, cf. [7], [17], [25}) recommend the Ornstein–Uhlenbeck mean-reverting process/Vasicek process to model the dynamics of inflation, although they admit that other options exist: the autoregressive process, the moving average process, the autoregressive moving average process and the autoregressive integrated moving average process are also popular alternatives. According to [7], pp 128-129 and pp 136-138, the preference for the Vasicek process is justified by the fact that the ultimate purpose is to develop a term structure of inflation that reflects expected inflation rates over different time horizons, and the Vasicek process allows closed or semi-closed formulas for the term structure of expected inflation to be derived.

4 Construction of Projected Lifetables for Pensioners

We analyze the changes in mortality as a function of age *x* and time *t*, following the notation in [4]. Hence, $\mu_x(t)$ denotes the force of mortality at age x during calendar year t, and D_{xt} will denote the number of deaths reported at age *x* during year *t*, from an exposure-to-risk E_{xt}. The probability of death of a life age x during year t is q_x and the probability of survival till age $x + 1$ is $p_x = 1 - q_x$. The central mortality rate is given by

$$m_x(t) = \frac{D_{xt}}{E_{xt}}\,. \tag{34}$$

Assuming that the force of mortality is constant within time and age interval but can vary between intervals, we obtain

$$q_x(t) = 1 - e^{-\mu_x(t)}. \tag{35}$$

[27] proved that $m_x(t) \approx \mu_{x+\frac{1}{2}}(t)$ and from (35) we can assume $m_x(t) = \mu_x(t)$. Then, the force of mortality can be written

$$\mu_x(t) = \frac{D_{xt}}{E_{xt}}. \tag{36}$$

To construct lifetables for pensioners is a challenging task, due to the fact that very often the data from the population of interest, the pension funds population, is scarce, making us to follow the 3-stage process, shown in Figure 3.

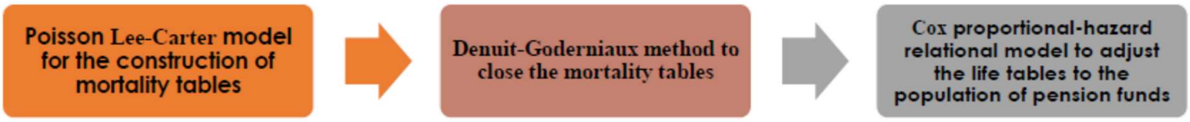

Figure 3: Building life tables for pensioners

4.1 Lifetable for the Reference Population - Poisson Lee-Carter Model

According to the classical Lee-Carter model [22],

$$\ln \hat{\mu}_x(t) = \alpha_x + \beta_x k_t + \epsilon_x(t)\ , \sum_x \beta_x = 1, \sum_t \kappa_t = 0,\ \ \epsilon_x(t) \sim N(0, \sigma_\epsilon^2)\ , \tag{37}$$

where $\hat{\mu}_x(t)$ represents the observed force of mortality at age x in year t, α_x represents the average mortality at age x over time, β_x denotes the age-specific pattern of mortality change, k_t represents time trend of mortality, and $\epsilon_x(t)$ is the homoscedastic centered error term.

Following [4], parameters are fitted to a matrix of age-specific observed force of mortality using singular value decomposition (SVD). After the estimation, Lee and Carter [22] use an ARIMA(0,1,0) times series model to perform projections and forecast the time trend of mortality, κ_t^* :

$$\kappa_t^* = \mu + \kappa_{t-1} + \varepsilon_t \tag{38}$$

[4] develop an extension of the Lee-Carter model, where $\epsilon_x(t)$ is replaced with the random variable D_{xt} such that

$$D_{xt} \sim Poisson\big(E_{xt}\mu_x(t)\big)\ ,\ \ \text{with}\ \ \mu_x(t) = \exp(\alpha_x + \beta_x k_t)\ . \tag{39}$$

The loglikelihood function is then

$$L(\alpha;\beta;\kappa) = \sum_{x,t} \{D_{xt}(\alpha_x + \beta_x \kappa_t) - E_{xt} \exp(\alpha_x + \beta_x \kappa_t)\} + constant\,. \tag{40}$$

[4] propose an iterative method, based on the Newton-Raphson algorithm, first developed by [19] for the estimation of $\beta_x \kappa_t$. The time trend projection is computed using ARIMA(0,1,0) like in the classical Lee-Carter model. With the estimates of α_x and β_x and the forecast of κ_t, κ_t^*, we can generate the force of mortality,

$$\mu_x(t) = \exp(\alpha_x + \beta_x \kappa_t^*)\ . \tag{41}$$

4.2 Closing the Lifetable for the Reference Population - Denuit-Goderniaux Method

According to [6], when data for older people lack the required quality for the construction of complete lifetables, as is the case in Portugal, one solution is to use the Denuit and Goderniaux method [11]. It is based on a logarithm quadratic regression,

$$\ln q_x(t) = a_t + b_t x + c_t x^2 + \varepsilon_{xt}, \qquad \mathcal{E}_{xt} \sim N(0, \sigma^2)\ , \tag{42}$$

fitted separately to each calendar year *t* and to a given age period, and imposing two constraints:

$$q_{x_{max}} = 1 \quad \text{and} \quad q'_{x_{max}} = 0, \tag{43}$$

where a, b and c are parameters to be estimated by OLS, x_{max} is a pre-defined highest attainable age and $q'_{x_{max}}$ is the first derivative with respect to age x of $q_{x_{max}}$. The first constraint imposes a maximum age for human life. The second one guarantees no decreasing death probabilities at older ages. Both guarantee a concave mortality curve with horizontal tangency at x_{max}. Inserting (43) into (42), then

$$\ln q_x(t) = c_t(x_{max} - x)^2 + \mathcal{E}_{xt}\,, \quad \mathcal{E}_{xt} \sim N(0, \sigma^2)\,. \tag{44}$$

4.3 Lifetable for the Population of Pensioners - Relational Models

Since the data from the pension funds population is scarce, it is necessary to apply relational models to relate the population under study with a reference population. We will use the Cox proportional-hazard model [8] based on previous work by [24], which provided the best fit for Portuguese pension funds data at the time.

The Cox proportional-hazard model assumes that the force of mortality of the population under study ($\mu_{x,t}^{est}$) is proportional to that of the reference population ($\mu_{x,t}^{ref}$), with the proportional factor a independent of age. Then

$$\mu_{x,t}^{est} = a\, \mu_{x,t}^{ref}\,, \qquad \mu_{x,t}^{j} = \frac{D_{x,t}^{j}}{E_{x,t}^{j}}, j = est, ref, \tag{45}$$

where a is estimated by linear regression.

5 Application

5.1 ESG Estimation

5.1.1 Interest Rate Model

The "All euro area central government bond yield curve" of ECB[6], was used to estimate the parameters of the model. To cover the short, medium and long run, we selected maturities of 1, 10 and 30 years of the daily spot rates from 2 January 2017 to 31 December 2021 (five years). The Nelson-Siegel-Svensson parameters of the first day (2 January 2017) were used to compute the deterministic function, which depends on the forward rate.

The bound limits imposed to the parameters are $a \in [0,1], b \in [0,1], \sigma \in [0,1]$, $\eta \in [0,1], \rho \in [-1,1], \lambda_1 \in [0,0.02], \lambda_2 \in [0,0.02]$, $h \in [0.0001,0.001]$, and the likelihood function is the one given by the Kalman filter method [10]. Setting the initial values for the parameters within the bound limits, an initial likelihood function is computed and used as starting value for the differential evolution algorithm. The process stops when convergence is achieved. Since it is an iterative process, the procedure is repeated numerous times and using different initial values to assess the quality of the convergence. Results can be seen in Table 1.

