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The Covid-19 Recession in Germany: A Macro-Epidemiological Analysis *

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Abstract

What are the drivers of output fluctuations in Germany during the COVID-19 pandemic? We develop a macro-epidemiological model based on the evidence that efficiency and labor wedges are the key distortions in the neoclassical growth model that account for the GDP dynamics during the period. We find that the consumption and labor-supply effects of containment policies and the endogenous responses of households to pandemic-associated health risks can account for almost all weekly dynamics of output in Germany between the first quarter of 2020 and the second quarter of 2021. The containment policies are found to be responsible for especially large output losses during the pandemic, but the endogenous household responses appear to play an important complementary role. We simulate a counterfactual, laissez-faire type of response to the pandemic and find that not only would it not have avoided a sizeable recession either, but it would also lead to substantially higher losses in human life and stress on the German health service.

Keywords: Covid-19, Germany, SIR-Macro, Dynamic General Equilibrium Model, Business Cycle Accounting.

JEL Classification: C63, E27, E32, I1, H0.

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1 Introduction

Causing almost 40 thousands deaths (RKI 2022*e*) and a 5% decline in real output in 2020 (OECD 2021), the Covid-19 pandemic had a large impact on the German economy. While lockdowns imposed severe restrictions on the economic decisions of households and firms alike, individuals were exposed to a constant health risk through previously trivial economic activities, such as going shopping or working. Furthermore, global value chains were disrupted by the temporary suspension of air travel and border closures and, in the face of lockdowns and highly-volatile infection dynamics, households and firms were subjected to a considerable degree of uncertainty. Amid falling stock markets and heightened financial stress, the German government implemented fiscal stimulus packages of historical proportions while the European Central Bank (ECB) injected liquidity into the system.

Amongst all these factors, what are the main drivers of Germany's Covid-19 recession? The aim of this paper is to answer this question, focusing on output fluctuations during the first one-and-a-half year of the pandemic, i.e. we are interested in understanding the pre-vaccination dynamics. We start our journey by applying the Business Cycle Accounting (BCA) method to enlighten our search for a promising model to understand the episode. We find that the labor wedge was the central driver of German output during Covid-19 (the efficiency wedge came in a distant second place as the most important driver). Given the nature of the episode, we rely on the recent macro-epidemiological literature and extend an off-the-shelf model to include lockdown shocks.

We introduce reduced-form lockdown policies as distortionary taxes on consumption, labor, and "social interactions" in the epidemiological New Keynesian (NK) framework developed by Eichenbaum et al. (2020). These mechanisms translate themselves into output fluctuations by introducing a time-varying wedge into the households' consumption-leisure decision. The reduced-form lockdown policies are designed to match the pattern of empirical measures of containment policies by Hale et al. (2021).

Our paper complements the works of Hinterlang et al. (2021), Clemens & Röger (2021), and Funke & Terasa (2020), differing from these contributions in three fundamental aspects. Firstly, we abstract from fiscal policy interventions, the primary goal of the above-mentioned papers. Secondly, we are more interested in the weekly dynamics of the pandemic, even though we show that the model has a good fit to the data when we aggregate the output of the simulations at a quarterly frequency. Thirdly, we incorporate explicit epidemiological dynamics into the DSGE model following the newly developed literature on macro-epidemiological modeling. Given the widely recognized importance of the direct interactions between Covid-19 dynamics and macroeconomic outcomes in both the theoretical (Atkeson (2020), Eichenbaum et al. (2021), Farboodi et al. (2021)) and the empirical literature (Goolsbee & Syverson (2021), Andersen et al. (2020), Baek et al. (2021), Faria-e Castro (2021)), we complement the existing literature on the German Covid-19 recession. Furthermore, as the bulk of publications on macro-epidemiological modeling focuses on the macroeconomic effects of Covid-19 in the US,[1] our paper also broadens this general branch of the literature to a more global context.

We find that output fluctuations in Germany are primarily driven by a combination of consumption and labor-supply effects from the endogenous responses of households to Covid-19 associated health risks and economic restrictions in the form of containment policies. The model is able to replicate the observed fluctuations in key macroeconomic and epidemiological variables with substantial accuracy. While the model identifies the back-and-forth between aggravation and relaxation of containment policies as the driver of especially severe output losses, it finds that the endogenous responses of households to the evolution of Covid-19 infection risks played a complementary role.

Given the performance of the model in replicating the dynamics of macroeconomic variables in Germany, we raise an additional question: what if there were no containment policies? We use the model to study the costs and benefits of containment policies. We find that even though the scenario with lockdowns imposes a more severe economic recession (in the counter-factual no-lockdown exercise, in the absence of containment policies, endogenous household responses would still produce a sizeable recession, though a milder one), we also find that it is also a situation in which more human lives are saved (the benefits).

The remainder of the paper is organized as follows: Section 2 provides an overview of the literature on macro-epidemiological modeling, as well as on the macroeconomic effects of Covid-19 in Germany. Section 3 briefly outlines the BCA method and its results for Germany. Section 4 provides a detailed description of the DSGE model, whose calibration is detailed in Section 5 and is used in the quantitive exercises whose results (and potential limitations) are presented in Section 6. We dedicate Section 7 to our final remarks.

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[1] See for example Acemoglu et al. (2020),Ascari et al. (2021),Brotherhood et al. (2020),Crucini & O'Flaherty (2020),Eichenbaum et al. (2021),Eichenbaum et al. (2022*b*),Eichenbaum et al. (2022*c*),Farboodi et al. (2021), Jones et al. (2021),Kaplan et al. (2020),Lepetit & Fuentes-Albero (2022)

2 Macro meets epidemiology

The Covid-19 crisis differs from previous economic crises along at least two fundamental dimensions. Being a highly infectious, airborne disease, the risk of a Covid-19 infection and potential, serious health complications became an omnipresent feature of any activity involving human interaction. As a substantial share of economic transactions relies on such interaction, especially in the services (e.g. travel and recreation sectors), this presented individuals with a constant trade-off between performing basic economic functions and possibly severe health consequences. Furthermore, in the absence of any effective vaccine or treatment at the beginning of the pandemic, policymakers were confronted with a tension between "saving lives and saving livelihoods" (Kaplan et al. 2020), as they had to design containment policies weighing economic losses against potential losses in human life.