[6] ECB – Euro area yield curves

Table 1 - Estimated parameters for the interest rate model

A	b	σ	H	P	λ_1	λ_2	h	loglike
0,04848	0,83339	0,00650	0,00861	-0.94892	0.00002	0.01866	0.001	-20631,99

The two factors of the model have almost perfect negative correlation. Only one of the market price of risk (λ_2) has a significant value and the volatility parameters (σ and η) have very low values. The comparison between the estimated yields and the observed ones is presented in Figure 4 and in general the estimated and the observed curves follow the same path. The errors, extracted from the Kalman filter, are mostly within - 25 and + 25 basis points. The statistical performance is summarized in Table 2, measured by the Root mean squared error (RMSE), the Mean absolute error (MAE) and the Adjusted root mean squared error (ARMSE).

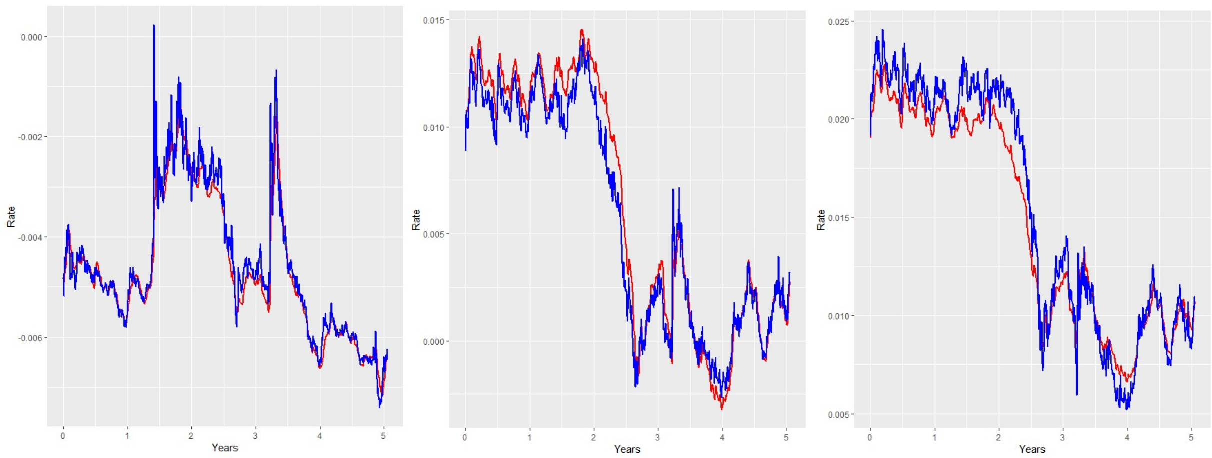

Figure 4: Estimated (red) vs observed yields (blue) for maturities 1-year (left), 10-year (center) and 30-year (right)

Table 2 - Error estimation of interest rate model

Maturity	RMSE	MAE	ARMSE
1-year	0,421	0,022	0,512
10-years	0,110	0,086	5,403
30-years	0,129	0,107	0,094

5.1.2 Equity Model

The index STOXX Europe 600 was used for the estimation without considering any country-specific risk premium [17], see Figure 5.

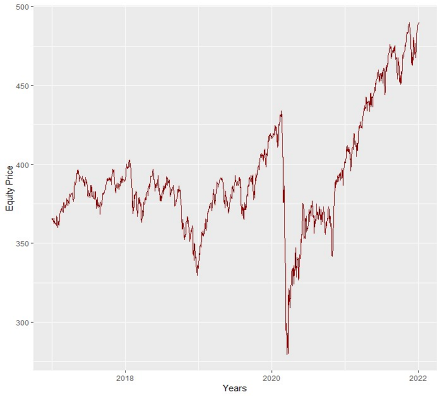

Figure 5: STOXX Europe 600 closed price

To apply Damodaran method, the 10-year yield rate of the ECB's "All euro area central government bond yield curve" on the reference date of 3 January 2022 (first day of the simulation) was considered, being equal to $R_f = 1{,}89\%$. The long-term growth EPS forecast, g, is a weighted average of the average growth rate of the next six years, where the values of 2022 were provided by Refinitiv[7] and for the following years were given by the risk-free rate, as presented in Table 3, from which $g = 6{,}14\%$ was derived.

Table 3 - EPS forecast

Year	2022	2023 to 2027
EPS forecast (%)	25.1	1,89

The dividend and buyback yield was provided by Bloomberg. The closed price of the index is used in the estimation to solve (22). The estimates of the parameters are in Table 4.

Table 4 – Estimated parameters for the equity risk premium

Parameter	R_f	P_0	g	γ
Value	1,89%	489.99	6,14%	3,74%

Imposing $P_0 = Value\ of\ Equity$ in (21), and using EXCEL solver, we obtain $E[R_m]$. From (19), we have that the equity risk premium $\lambda_{eq} = E[R_m] - R_f$ = 4,58%.
The yearly close price of the index, from the start of 2017 until the start of 2022 (Figure 5), was used to estimate the volatility of the GBm process, σ, considering the annualized standard deviation of the last ten years as a proxy for the volatility of the equity model. The estimated parameters for the equity index model are shown in Table 5.

Table 5 - Estimated parameters for the equity model

σ	λ_{eq}	P_0	$r(t)$
16,38%	4,58%	489.99	Short rate

[7] REFINITIV – Financial Technology, Data and Expertise

5.1.3 Inflation Model

Parameter θ, which represents the mean at long-run, is given by the ECB target inflation of 2%. For the estimation of σ, the monthly Yo-Yo from HICP[8] (1999-2021) time series was considered, where the estimate of $\hat{\sigma}$ equals the standard deviation of the time series, see Figure 6 (left). The initial value of the inflation rate model, i_0, is the first value of monthly Yo-Yo from HICP (1999-2021) time series.

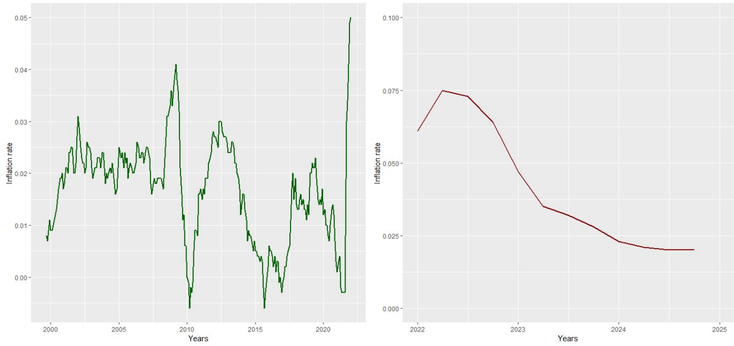

Figure 6: Yo-Yo monthly inflation rates, 1999-2021(left) and inflation projection, 2022-2024 (right)

The macro-economic inflation projection made by the European Commission, see Figure 6 (right), was used to estimate the speed to the mean reversion, k. As we have already obtained $\hat{\theta}$ and $\hat{\sigma}$, only need to estimate k, applying (30). Results are in Table 6.

Table 6 - Estimated parameters for the inflation model

$\hat{\theta}$	$\hat{\sigma}$	$\hat{k}$	i_0
0.02	0.0100284	0.4712229	0.008

5.2 Lifetables Projection

5.2.1 Data

The construction of the projected lifetables needs to make use of data exclusively from Portugal: from the Portuguese general population and from the Portuguese pension funds population, by age and gender. For the general population the source was the Human Mortality Database[9], considering ages between 0 and 90 ($x \epsilon \{0,90\}$) and years between 1970 and 2018 ($t \epsilon \{1970,2018\}$). For the pension schemes population, data was provided by ASF. From the analysis of the pension funds data, it was decided to choose ages between 60 and 90 since this is the interval with sufficient quality to conduct the mortality analysis. The Portuguese pension schemes are mainly composed of members within this age interval (see [24]).

5.2.2 Poisson Lee-Carter Model

Three steps were taken to estimate the model:

[8] HICP – ECB Statistical Data Warehouse

[9] Human Mortality Database

1. Estimation of the parameters (α, β, κ), where $\alpha = \{\alpha_x, x = (0, \dots, 90)\}$, $\beta = \{\beta_x, x = 0, \dots, 90\}$ and $\kappa = \{\kappa_t, t = 1970, \dots, 2018\}$.The estimated parameters are in Figure 7, whose shapes are similar to the ones obtained by [24].

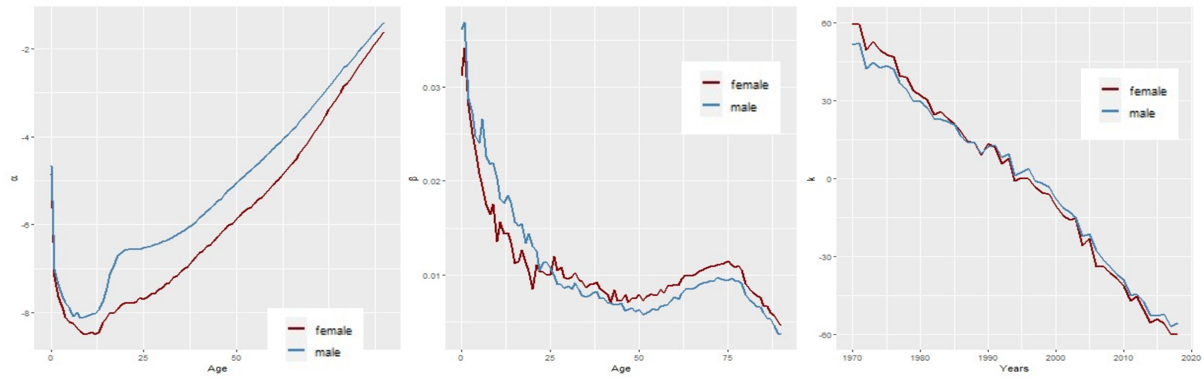

Figure 7: Poisson Lee-Carter parameters, α (left), β (middle), k (right) - Males (blue), females (red)

2. Maximum likelihood estimation of the ARIMA (0,1,0) model, cf. Table 7 for results.

Table 7 – Estimated parameters for ARIMA (0,1,0)

	Males	Females
Drift, μ	-2.230012	-2.492037
Variance, σ^2	8.124448	11.921601

3. Projection of κ_t over 125 years, from 2018 till 2143.

After obtaining these estimates, (41) was applied to build two matrices with projected values of the force of mortality for females and males, respectively denoted $[\mu_{F,x}^{ref}(t)]_{91x126}$ and $[\mu_{M,x}^{ref}(t)]_{91x126}$, $x \in \{0,1, \dots 90\}$ and $t \in \{2018, \dots, 2143\}$, see Figure 8 for a few illustrations.

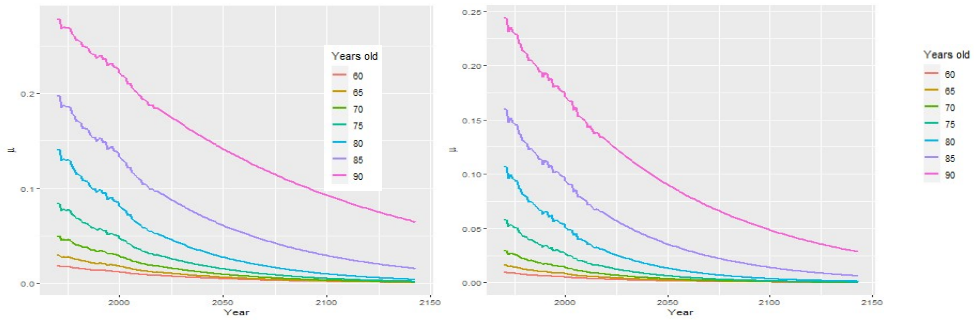

Figure 8 – Force of mortality μ for females (left) and males (right), $x \epsilon \{60, 65, \ldots, 90\}, t \leq 2143$

5.2.3 Denuit and Goderniaux Method

The preliminary task is to calculate two other matrices from $[\mu_{F,x}^{ref}(t)]_{91x126}$ and $[\mu_{M,x}^{ref}(t)]_{91x126}$, denoted $[q_{F,x}^{ref}(t)]_{91x126}$ and $[q_{M,x}^{ref}(t)]_{91x126}$, with the mortality rates. Afterwards, in order to close the lifetables, (44) is applied with $x_{max} = 125$. A separate log-quadratic regression is fitted to each calendar year t and to ages (x) 90 and over; mortality rates for ages $x \epsilon \{0, \ldots, 125\}$ and calendar years $t \epsilon \{2018, \ldots, 2143\}$ are obtained, allowing to build complete matrices $Q_F^{ref} = [q_{F,x}(t)]_{126x126}$ and $Q_M^{ref} = [q_{M,x}(t)]_{126x1}$, for females and males, respectively. Figure 9 exemplifies some cases.

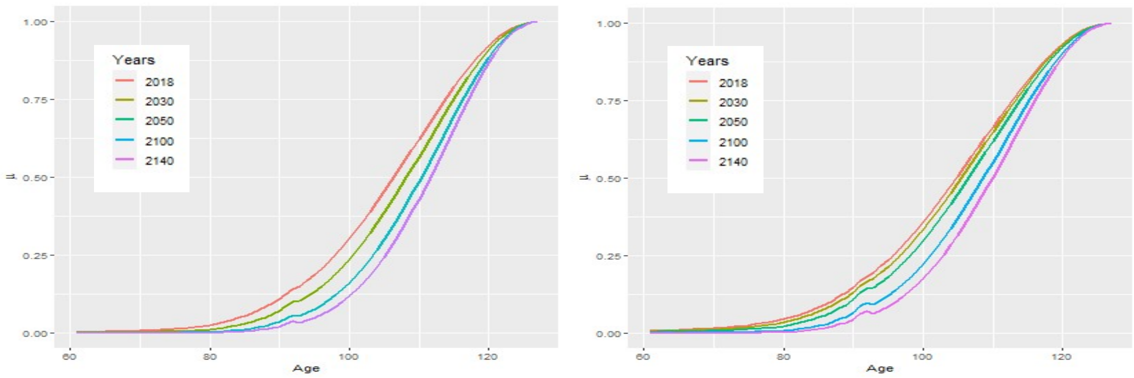

Figure 9: Closing Projected Lifetables, females (left) and males (right)

5.2.4 Relational Model

At this final stage, the projected lifetables for the pension schemes population (PF) are built, using Cox proportional hazard and the linear regression in (45), see Table 8.

Table 8 – Estimated parameter for the Cox proportional-hazard model

	Males	Females
Parameter $\boldsymbol{a}$	0.6969615	0.748895
Standard error $\boldsymbol{\sigma_\varepsilon}$	0.0133271	0.012929

The computation of the matrices with the mortality rates for the pension funds, $Q_F^{PF} = [q_{F,x}^{PF}(t)]_{126x126}$ and $Q_M^{PF} = [q_{M,x}^{PF}(t)]_{126x1}$, is then straightforward.

5.3 The Tool

Now that all parts are available, the calculation tool is assembled using R and Excel. It will project the benefits provided by DC pension schemes. It foresees a set of open fields, allowing the user to choose the key assumptions of the projection. We start by describing the required inputs, followed by the specification of the simulation. Afterwards, the risk and performance measures to assess the results of the projection will be discussed. In accordance to the EU guidelines, the tool allows users to take a stochastic or a deterministic approach.

5.3.1 Inputs to the Calculation Tool

In terms of inputs related to the accumulation phase, we have: (1) Age at the beginning of the projection; (2) Retirement age; (3) Member's contribution rate; (4) Annual fee that is charged to the pension fund. In relation to the decumulation phase, the user can choose the mortality table and the interest rate, to express the benefit as an annuity. It is implicitly assumed that the annuity is a constant lifelong annuity.