As conventional macroeconomic models were unable to quantify these trade-offs between economic activity and health risks faced by both individuals and policymakers, economists quickly recognized the need for adaptation in those models (e.g. Atkeson (2020)). The recent macro-epidemiological literature merges a broad range of DSGE models with different variations of Kermack & McKendrick's (1927) canonical SIR framework. The intersection between macroeconomic and epidemiological dynamics enabled researchers to analyze the outcomes and individual decision-making during the pandemic, as well as to formulate optimal containment policies while taking into account both potential economic and human losses.

Eichenbaum et al. (2021, 2022*a*) are examples of such contributions. The authors link the epidemiological structure of a discrete-time SIR model to the consumption and labor-supply decisions of a representative household in an otherwise standard Real Business Cycles and New Keynesian environments. Their work was extended by Acharya et al. (2020), Ascari et al. (2021), Crucini & O'Flaherty (2020), Giagheddu & Papetti (2020), Krueger et al. (2022) and Rubini (2020). While Ascari et al. (2021) and Krueger et al. (2022) append Eichenbaum et al.'s (2021,2022*a*) model to analyze the role of sectoral heterogeneity in the transmission of the Covid-19 crisis, Crucini & O'Flaherty (2020) and Acharya et al. (2020) investigate the role of geographical disparities. Similarly, Giagheddu & Papetti (2020) analyze the importance of age-heterogeneity, whereas Rubini (2020) assesses the differential response of households to the pandemic in low-income countries. The reader could also refer to Acemoglu et al. (2020), Alvarez et al. (2020), Brotherhood et al. (2020), Farboodi et al. (2021), Jones et al. (2021), and Kaplan et al. (2020) for different approaches.

The literature on the Covid-19 crisis at the time we write this paper focused more on the US, with a few exceptions concerned with other regions such as Krueger et al. (2022), Rubini (2020), Cakmakli et al. (2020), Giagheddu & Papetti (2020) and Alon et al. (2020).

Analyzing the Swedish laissez-faire response to the pandemic, Krueger et al. (2022) show that, in the absence of containment policies, sectoral heterogeneity in infection risks can function as a potent mechanism to avoid large economic and human losses. Investigating the impact of Covid-19 on emerging economies by looking at the example of Turkey, Cakmakli et al. (2020) find that open-economy channels can substantially amplify the economic costs of domestic containment policies. For instance, due to lower working-from-home capacities and subsistence levels in consumption, households in low-income countries react significantly differently to the pandemic than those in advanced economies (Rubini 2020). However, there is some evidence that conventional lockdowns are less effective in developing countries (Alon et al. 2020). Finally, Giagheddu & Papetti (2020) point out the importance of age-heterogeneity for the optimal design of containment policies by analysing the impact of Covid-19 in Italy.

Clemens & Röger (2021), Funke & Terasa (2020), and Hinterlang et al. (2021) analyse the German Covid-19 recession through the lens of DSGE models. Clemens & Röger (2021) employ a deterministic, non-linear DSGE model featuring a zero-lower bound with the objective of evaluating the effectiveness of Germany's temporary VAT cut as a fiscal stimulus measure during the pandemic. They find that the temporary VAT cut, which marks Germany's largest stimulus measure in fiscal terms, indeed helped to stabilize output through its significant effect on durable goods consumption. Nevertheless, Clemens & Röger (2021) also report that, besides the cushioning effects of the VAT cut, lockdowns can be regarded as the main driver of Germany's severe output losses during 2020 and 2021. Funke & Terasa (2020) reach a similar conclusion regarding the overall effectiveness of Germany's VAT policy in stimulating output while finding lower effects on consumption demand. However, their analysis refrains from a distinction between durable and non-durable goods.

Finally, developing a large-scale DSGE model featuring sectoral heterogeneity, financial frictions, trade, and many other frictions, Hinterlang et al. (2021) provide a comprehensive analysis of the entire fiscal package adopted by the German government. In line with the results by Clemens & Röger (2021) and Funke & Terasa (2020), their study reveals that Germany's temporary VAT reduction helped to stabilize consumption demand and output. Moreover, Hinterlang et al. (2021) find that while large subsidy payments to especially affected industries temporarily decreased firms' default probabilities, their cushioning effect on output losses was negligible. From a cost-effectiveness standpoint, they argue that public investment has proven to be the most impactful fiscal response to the pandemic.

Bauer & Weber (2021) estimate the causal effects of lockdowns on unemployment at the onset of the pandemic with difference-in-difference estimators. They find that the majority of the flow from employment to unemployment (60%) was indeed the result of lockdown policies. However, this flow was considerably reduced by Germany's employee retention scheme *Kurzarbeit*, as it lowered unemployment by up to 3 percentage points during the trough of the recession (Aiyar & Dao 2021). Covid-19 initially lead to a sizeable reduction in earnings across all income classes, but Germany's fiscal response to the pandemic caused an increase in the income of low-income households and reduced that of high-income ones (Bruckmeier et al. 2021, Christl et al. 2022).

3 Business Cycle Accounting Analysis

We start our journey agnostic in the sense that we impose as little theoretical structure as possible. With that in mind, we apply the BCA method developed by Chari et al. (2007) to identify the promising model specifications that are relevant to analyzing the sources of macroeconomic fluctuations during the Covid-19 pandemic in Germany. The method has two dimensions: the accounting procedure and the equivalence procedure. We focus our attention on the former.