The tool allows the user to introduce three types of portfolios with a mix of bonds and equities: rebalanced, lifecycle and fixed portfolios. The rebalanced portfolios correspond to an investment strategy ensuring that the level of risk is kept within a certain desirable range, which is done by allowing the user to define the lower and upper bounds of equity weights throughout the projection horizon. In the lifecycle portfolios the user can set the weight of equities for each year of the projection, applying an investment strategy that reduces the share in risky assets (equity) as the member approaches the retirement age. It is used to mitigate the risk of a reduction in retirement income, in case a negative shock in equity markets occurs near retirement age. Finally, the fixed portfolios express an investment strategy without risk mitigation.

5.3.2 Risk Profile and Performance Assessment

Apart from the results in euros, the tool produces a set of indicators which allow an analysis of the risk profile and an assessment of the fund performance.

Replacement Rate

The replacement rate (RR) is one of the measures commonly used by regulators and policy makers to assess the adequacy of retirement income. It corresponds to the ratio (in percentage) between the first estimated monthly benefit and the wage of the member at retirement.

$$RR = \frac{Estimated\ monthly\ benefit}{Final\ wage} \times 100. \tag{46}$$

As it measures the percentage of the worker income that will be replaced by the expected outcome of a particular pension scheme, this indicator can be used as a benchmark by the member to help track how the scheme is doing in comparison to expectations.

Expected and median lump sum

The potential performance of the investment strategy can be measured by the median or the expected value of the distribution of the lump sum (accumulated value) at retirement. The median represents a more robust measure since it is less sensitive to extreme values.

Risk of a low lump sum

The empiric distribution of the lump sum gives valuable information about the different levels of outcomes members will achieve and can be used to assess the investment performance, e.g., by calculating different percentiles.

Probability of recoup capital

This risk measure corresponds to the probability of the investment strategy reaching at least the sum of all contributions at retirement age. It is computed as the proportion of simulated scenarios where the lump sum is greater than the total contributions.

Expected shortfall when not recouping capital

The expected shortfall measures the average difference between the lump sum and total contributions, conditional on not recouping the capital. The greater the expected shortfall, the greater the risk that members will have benefits below the needed level.

Probability to reach a given goal

The performance of the investment strategy can also be measured by comparing the average return achieved with specific rates of return, which represent the level of ambition. For the results presented next, similar to the analysis performed in [17], the ultimate forward rate (UFR) published by EIOPA, and equal to 3,45% in 2022, was used as a proxy for the long-term risk-free rate, although the calculation tool allows the user to introduce other rates as benchmark (we will introduce and discuss a more ambitious goal, 5%).

Joint risk-performance assessment

The combination of risk and performance measures can be applied to assess the risk-performance profile of the investment strategies. With both dimensions, the difference between investment strategies becomes more visible. We will use the standard deviation of returns as risk measure and the mean of returns over total contributions as the performance measure.

5.4 Using the tool

5.4.1 Inputs

For the case study, the inputs related to the accumulation and decumulation phases, specific to a paradigmatic member, are in Table 9.

Table 9 - Inputs of the accumulation and decumulation phases

Age	Retirement age	Initial wage	Contribution rate	Fee	Time interval
30	70	1000	10%	1%	Month

Mortality table	Interest rate 1	Interest rate 2
Projected lifetable	1%	3%

We have considered nine scenarios, associated to three rebalanced portfolios, three lifecycle portfolios and three fixed portfolios, in an attempt to capture different investment strategies and assess the respective risk and potential performance. The lower and upper bounds for the rebalanced portfolios (RB1, RB2 and RB3) are in Table 10.

Table 10 – Weight of equities in the rebalanced portfolios (%)

Scenario/Portfolio	Initial equity weight	Lower limit	Upper limit
RB1	30%	0%	30%
RB2	50%	0 %	50 %
RB3	70%	0%	70%

The lifecycle portfolios (LC1, LC2 and LC3) have fixed equity weights in the first 20 years, which will decrease in the last 20 years, at 1% per year, see Table 11.

Table 11 – Weight of equities in the lifecycle portfolios (%)

Scenario/Portfolio	From year 0 to 20	From year 21 to 40
LC1	30 %	reduces 1% each year
LC2	50 %	reduces 1% each year
LC3	70%	reduces 1% each year

The fixed portfolios (FP1, FP2 and FP3) have constant equity weights during the whole horizon, as shown in Table 12.

Table 12 – Weight of equities in the fixed portfolios (%)

Scenario/Portfolio	Equity weight
FP1	30%
FP2	50%
FP3	70%

10 000 Monte Carlo simulations will be performed. Each one provides one possible outcome during the accumulation phase for the bond and equity returns, inflation rate and real wage growth rate and, consequently, one possible outcome for the accumulated value in the fund at retirement. The reference data is the first date of the simulation, 3-1-2022, and the results are simulated considering a monthly interval ($dt = 1/12$).

In addition to the results for the fixed, rebalanced and lifecycle portfolios, and the UFR (annual return of 3.45%) and RATE1 (annual return of 5%) scenarios, a scenario considering an accumulated value equal to the sum of contributions (i.e., with 0% return), identified as 'CONTRIB', is also included. In total, we explore 12 scenarios, obtaining 120 000 results.

In the following, where applicable, results are presented in terms of the mean and also considering three settings, based on the percentiles of the distributions obtained: (1) pessimistic setting: 15th percentile; (2) Intermediate setting: median; (3) Optimistic setting: 85th percentile.

5.4.2 Lump Sum

Results obtained for the lump sum in the three settings can be seen in Tables 13 and 14 and in Figure 10. For the optimistic setting, the higher lump sum is given by scenario FP3, followed by scenarios RB3 and LC3, which correspond to the portfolios with higher equity exposure. In particular, RB3 is the only one that achieves a median lump sum greater than the one obtained by a portfolio with average return equal to the UFR. In the optimistic scenario all nine portfolios give lump sums higher than UFR but only RB3, LC3, FP3 and FP2 outperform RATE1. Regarding the pessimistic setting, all rebalanced and lifecycle portfolios achieve better results than CONTRIB, suggesting that the probability of these investment strategies to yield a negative return could be low, even in this setting - which is the opposite to the results for the three fixed portfolios.

Table 13 - Lump sum (euros)

Scen/Port	Mean	p15	Median	p85
RB1	100985	76788	97004	125624
RB2	115292	77250	106428	154566
RB3	135381	74304	116671	195194
LC1	95704	74079	92013	116547
LC2	114376	76724	101746	148348
LC3	133048	75411	110111	185828
FP1	98592	63809	84819	131213
FP2	121774	64658	98063	174937
FP3	144955	65297	111416	219055
CONTRIB	70464	67321	70501	73511
UFR	114162	109645	114290	118487
RATE1	159447	153498	159671	165110

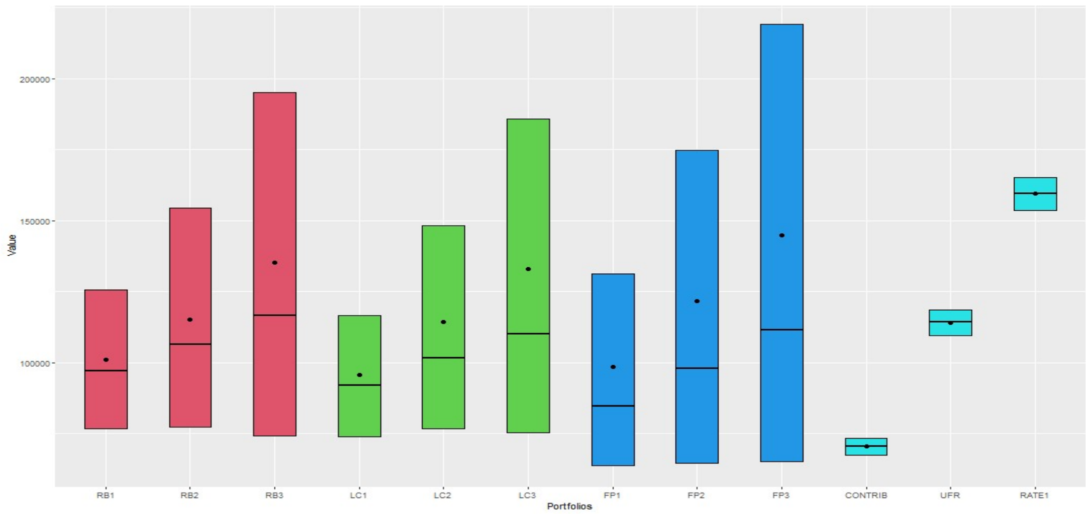

Figure 10: Lump sum (euros - the black points represent the mean, the top bars the 85th percentile, the middle bars the median and the bottom bars the 15th percentile)

Table 14 - Lump sum as a percentage of total contributions

Scen/Port	Mean	15th perc	Median	85th perc
RB1	143	114	138	171
RB2	164	115	151	210
RB3	192	110	165	266
LC1	136	110	131	159
LC2	162	114	144	202
LC3	189	112	156	253
FP1	140	95	120	178
FP2	173	96	139	238
FP3	206	97	158	298
CONTRIB	100	100	100	100

UFR	162	163	162	161
RATE1	226	228	226	225

5.4.3 Annuities

Following the good practices prescribed in Section 2, as a way to help members to better understand their purchasing power after retirement, annuities corresponding to the lump sums were calculated in nominal and real terms.

The annual amounts in Tables 15 and 16 are for a whole life annuity, assuming that the member retires in 2062. In each case, the value of the annual payment made by the whole life annuity is obtained dividing the lump sum in each scenario by the expected present value of a whole life annuity paying 1 per year, given the interest rate (i_1=1% or i_2=3%) and the lifetables projected in Section 5.2.

Table 15 – Annuity payments to a male life and a female life at i_1=1% (euros)

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	5816	4423	5587	7235	4805	3653	4615	5977
RB2	6640	4449	6130	8902	5485	3675	5064	7354
RB3	7797	4280	6720	11242	6441	3535	5551	9287
LC1	5512	4267	5300	6713	4553	3525	4378	5545
LC2	6588	4419	5860	8544	5442	3650	4841	7058
LC3	7663	4343	6342	10703	6330	3588	5239	8841
FP1	5678	3675	4885	7557	4691	3036	4036	6243
FP2	7014	3724	5648	10076	5794	3076	4666	8323
FP3	8349	3761	6417	12617	6897	3107	5301	10422
CONTRIB	4058	3877	4061	4234	3353	3203	3354	3498
UFR	6575	6315	6583	6824	5432	5217	5438	5637
RATE1	9183	8841	9196	9510	7586	7303	7597	7856

Table 16 – Annuity payments to a male life and a female life at i_2=3% (euros)

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	7299	5550	7012	9080	6161	4685	5918	7665
RB2	8333	5584	7693	11172	7034	4713	6493	9430
RB3	9785	5371	8433	14109	8260	4533	7118	11909
LC1	6918	5354	6651	8424	5839	4520	5614	7111
LC2	8267	5546	7354	10723	6978	4681	6208	9051
LC3	9617	5451	7959	13432	8117	4601	6718	11338
FP1	7126	4612	6131	9484	6015	3893	5175	8005
FP2	8802	4673	7088	12644	7430	3945	5983	10673
FP3	10477	4720	8053	15833	8844	3984	6798	13365
CONTRIB	5093	4866	5096	5313	4299	4107	4301	4485
UFR	8252	7925	8261	8564	6965	6690	6973	7229
RATE1	11525	11095	11541	11934	9728	9365	9742	10074

Continuing to follow the good practices, the inflation-adjusted values for the annuities are in Tables17 and 18. Instead of calculating the expected present values of the annuities using directly the flat term structures adopted, they are calculated with the ‘real’ rates, i.e. the rates calculated after correcting those returns from the effects of inflation.

We observe a significant reduction of the annuities’ values, showing that in the long-term inflation has a significant effect on the adequacy of retirement income.

Table 17 - Annuity payments to a male life and a female life at i_1=1% (euros, inflation-adjusted)

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	2459	1795	2348	3131	2031	1483	1939	2587
RB2	2807	1817	2573	3816	2319	1501	2126	3152
RB3	3296	1755	2828	4841	2722	1450	2336	3999
LC1	2330	1731	2232	2927	1925	1430	1844	2418
LC2	2784	1804	2475	3681	2300	1491	2045	3040
LC3	3238	1779	2670	4605	2675	1470	2206	3804
FP1	2400	1501	2074	3236	1982	1240	1713	2673
FP2	2963	1531	2389	4291	2448	1265	1974	3544
FP3	3527	1552	2717	5378	2914	1282	2245	4443
CONTRIB	1653	1437	1637	1868	1365	1187	1352	1543
UFR	2781	2406	2755	3159	2297	1988	2276	2610
RATE1	3884	3364	3847	4408	3208	2779	3178	3641

Table 18 - Annuity payments to a male life and a female life at i_2=3% (euros, inflation-adjusted)

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	3086	2253	2946	3930	2605	1901	2487	3317
RB2	3523	2280	3229	4789	2974	1925	2726	4042
RB3	4136	2203	3550	6075	3491	1859	2996	5128
LC1	2924	2172	2801	3673	2468	1834	2364	3100
LC2	3494	2264	3106	4619	2949	1911	2622	3899
LC3	4063	2233	3351	5779	3430	1885	2828	4878
FP1	3012	1884	2603	4061	2542	1590	2197	3428
FP2	3719	1922	2999	5384	3139	1622	2531	4545
FP3	4426	1948	3410	6749	3736	1644	2878	5697
CONTRIB	2074	1804	2054	2345	1751	1523	1734	1979
UFR	3490	3020	3458	3965	2946	2549	2919	3346
RATE1	4874	4221	4827	5532	4114	3563	4075	4669

5.4.4 Replacement Rate

The replacement rates, see equation (46), are presented separately for the male and female cases, since this is more explanatory. Results for the two interest rates i_1=1%, i_2=3% will be analyzed.

For i_1=1%, a significant dispersion in the results can be observed. The main highlights are, cf. Table 19 and Figure 11:

- The best *RR* are 66% (male, scenario FP3, optimistic setting) and 54% (female, scenario FP3, optimistic setting); the lowest are 19% (male, scenario FP2, pessimistic setting) and 16% (female, scenarios FP1, FP2, FP3, pessimistic setting). Differences are impressive.
- In the intermediate setting, three scenarios (RB3, LC3 and FP3) achieve *RR* higher than the one in the UFR scenario.

- In the optimistic setting, only one portfolio (LC1) obtains *RR* lower than the one in UFR.
- In the pessimistic setting, the fixed portfolios are the only ones that do not perform better than the CONTRIB scenario.