We start with a "prototype economy," the neoclassical growth model, with four time-varying distortions in its equilibrium conditions. These distortions look like (but they are not, necessarily) a tax on labor income (labor wedge), a tax on investment (investment wedge), aggregate productivity (efficiency wedge), and a term in the aggregate resource constraint (government wedge).

We rely on observed data on output, hours worked, investment, government consumption, and net exports to estimate the four wedges.[2]

Wedges in Germany

Starting from the prototype economy and its respective distortions, we estimate the wedges for Germany between the first quarter of 2020 and the

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[2] Output was adjusted for taxes on consumption and the consumption of durable goods. See Brinca et al. (2020) for a detailed explanation of the method and a comprehensive summary of results regarding the role of each wedge in different episodes, the detailed frictions in a fully specified model that can be related to each of the wedges within the prototype economy (the equivalence procedure), and the extensions of the method.

Table 1: Relative contributions of the wedges to output changes in Germany during Covid

ϕ_e	ϕ_l	ϕ_i	ϕ_g
Individual Wedges			
18%	66%	8%	8%
All-but-one Wedge			
7%	2%	25%	66%

Notes: This table presents the ϕ-statistics, the relative contribution of each wedge with two sets of simulations: individual wedges and all-but-one wedge. In the former, we simulate the model with only one wedge varying and the others remaining fixed at their steady-state levels. In the latter, we simulate the model with only one wedge constant at its steady-state level; ϕ_e is for the efficiency wedge, ϕ_l for the labor wedge, ϕ_i for the investment wedge, and ϕ_g for the government wedge.

second quarter of 2021. A detailed description of the underlying data can be found in Appendix A.

In Table 1 we present the ϕ-statistics for output, which measures the relative contribution of each wedge in the total variation of output. By construction, the four wedges account for all output variation. We measure how much a specific wedge in isolation (individual wedges), accounts for the dynamics of output by simulating the model allowing only that wedge to vary (the others remain constant at their steady-state level). Analogously, we can simulate a combination of three wedges (all-but-one wedge) by holding one wedge fixed at its steady-state value and feeding into the model the estimated path for the other three.

As can be seen in Table 1, the variation in output is dominated by the influence of the labor (ϕ_l) wedge, as it individually explains about 66% of observed output fluctuations. Moreover, explaining about 18% of total fluctuations, the efficiency wedge (ϕ_l) appears to be of moderate importance, while the role of the investment (ϕ_i) and the government wedge (ϕ_g) is quite low, with each accounting for only about 8% of total output variation.

Inverting the analysis by simulating the prototype economy with all but one wedge at a time, yields a similar conclusion. The model without the changes in the government wedge accounts for almost 66% of the output dynamics, whereas if we remove the investment, the efficiency or the labor wedge the model can only explain 25%, 7% or 2%, respectively.

In summary, according to the BCA exercise, the labor wedge is the most important wedge to explain German output dynamics during Covid-

19. Hence, models that exhibit frictions that manifest themselves as a time-varying distortion to the consumption-leisure decisions of households are promising candidates for explaining output fluctuations in Germany during the pandemic.

Furthermore, the result that the government wedge only explains a small fraction of output variation indicates that frictions associated with an open economy and government spending dynamics are of relatively little relevance, as these two channels are typically associated with the government wedge. This observation also provides valuable insight into the relevant classes of frictions for the analysis of the German Covid-19 recession, given the traditionally strong dependency of Germany on its exports.

We are aware that an open-economy framework is usually used for modeling the German economy. However, due to the BCA results, we consider that we can rely on a different setup in this paper (the one presented in the next section). Of course, we also acknowledge that being able to omit this feature in the case of the Covid-19 recession greatly simplifies the analysis.

4 The Model

We extend a representative-agent, DSGE model with complete capital markets, monopolistically-competitive firms, exogenous government spending (with a balanced budget at all periods), with physical capital, and an epidemiological block in the form of a modified version of Kermack & McKendrick's (1927) canonical SIR model developed by Eichenbaum et al. (2020), in order to include containment policies.

Since this extension only alters the model's household structure, the next section focus on the description of the epidemiological dynamics and the household's problem. We present the complete derivation of the model in Appendix B.

4.1 Epidemiological Dynamics

The SIR framework is a dynamic, epidemiological model that analyses the propagation of an infectious disease within a given population (Kermack & McKendrick 1927). It begins by assuming that at time t = 0, an initial share of a population is infected with a contagious disease. The population is thus divided into four sub-groups: individuals that are either susceptible to the disease, currently infected with it, or that have recovered or died from it. The relative size of these sub-groups is assumed to evolve dynamically over time, as susceptible individuals eventually become infected, and infected ones, after remaining as such for some time, either recover or die. The dynamic evolution of the sub-groups sizes is described through a set of interdependent difference equations that reflect the disease's fundamental properties regarding its contagiousness and lethality:

$$S_{t+1} = S_t - T_t, \tag{1}$$

$$I_{t+1} = I_t - (\pi_r + \pi_d) I_t + T_t, \tag{2}$$

$$R_{t+1} = R_t + \pi_r I_t, \tag{3}$$

$$D_{t+1} = D_t + \pi_d I_t, \tag{4}$$

where S_t, I_t, R_t and D_t denote the shares of susceptible, infected, recovered, and deceased individuals in the *total* population, respectively; π_r and π_d are parameters describing the probabilities of individuals either recovering or dying from the disease in each period and T_t denotes the share of *newly* infected population members.

Following the exogenous infection of an initial share (I_0) of the population at time $t = 0$, equations (1) to (4) denote the laws of motion which govern the development of aggregate health conditions in the population across periods. Equation (1) simply states that next period's share of susceptible population members is composed of the current share of susceptibles minus the proportion of newly infected in the current period. According to equation (2), the share of infected individuals in the following period is composed of the shares of already and newly infected ones in the current period minus the proportion of infected individuals that either recover or die during this period.