Table 19 - Replacement Rate at i_1=1%

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	30	23	29	38	25	19	24	31
RB2	35	23	32	47	29	19	26	38
RB3	41	22	35	59	34	19	29	49
LC1	29	22	28	35	24	18	23	29
LC2	34	23	31	45	28	19	25	37
LC3	40	23	33	56	33	19	27	46
FP1	30	19	26	40	25	16	21	33
FP2	37	19	30	53	30	16	24	44
FP3	44	20	34	66	36	16	28	54
CONTRIB	21	20	21	22	18	17	18	18
UFR	34	33	34	36	28	27	28	29
RATE1	48	46	48	50	40	38	40	41

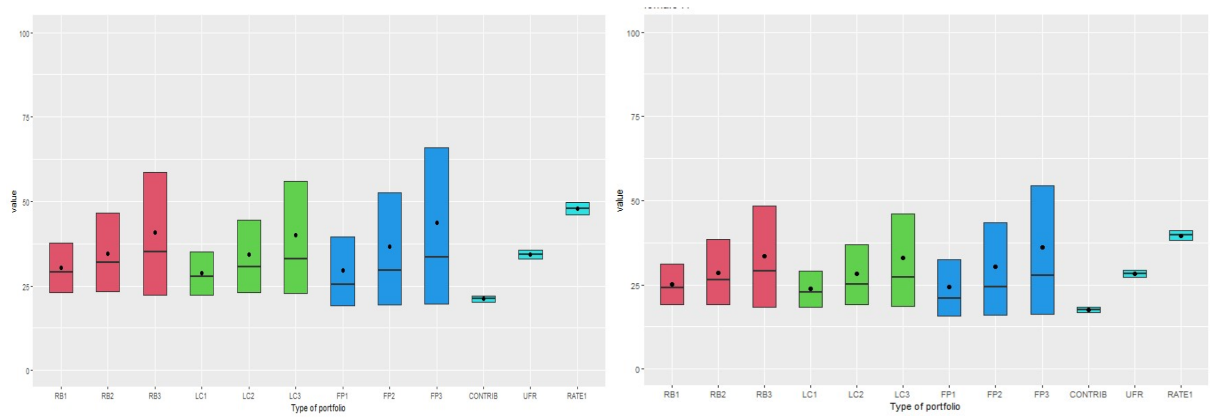

Figure 11: Replacement Rate at i_1=1% (males on left and females on right)

In the case of i_2=3%, conclusions are similar, but results display even higher dispersion than the ones obtained with i_1=1%, cf. Table 20 and Figure 12. Also, as expected, for both i_1 and i_2, the portfolios with higher equity exposure allow to achieve higher replacement rates but also present more uncertainty.

Table 20 - Replacement Rate at i_2=3%

	Male				Female			
Scen/Port	**Mean**	**15th perc**	**Median**	**85th perc**	**Mean**	**15th perc**	**Median**	**85th perc**
RB1	38	29	37	47	32	25	31	40
RB2	44	29	40	58	37	25	34	49
RB3	51	28	44	74	43	24	37	62
LC1	36	28	35	44	31	24	29	37

LC2	43	29	38	56	36	24	32	47
LC3	50	29	42	70	42	24	35	59
FP1	37	24	32	50	31	20	27	42
FP2	46	24	37	66	39	21	31	56
FP3	55	25	42	83	46	21	36	70
CONTRIB	27	25	27	28	22	21	23	23
UFR	43	41	43	45	36	35	36	38
RATE1	60	58	60	62	51	49	51	53

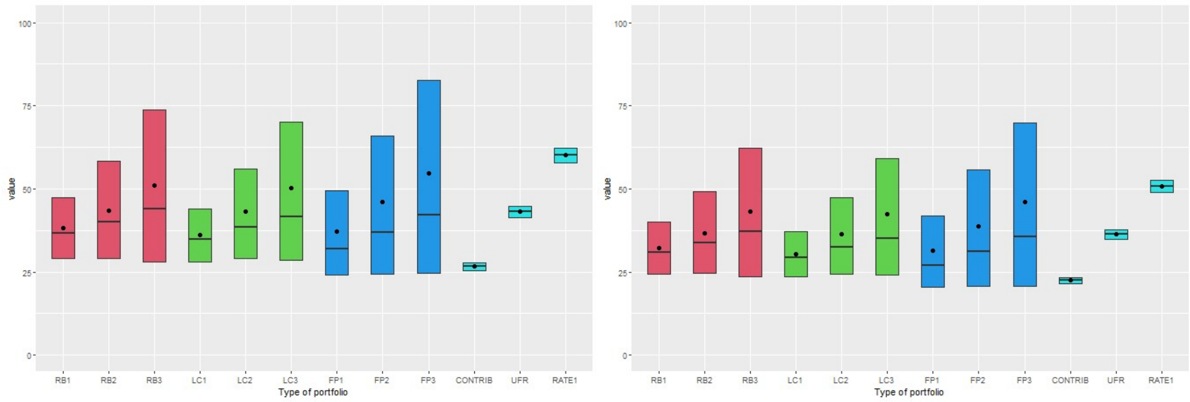

Figure 12: Replacement Rate at i_2=3% (males on left and females on right)

The replacement rates calculated based on inflation-adjusted values are similar to the ones obtained without inflation, as expected, as inflation affects equally both the numerator and denominator in (46).

5.4.5 Risk and Performance Analysis

The risk analysis starts by assessing the probability of the portfolios performing better than the CONTRIB scenario. Secondly, it analyzes what is the expected potential loss if the lump sum does not achieve at least the sum of all contributions. Finally, the standard deviation over the total contributions is computed and analyzed.

The scenarios that accommodate risk mitigation concerns (RBs and LCs) are the ones with greater probabilities to recouping the contributions made to the pension scheme (around 90%). Within these portfolios, higher exposures to equity imply lower probabilities of achieving at least the contributions made. For the same reason the fixed scenarios FP1, FP2, FP3 (no mitigation concerns) have lower probabilities to recoup contributions (Figure 13, left). The expected shortfall is consequently greater for FP1, FP2, and FP3 - and smaller for LC1, the lifecycle scenario with the lowest equity exposure (Figure 13, right). This indicator depends on the equity exposure but also on the type of portfolio.

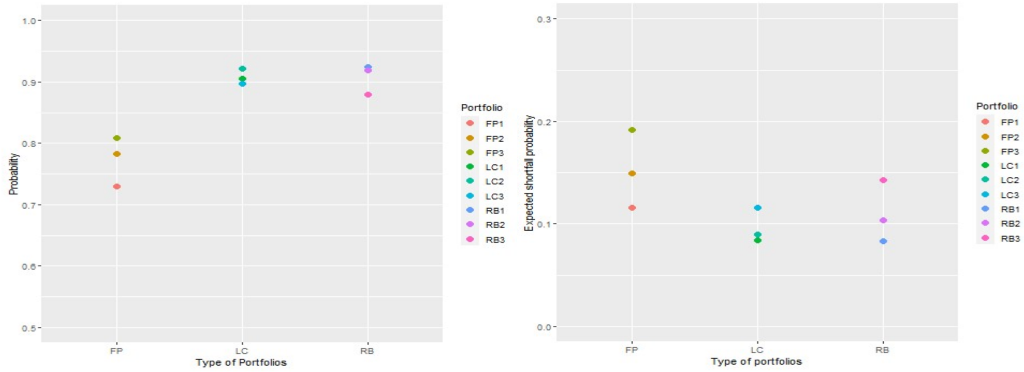

Figure 13: Probability of outperforming scenario CONTRIB (left) and expected shortfall (right)

The 5th percentile can be seen as a proxy for a worst-case setting (worse than the pessimistic setting). The 5% worst outcomes give lump sums that vary between 70% and 93% of the total contributions. Investment strategies including risk mitigation (RBs and LCs) with lower equity exposure obtain the best results. For example, in this worst-case setting, RB1 and LC1 have higher probability of recouping contributions than RB3 and LC3.

Regarding the standard deviation, the highest value is obtained for FP3. When comparing portfolios with similar equity exposure, we observe that the fixed portfolios have higher standard deviation than the lifecycle and rebalanced portfolios, and between these two, the lifecycle portfolios have higher standard deviation. That can be seen by comparing FP3 with LC3 and RB3 (or FP2 with LC2 and RB2). If we assume that the standard deviation is a proxy of the risk of a portfolio, we can conclude that the rebalanced portfolios have less risk than the lifecycle and the fixed portfolios, for the same equity exposure (Figure 14).