Equations (3) and (4) imply that the cumulative shares of recovered/deceased population members next period simply consist of the shares of recovered/deceased individuals in the current period plus the shares of newly recovered/deceased. Equations (1) to (4) are related to the development of the pandemic. But how is this related to (macro)economic decisions?

According to equation (5), new infections in the economy are generated by three additively separable terms, with each of these terms carrying a different assumption about how the pandemic is able to spread among the economy's population:

$$T_t = \pi_1 \left(S_t C_t^S \right) \left(I_t C_t^I \right) + \pi_2 \left(S_t N_t^S \right) \left(I_t N_t^I \right) + \pi_3 \left(S_t I_t \right) \left(1 - \mu_t^l \right). \tag{5}$$

The first expression states that susceptible members of the population can be infected through consumption activities. The magnitude by which this occurs is given by the term of the aggregate consumption of susceptible individuals, $S_t C_t^S$, with the total consumption of infected individuals, $I_t C_t^I$, and the parameter π_1. Following Eichenbaum et al. (2020), the parameter π_1 is related to both the time spent on consumption activities, as well as the probability of becoming infected through such.

Similarly, the second term in equation (5) relates workplace interactions to the creation of new infections. Susceptible individuals are assumed to meet infected ones at work, leading to new infections proportional to the product of the aggregate labor supply of susceptibles, $S_t N_t^S$, with the aggregate labor supply of infected, $I_t N_t^I$, and the parameter π_2. Here, the parameter π_2 denotes the "pure" probability of becoming infected in the workplace, as time spent on work is already captured by the fact that labor supply is expressed in hours worked.

Finally, the third term of equation (5) rationalizes that apart from working and consuming, susceptible population members can also be infected through social interactions with infected individuals that are unrelated to economic activity. Thereby, the term $S_t I_t$ describes the magnitude of these interactions, whereas π_3 governs the probability of infection through such meetings. The final expression of the term, $(1 - \mu_t^l)$, is related to potential government interventions that aim at reducing these types of interactions between individuals.

In summary, equation (5) integrates epidemiological and economic dynamics by specifying the share of newly infected individuals as a non-linear function of the aggregate consumption and labor supply of susceptible and infected individuals. The fundamental assumption that underlies this specification is that both consumption and work activities involve a degree of physical proximity between population members that facilitates the propagation of an infectious disease within the economy.

4.2 The Household's Problem

Following Eichenbaum et al. (2020), the model features an infinitely-lived, representative household which is normalized to measure one in size. Its members are divided into different groups based on their respective health status, with s_t, i_t, and r_t denoting the share of the *household's* members that are either susceptible, infected, or recovered at time t. We assume that the household consumes and supplies labor contingent on the respective health conditions of its members. Therefore, its consumption (c_t^s, c_t^i, c_t^r) and labour supply (n_t^s, n_t^i, n_t^r) are differentiated by the respective superscripts r, i, s. Deceased individuals do not enter the household structure anymore, as they can neither supply labor nor consume.

The household *as a whole* is further assumed to invest in capital k_t and bonds B_t, which yield a nominal interest rate of R_t^k and R_{t-1}^b, as well as paying lump-sum taxes ψ_t and receiving the profits (ϕ_t) of monopolistically competitive firms. Additionally, it is subjected to distortionary taxes on labor (μ_t^n) and consumption (μ_t^c), as well as restrictions on random interactions ($1-\mu_t^l$), which altogether serve as reduced-form representations of containment policies. They will be discussed in detail in Section 4.4.

The household's maximization problem can be written as follows:

$$\begin{aligned}\max_{c_t^s,c_t^i,c_t^r,n_t^s,n_t^i,n_t^r,s_{t+1},i_{t+1},r_{t+1},\tau_t,k_{t+1},B_{t+1}} \quad & U = \sum_{t=0}^{\infty}\beta^t\Big\{s_t\big(log(c_t^s) - \frac{\theta}{2}(n_t^s)^2\big) \\ & + i_t\big(log(c_t^i) - \frac{\theta}{2}(n_t^i)^2\big) + r_t\big(log(c_t^r) - \frac{\theta}{2}(n_t^r)^2\big)\Big\},\end{aligned}$$

subject to:

$$\begin{aligned}(1+\mu_t^c)P_t(s_tc_t^s + i_tc_t^i + r_tc_t^r) + P_tx_t + \psi_t + B_{t+1} = \\ (1-\mu_t^n)W_t(s_tn_t^s + i_tn_t^i + r_tn_t^r) + R_t^kk_t + R_{t-1}^bB_t + \phi_t,\end{aligned} \tag{6}$$

$$k_{t+1} = x_t + (1-\sigma)k_t, \tag{7}$$

$$\tau_t = \pi_1 s_t c_t^s(I_tC_t^I) + \pi_2 s_t n_t^s(I_tN_t^I) + \pi_3 s_t I_t(1-\mu_t^l), \tag{8}$$

$$s_{t+1} = s_t - \tau_t, \tag{9}$$

$$i_{t+1} = i_t + \tau_t - (\pi_r + \pi_d)i_t, \tag{10}$$

$$r_{t+1} = r_t + \pi_r i_t, \tag{11}$$

where β denotes the psychological discount factor of the household, θ a labor-scaling parameter, and σ the weekly depreciation rate of capital; P_t and W_t refer to the aggregate price level and the nominal wage, respectively, whereas x_t denotes investment and τ_t is the share of newly infected individuals in the population at each period.

The household's budget constraint is defined by equation (6) while equation (7) defines a standard law of motion for capital. Equations (8) to (11) denote the epidemiological dynamics described in the previous section applied to the household level.

A central assumption of the model is that its representative household is not passively subjected to the pandemic, but can actively choose its development by optimizing its behavior with regard to the variables s_{t+1}, i_{t+1},r_{t+1} and τ_t. As these four variables refer to the share of infected individuals on a *household level*, this implies that the household is aware of the health statuses of its own members and is able to influence them through its respective behavior. This can arguably be regarded as an admissible assumption since individuals *within a given household* should be able to observe and coordinate their respective health conditions.

However, in contrast to its ability to choose epidemiological variables within itself, the individual household takes the aggregate variables S_{t+1},I_{t+1}, R_{t+1},T_{t+1} as given. Nevertheless, due to the assumption of a representative household, it follows after aggregation that $s_{t+1} = S_{t+1}$, $i_{t+1} = I_{t+1}$, $r_{t+1} = R_{t+1}$ and $\tau_{t+1} = T_{t+1}$. Hence, even though epidemiological variables on the household and the aggregate level are conceptually different, they are quantitatively equivalent.

Related to the conceptual distinction between the household and the aggregate, it should also be noted that in equation (8), I_tC^I and I_tN^I refer to the aggregate consumption and labor-supply of infected individuals in the economy. This implies that new infections are generated not only by the interaction of susceptible individuals with infected ones within their own household but by the interaction of susceptibles with infected ones in the entire economy.

Initial Contagion

Following Eichenbaum et al. (2020), we assume that initially, the economy is at a steady state with $s_t = 1$ and $i_t = r_t = d_t = 0$. Then, an exogenous share $i_0 = \epsilon$ of the household's population becomes infected at some initial period $t = 0$. As of then, the pandemic starts running its course, dividing the household according to the health status of its members and forcing it to adjust its optimal behaviour with regard to the additional variables s_t, i_t, r_t, τ_t and the additional equations (8) to (11). Starting from this initial shock to the households behavioural structure triggered by $i_0 = \epsilon$, the model simulates the economy's transition from the aforementioned pre-pandemic to an endogenous, post-pandemic steady-state in a perfect-foresight, non-linear framework. The frequency of the model in this simulation framework is calibrated such that one period in the model corresponds to one week.

"Distorted" optimal choices

In order to illustrate the model's main mechanism, we focus on the first-order conditions with respect to consumption (c_t^s) and labor supply (n_t^s) of susceptible individuals, which are defined by equations (12) and (13), respectively:

$$\frac{1}{c_t^s} = \tilde{\lambda}_t^b(1 + \mu_t^c) - \lambda_t^\tau \pi_1(I_t C_t^I), \tag{12}$$

$$\theta n_t^s = \tilde{\lambda}_t^b(1-\mu_t^n)w_t + \lambda_t^\tau \pi_2(I_t N_t^I), \tag{13}$$

where $\tilde{\lambda}_t^b$ denotes the scaled Lagrange-multiplier of the households budget constraint and w_t refers to the real wage.[3]

Equations (12) and (13) demonstrate that the presence of explicit, epidemiological dynamics in the household's optimization problem introduces distortions (wedges) into the consumption and labor supply conditions for susceptibles in the form of $\lambda_t^\tau \pi_1(I_t C_t^I)$ and $\lambda_t^\tau \pi_2(I_t N_t^I)$. From their definition, it is easy to observe that if I_t increases, *ceteris paribus*, these distortions increase as well.

Following equation (12), an increase in $\lambda_t^\tau \pi_1(I_t C_t^I)$ reduces c_t^s, as with a rise in the right-hand side of the equation (given that $\lambda_t^\tau < 0$), c_t^s has to decline for the optimality condition to hold again. By an equivalent logic, equation (13) implies that an increase in $\lambda_t^\tau \pi_2(I_t N_t^I)$ triggers a decrease in n_t^s.

Consequently, while consumption and labor supply decisions of the household feed into the development of the pandemic through equation (8), the pandemic feeds back into macroeconomic outcomes through equations (12) and (13). Following Eichenbaum et al. (2020), this feedback effect of the pandemic on household behavior can be interpreted as voluntary self-containment. When infections in the economy are rising, the risk of contracting the virus through consumption- and work-related activities also temporarily increases. As the possibility of contracting the virus and eventually dying from it implies a loss of future lifetime utility, susceptible individuals respond by *voluntarily* reducing their consumption and labor supply to mitigate the elevated infection risk.

From these simultaneous consumption and labor supply effects of voluntary mitigation behavior, it follows that Covid-19 affects the economy through a combination of negative demand and supply shocks. These shocks have opposing effects on investment. While the negative demand shock resulting from a decrease in consumption stimulates investment, the negative supply shock in the form of reduced labor supply decreases it. In the quantitative exercises of Section 5, the negative supply shock dominates as in Eichenbaum et al. (2020), implying that the Covid-19 pandemic is also associated with a significant decrease in investment.

With consumption, hours worked and investment all declining simultaneously, it ultimately follows that output decreases as well. As this fall in output is the result of the initial consumption and labor supply responses of susceptibles to Covid-19 associated health risks, the model rationalizes that

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[3] The scaling factor of $\tilde{\lambda}_t^b$ is the aggregate price level. While λ_t^b denotes the "pure" Lagrange-multiplier, $\tilde{\lambda}_t^b$ is defined as $\tilde{\lambda}_t^b = \lambda_t^b P_t$.

negative output movements in response to the pandemic are primarily driven by the voluntary containment behaviour of households.

This mechanism is in line with the results of the BCA exercises. Equations (12) and (13) yield the following equilibrium condition:

$$-\frac{U_{n_t^s} - s_t \lambda_t^\tau \pi_2 (I_t N_t^I)}{U_{c_t^s} + s_t \lambda_t^\tau \pi_1 (I_t C_t^I)} = \frac{(1-\mu_t^n)}{(1+\mu_t^c)} mc_t AF_{N,t}, \tag{14}$$

where $U_{n_t^s}$ and $U_{c_t^s}$ denote the marginal utilities of consumption and labor of susceptible individuals, $AF_{N,t}$ refers to the marginal product of labor, and mc_t denotes the marginal costs of intermediate-good firms. Equation (14) is derived by eliminating the Lagrange-multiplier $\tilde{\lambda}_t^b$ in equations (12) and (13), and substituting the real wage rate w_t with the labor demand equation of intermediate-good firms.[4] Abstracting from the term $\frac{(1-\mu_t^n)}{(1+\mu_t^c)}$ for a moment, which is the result of incorporating consumption and labor income "taxes" into the model, equation (14) illustrates an important property of the model's endogenous containment mechanism. It shows that, together with the marginal-cost term, voluntary containment drives a wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor. Hence, it introduces frictions into the household's consumption-leisure decision, which are conceptually similar to those associated with a prototypical labor wedge in the BCA analysis. Given that, in conjunction with the efficiency wedge, the labor wedge was found to be the most significant driver of German GDP dynamics. This property of the model qualifies it as a potentially promising candidate to explain German output fluctuations during Covid-19.