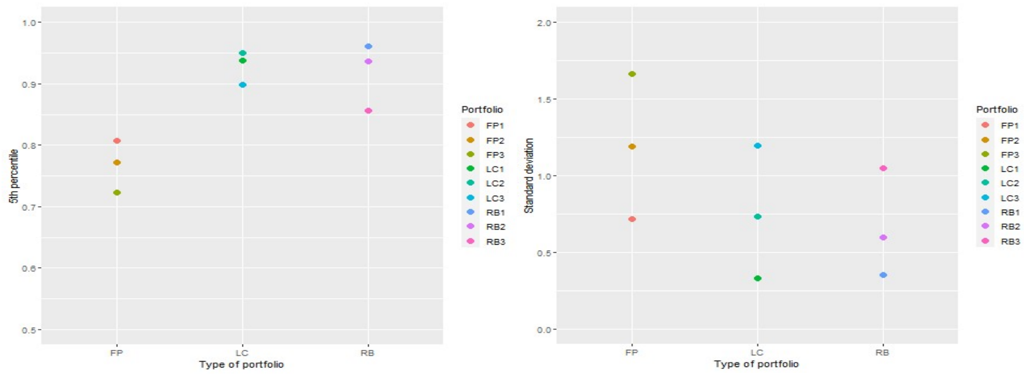

Figure 14:1 5th percentile (left) and standard deviation (right)

With respect to performance, the mean is more sensitive to extreme values than the median, as expected - see Figure 15. Naturally, in terms of ranking of the portfolios, the mean and the median produce similar results.

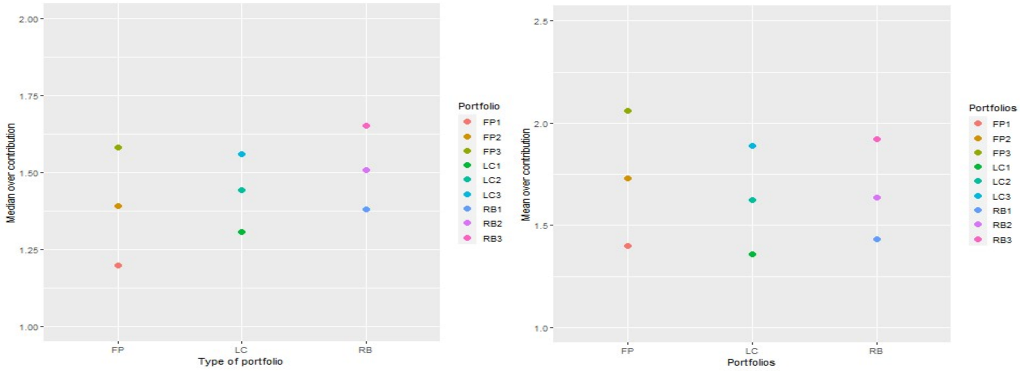

Figure 15: Median (left) and Mean (right) over the total contributions

To further measure the performance of the scenarios associated to different investment strategies, we will compare their corresponding lump sums with the ones accumulated in scenarios UFR and RATE1. Figure 16 (left) shows that the probability of the lump sum being greater than the one obtained with UFR or RATE1 increases, as equity exposure increases. Scenarios with very low equity exposure have a very low probability of reaching an average return of 3.45% (and even lower of achieving 5%, obviously). Best results are in RB3, followed by FP3 and LC3. LC1 is the portfolio with less probability of reaching 3,45% of average return. On the other hand, the scenario with greater probability of achieving a 5% average return is FP3, followed by RB3 and LC3 (Figure 16, right).

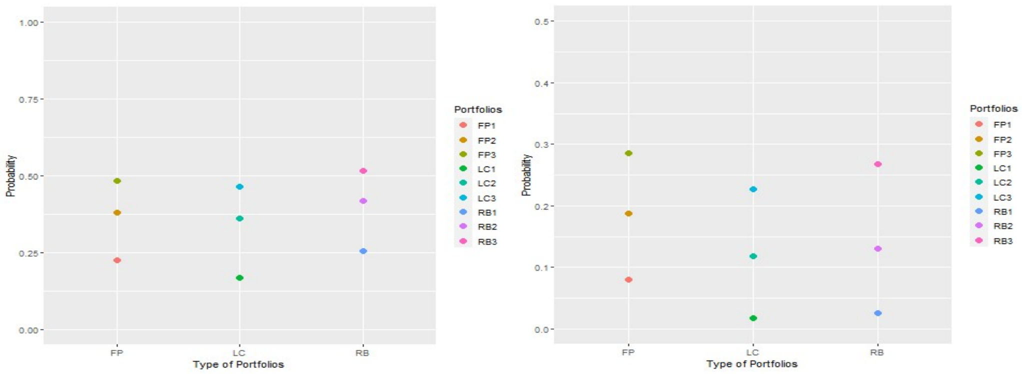

Figure 16 - Probability of lump sum greater than UFR (left) and probability of lump sum greater than RATE1 (right)

When computing jointly the mean and the standard deviation it is obvious the relationship between risk and potential performance (Figure 17, left), with higher return (measured by mean) related to higher risk (measured by the standard deviation).
The same can be seen in Figure 17 (right), where the risk is given by the 5th percentile. In the case of the fixed portfolios, we can see more clearly that the same level of performance is linked to higher risk. For the rebalanced and lifecycle portfolios, it is interesting to realize that increasing the equity exposure increases the risk.

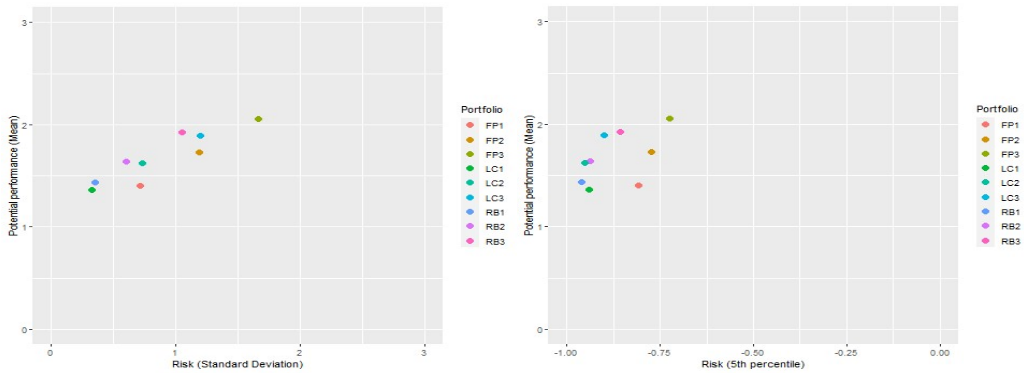

Figure 17:2 Risk-performance analysis, mean vs standard deviation (left) and mean vs 5th percentile (right)

5.4.6 Communication and explanation of results

In this section, we follow closely the European and Portuguese Regulation (European Parliament & Council, 2019[10] [14]; Law no. 27/2020 [23]).

First, before signing the contract with the pension fund management entity, savers must receive advice on the investment options that best suit their personal profiles. As stated in the Regulation, p. 6, “Advice should particularly aim at informing a saver about the features of the investment options, the level of capital protection and the forms of out-payments.” In addition to the advice, there are two standardized information documents that must be provided to savers: the “Key Information Document - KID” and the “Pension Benefit Statement - PBS”, with a clear definition of all costs involved in order to keep everything as transparent as it is intended to be.