4.3 Government

The government is assumed to finance a constant stream of public consumption (G) through its revenues from lump-sum (ψ_t) and distortionary taxes (μ_t^c, μ_t^n). Its budget constraint in nominal terms is defined as

$$\psi_t + \mu_t^c P_t C_t + \mu_t^n W_t N_t = P_t G, \tag{15}$$

From the assumption of a constant government consumption stream in equation (15), it follows that when revenues through distortionary taxes increase, lump-sum taxes must decrease by an equal amount. This implies that all revenues from distortionary taxes are rebated as a lump-sum to the household. Consequently, while μ_t^c and μ_t^n affect the household's behavior

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[4] See equation 43 in the Appendix.

by changing the relative costs of its consumption and labor supply decisions, they leave government expenditures unaffected. This specification is similar to the one used by Eichenbaum et al. (2021).

In addition to a governmental authority directing fiscal policy measures, the model also features a monetary authority that conducts interest rate policies according to the following Taylor rule:

$$log\Big(\frac{R_t^b}{R^b}\Big) = r_\pi log\Big(\frac{\pi_t}{\pi}\Big) + r_x log\Big(\frac{y_t}{y_t^f}\Big), \tag{16}$$

where R_t^b denotes the nominal interest rate on bonds, π_t refers to the gross inflation rate, y_t denotes real output, and y_t^f represents hypothetical real output in a flexible-price economy. Furthermore, R^b and π refer to the pre-pandemic steady-state values for the interest rate on bonds and inflation, respectively, and parameters r_π and r_x govern the magnitude by which monetary policy responds to deviations in output and inflation from their corresponding (flexible-prices) steady-state values.

Following equation 16, the monetary authority adjusts the interest rate on bonds (R_t^b) relative to its target value (R^b) proportional to the percentage change of gross inflation (π_t) relative to its desired value (π) and the percentage difference between actual output y_t and its hypothetical value in a flexible-price economy y_t^f.

We are aware of the fact that Germany has no autonomous monetary policy. However, given its size within the euro area, we consider this assumption a better approximation than for a smaller euro-area country (i.e. we assume that the ECB reacts more to German output gaps and inflation gaps than, for instance, to the Portuguese ones).

4.4 Containment Policies

We incorporate containment policies into the model through a *combination* of the distortionary taxes μ_t^c and μ_t^n, as well as the "tax" on social interactions μ_t^l. As can be seen in 15, these taxes enter the government budget constraint. However, as Government spending is assumed to be constant, all revenue from these taxes is rebated to the households via the lump-sum tax T_t, which is similar to the formulation of containment policies in Eichenbaum et al. (2021). By modelling containment policies specifically as distortionary taxes on consumption and labor we assume that similar to the endogenous containment behavior of individuals, those policies affect aggregate output primarily through their consumption and labor supply effects. This property is clearly illustrated by the first-order conditions with respect to consumption and labor supply of infected and recovered individuals:

$$\frac{1}{c_t^j} = \tilde{\lambda}_t^b(1+\mu_t^c), \tag{17}$$

$$\theta n_t^j = \tilde{\lambda}_t^b(1-\mu_t^n)w_t, \tag{18}$$

where the superscript $j \in \{i, r\}$ refers to infected/recovered individuals. It is easy to see that an increase in μ_t^c, *ceteris paribus*, leads to a decline in the consumption of all individuals. Analogously, increases in μ_t^n negatively affect labor supply. Finally, these consumption and labor supply effects are complemented by the term μ_t^l, which directly reduces the number of social interactions $s_t I_t$ in equation (8).

The representation of containment policies by the variables μ_t^c, μ_t^n, and μ_t^l is supported by mobility data, which illustrates how containment policies affected economic and social activity in Germany. Firstly, by closing bars and restaurants or confining people to their home (Bundesregierung 2022), individuals were *physically* restrained from consuming certain types of goods and services, which is reflected in the top panel of Figure 1. It shows that the implementation of general lockdowns in Germany (as marked by the orange bars) coincided with a steep decrease in consumption traffic in the retail and recreation sectors, relative to a pre-Covid baseline. As not all goods and services in the retail and recreation sector have an equivalent, non-physical substitute, these forced reductions in physical consumption traffic imply that containment policies imposed temporary, quantitative restrictions on the consumption possibilities of individuals.

Secondly, the bottom panel of Figure 1 shows that strict containment policies in Germany were further associated with a strong decline in workplace traffic. Since, according to Dingel & Neiman (2020), only a subset of jobs in Germany can effectively be performed from home. These forced reductions in physical workplace presence also imply a quantitative restriction on the labor hours that individuals were able to supply.[5] Nevertheless, as containment policies in Germany also specifically restricted the number of people allowed within a 2-meter radius of each other, independent of their respective location (Bundesregierung 2022), Figure 2 also implies a relationship between containment policies and restricting social interactions that were unrelated to economic activity.

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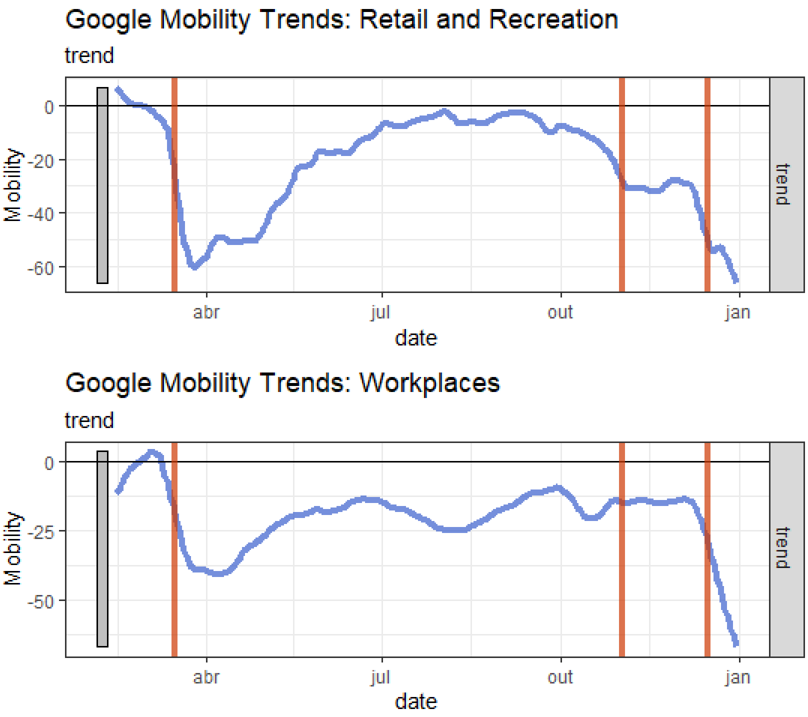
[5] Figure 2 illustrates that according to the Covid-19 Mobility Project (2022), average social contacts of individuals in Germany also experienced a drastic decline in response to lockdowns. This observation reflects to a certain extent the reductions in consumption and workplace traffic since workplace interactions constitute a significant amount of individuals' daily social contacts.

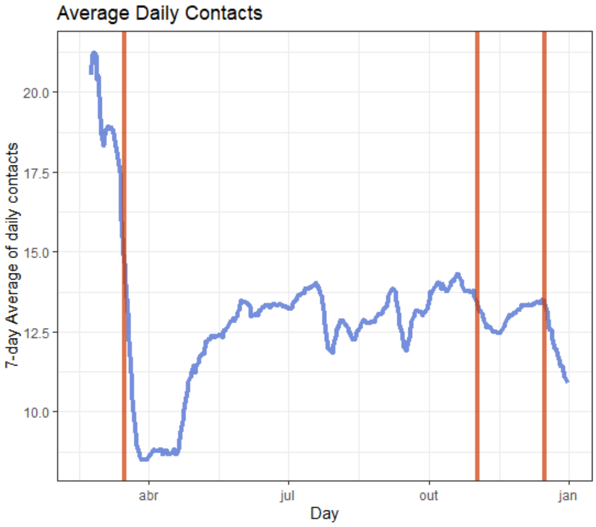
Figure 1: Consumption and Labor Mobility in Germany: January to December 2020



In summary, based on their impact on economic and social mobility, containment policies in Germany can be understood as temporary, quantitative restrictions which simultaneously reduce consumption, labor hours, and social activity.

With an increase in the price of consumption goods through μ_t^c, households can afford less of these goods for any fixed budget which, following equations (12) and (17) reduces consumption. This mechanism is approximately equivalent to a direct, quantitative restriction, as regardless of whether the household is physically restricted from buying the goods or simply cannot afford them, it is constrained in its consumption possibilities. A similar mechanism applies for μ_t^n. With an increase in μ_t^n, households have to give up relatively more leisure to achieve a certain amount of labor income. Since giving up leisure implies disutility, a rise in μ_t^n increases the costs of supplying labor, which ultimately reduces its supply via equations (13) and (18). Hence, the household is again constrained in its choices, as the amount of labor that was optimal prior to the tax is now "unaffordable" due to the increase in its associated utility costs. Finally, the term μ_t^l has a more straightforward interpretation as, since opposed to rendering some consumption-leisure choices unaffordable, it directly restricts social interactions by diminishing the term

Figure 2: Social Contacts in Germany: January to December 2020



$s_t I_t$.

4.5 Containment Policies and the Labour Wedge

Apart from matching empirical features of containment policies, the theoretical representation of them as simultaneous consumption and labor-supply restrictions also fits the results of the BCA exercise. This follows from combining the first-order conditirions with respect to consumption and labor supply of infected and recovered individuals into the respective, single equilibrium condition expressed by equation (19):

$$\frac{U_{n_t^j}}{U_{c_t^j}} = \frac{1-\mu_t^n}{1+\mu_t^c} mc_t A F_{N_t}, \tag{19}$$

where $U_{n_t^j}$, $U_{c_t^j}$ with $j \in \{i, r\}$ denote the marginal utilities of consumption and labor supply of susceptible or infected individuals.

Equation (19) is derived analogously to equation (14). It illustrates that by introducing a distortion into the relationship between the marginal rate of substitution of consumption of labor, and the marginal product of labor, containment policies resemble a labor wedge for infected and recovered individuals.

A similar observation follows from equation (14) for susceptible individuals. However, for susceptible individuals, the frictions induced by containment policies mix with those induced by voluntary containment to form one composite labor wedge. Furthermore, given the negative effects of μ_t^c, μ_t^n, and μ_t^l on consumption, labor supply, and social interactions, and the direct link of infections with these variables, it follows that, within this composite wedge, exogenous and endogenous containment effects interact with each other.

Therefore, by reducing consumption, labor supply, and social interactions of susceptible individuals, containment policies directly suppress new infections through equation (8). This, in turn, stimulates the willingness of susceptibles to consume and supply labor again, since with lower infections the respective health risks associated with those activities diminish as well. Hence, once containment policies are being eased, the economy experiences a boost in consumption demand and labor supply. However, as with higher economic and social activity infections start to rise again, this boost is only short-lived and the initial stimulus it provides is eventually reversed.