The KID embodies pre-contractual information about the long-term objectives of the plan and the way to reach them, including a description of the underlying instruments or reference values, and in which markets the provider invests, as well as an explanation of how the return is calculated; information on the portability service; all costs associated with investments. The PBS is a document to be provided *annually*, during the accumulation phase, containing key information that takes into consideration the specific nature of national pension systems and of any relevant laws, including national social, labour and tax laws. Articles 36 and 37 of the Regulation enumerate the contents of the PBS, namely the following: the earliest date on which the decumulation phase may start; information on the contributions paid by the saver or any third party into the account over the previous 12 months; description of the costs incurred in the last 12 months (management, asset safekeeping, related to portfolio transactions and others, and estimated final costs); the nature and the mechanism of the guarantee or risk mitigation techniques, if applicable; the total amount in the account of the saver on the date of the statement; information on the past performance of the saver’s investment option covering a minimum of 10 years, or all the years for which the product has been provided, if less than 10; i*nformation on pension benefit projections based on the earliest date on which the decumulation phase may start, and a disclaimer that those projections may differ from*

[10] See https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32019R1238&from=EN

the final value of the benefits received; summary information on the investment policy relating to ESG factors; how and where to get additional information.
Moreover, concerning the presentation of results of projections to members, good practices also in the scope of the more recent proposals by the International Organisation of Pension Supervisors, IOPS[11], tend to favour the display of results in real terms, in order to help members to better understand their purchasing power after retirement. For this, assumptions on the evolution of inflation are needed. IOPS also advises that replacement rates may be presented.

6 Conclusions

The main objective of this work was to present a tool to project the benefits in the Portuguese DC pension schemes. The case study illustrates the type of analysis that can be conducted with this tool and allows to understand how the different inputs and scenarios impact on the results.
Two key points should be highlighted, especially considering the long-term nature of the projections. First, the estimation process of the stochastic models is both vital and challenging, requiring a view of the future economic development and expert judgement, to determine the reasonableness of the scenarios. It should be reviewed on a regular basis, as economic and financial environment changes. Second, the use of static mortality tables can lead to significant differences in annuity values, while projected dynamic lifetables allow to better consider the expected future mortality improvements.
The current interest rate environment and market volatility are reflected in the results. To achieve medium to high level of replacement rates, the equity exposure needs to be significant, which increases the dispersion and therefore the uncertainty around the final outputs.
Some improvements could be introduced in the model, to enable a more realistic simulation. In particular, the calculation of the lump sum only considers returns from investments in government bonds and an equity index, but adding investment returns from corporate bonds, through a credit risk model, could result in a more complete representation of the investment scenarios. On the other hand, regarding projected lifetables, further studies could include possible adjustments to the model, for instance, to consider that past observed mortality improvements will slowdown in the long-term, and some assumption on that should be set. It is important to mention the impact of Covid-19 pandemic. Covid-19 caused excess mortality both directly and indirectly by increasing deaths from other diseases. Further studies should be carried to assess the impact on mortality and longevity assumptions due to it. Also, at a different level, further research can be done, comparing this 'Eiopean' tool with alternative tools, with different modelling options for the various components.
In general, this work allowed to better understand how to make pension projections and the challenges that such projections create in terms of calculations and presentation of the results. Still,

[11] IOPS – Good Practices for designing, presenting and supervising pension projections, 2021

as the DC schemes are more and more prevalent, and also to obey legislation, the development of such tools is of utmost importance, not only in Portugal, but all over the EU.

References

1. Antolin P, Payet S (2010) Assessing the Labour, Financial and Demographic Risks to Retirement Income from Defined-Contribution Pensions. OECD Journal Financial Market Trends 2010(2), 8-8

2. Björk T (2020) Arbitrage Theory in Continuous Time. Fourth Edition, Oxford University Press

3. Brigo D, Mercurio F (2006) Interest rate models – Theory and Practice (with smile, inflation and credit). Springer Finance

4. Brouhns N, Denuit M, Vermunt JK (2002) A Poisson log-linear regression approach the construction of projected lifetables. Insurance: Mathematics and Economics 31, 373-393

5. Bosworth B, Burtless G, Steuerle CE (2000) Lifetime Earnings Patterns, the Distribution of Future Social Security Benefits, and the Impact of Pension Reform. Social Security Bulletin 63(4),74-98

6. Coelho E, Bravo J, Magalhães MG (2008) Mortality and longevity projections for the oldest-old in Portugal. Proceedings of the European Population Conference 2008, Barcelona, Spain

7. Conning (2020) A User's Guide to Economic Scenario Generation in Property/Casualty Insurance. *Casualty Actuarial Society*, CAS Research Papers.

8. Cox DR (1972) Regression models and lifetables. Journal of the Royal Statistical Society – Series B (Methodology) 34(2), 187-220

9. Damodaran A (2021) Equity Risk Premiums: Determinants, Estimation, and Implications – The 2021 Edition, NYU Stern School of Business

10. De Jong F (2000) Time Series and Cross-section Information in Affine Term-Structure Models. Journal of Business & Economic Statistics 18(3), 300-314

11. Denuit M, Goderniaux AC (2005) Closing and projecting lifetables using log-linear models. Bulletin de l'Association Suisse des Actuaires 1, 29-49

12. Duffie D, Kan R (1996) A yield-factor model of interest rates. Mathematical finance 6(4), 379-406

13. European Parliament & Council (2016) Directive 2016/2341 of the European Parliament and of the Council of 14 December 2016 on the activities and supervision of institutions for occupational retirement provision (IORP)

14. European Parliament & Council (2019) Regulation 2019/1238 of the European Parliament and of the Council of 20 June 2019 on a pan-European Personal Pension Product (PEPP)

15. EIOPA (2018) Report on the pension benefit statement: Guidance and principles based on current practices

16. EIOPA (2019) Consultation paper on the proposed approaches and considerations for EIOPA´s technical advice, implementing and regulatory technical standards under Regulation (EU) 2019/1238 on a Pan-European Personal Pension Product (PEPP)

17. EIOPA (2020) Pan-European Personal Pension Product (PEPP): EIOPA'S stochastic model for a holistic assessment of the risk profile and potential performance
18. Fergusson K, Platen E (2015) Application of maximum Likelihood Estimation to Stochastic Short Rate Models. Annals of Financial Economics 10 (2), 1-26
19. Goodman LA (1979) Simple models for the analysis of association in cross-classifications having ordered categories. Journal of the American Statistical Association 74 (367), 537-552
20. Haastrecht A, Lord R, Pelsser AJ, Schrager D (2009) Pricing Long-Maturity Equity and FX Derivatives with Stochastic Interest Rates and Stochastic Volatility. Insurance: Mathematics and Economics 45(3), 436-448
21. Hull J, White A (1993) One-Factor Interest-Rate Models and the Valuation of Interest-Rate Derivative Securities. Journal of Financial and Quantitative Analysis 28(2), 235-254
22. Lee R, Carter L (1992) Modeling and forecasting U.S. mortality. Journal of the American Statistical Association 87(419), 659-671
23. Lei nº 27/2020, de 23 de julho, Aprova o regime jurídico da constituição e do funcionamento dos fundos de pensões e das entidades gestoras de fundos de pensões, transpondo a Diretiva (UE) 2016/2341 do Parlamento Europeu e do Conselho, de 14 de dezembro de 2016
24. Pateiro D (2013) Construção de Tábuas de Mortalidade Dinâmicas para o Mercado Português de Fundos de Pensões e Avaliação do Risco de Longevidade. Dissertação de Mestrado ISEG-ULisboa
25. Svensson LE (1994) Estimation and Interpreting Forward Interest Rates: Sweden 1992-1994. Working Paper No. WP/94/114. International Monetary Fund, Washington, DC
26. SOA (2016) Economic Scenario Generator, A Practical Guide
27. Thatcher AR (1999) The long-term pattern of adult mortality and the highest attained age. Journal of the Royal Statistical Society Series A162, 5-43
28. Vasicek O (1977) An Equilibrium Characterization of the Term Structure. Journal of Financial Economics 5(2), 177-188
29. Zhang J, Sanderson A (2009) Adaptive Differential Evolution. Springer-Verlag