In summary, the aforementioned properties of equations (14) and (19) have two important implications: (i) they demonstrate that the exogenous containment policies in the form of μ_t^c and μ_t^n introduce frictions into the model that are consistent with those highlighted by the BCA analysis; (ii) they illustrate that the consumption and labor-supply effects of endogenous containment behavior interact in complex ways with those associated with exogenous containment policies through the development of new infections in the economy.

5 Quantitative Analysis

In Section 3, we documented the results of the BCA exercise that informed us about the direction to follow in searching for a more detailed DSGE model. In this section, we calibrate and use the model presented in Section 4 to answer the main research question of this paper: what are the main drivers of output in Germany during the Covid-19 crisis?

In our parametrization strategy, we rely on three sets of parameters: those related to the containment dynamics (i.e. how do we model lockdowns), those required in the epidemiological block of the model (i.e. the SIR model), and those related to economic decisions by agents (i.e households, firms, and the government). All the parameter choices are detailed in the following three sub-sections.

5.1 Containment Dynamics

As containment policies in Germany closely followed the development of the pandemic, they displayed a highly dynamic pattern with measures being tightened and relaxed according to the number of cases and deaths in the population. Therefore, we assume that the consumption, labor, and social restrictions associated with these policies evolve dynamically as well, following a set of autoregressive processes of order one:

$$\mu_t^c = \rho_t \mu_{t-1}^c + a_t^{\mu,c}, \tag{20}$$
$$\mu_t^n = \rho_t \mu_{t-1}^n + a_t^{\mu,n}, \tag{21}$$
$$\mu_t^l = \rho_t \mu_{t-1}^l + a_t^{\mu,l}, \tag{22}$$

where ρ_t is a common, time-dependent persistence parameter, which itself is assumed to follow an exogenous process defined as

$$\rho_t = \rho_{t-1} + a_t^{\rho}, \tag{23}$$

where $a_t^{\mu,j}$ (for $j = c, n, l$) and a_t^{ρ} denote exogenous innovations.

Having defined these general processes for the containment variables, we set the starting date of the model simulation ($t = 0$) to be the first week of 2020.[6] Then, we calibrate the timing and the magnitude of the exogenous shocks $a_t^{\mu,j}$ and a_t^{ρ} such that the time-series patterns of μ_t^c, μ_t^n, and μ_t^l are consistent with the major empirical trends of containment policies in Germany between 2020:Q1 and 2021:Q2. For this calibration procedure, we rely on press announcements of the German federal government (Bundesregierung 2022) and the Oxforford Covid-19 Stringency Index (OCSI henceforth) of Hale et al. (2021) as primary sources.

The OCSI essentially provides a cumulative measure for the severity of government-imposed restrictions in response to Covid-19. It is computed as the arithmetic mean of various sub-indices, with each sub-index relating to the intensity of a particular restriction on economic or social activity, such as the closing of office spaces, limitations on social gatherings, or constraints on individual mobility. Consequently, the index reflects the intensity of a variety of containment measures through one composite index. Moreover, it also accounts for whether measures were implemented locally or nationwide, by

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[6] Hence, we assume that the initial outbreak of Covid-19 in Germany occurred in the first week of 2020, which constitutes a reasonable assumption based on the fact that the first *clinically confirmed* Covid-19 infection in Germany occurred on the 27^{th} of January (Bundesministerium für Gesundheit 2022).

assigning a higher index value in case of the latter. This aggregate interpretation of the OCSI makes it a good reference for the behavior of μ_t^c, μ_t^n, and μ_t^l, since, given the aggregate nature of the model, these variables also reflect the cumulative restrictions that containment policies pose to economic and social behavior, rather than referring to any specific measure in isolation.

Figure 3: Lockdown Stringency and Daily Infections

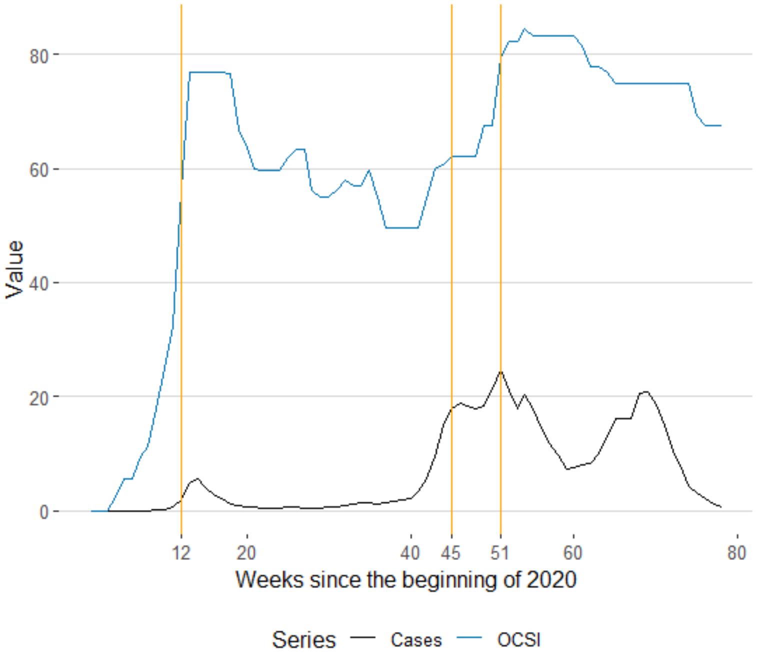

Figure 3 presents the OCSI's weekly average for Germany from 2020:Q1 to 2021:Q2, along with a weekly average of nationwide infections (in thousands). In addition, it also displays three orange bars at the 12th, the 45th, and the 51st week of 2020, which mark the particular dates at which, according to federal press announcements (Bundesregierung 2022), the German government adopted especially severe containment measures.

The two bars on the 12th and 51st weeks in Figure 3 mark the implementation of full, nationwide lockdowns in Germany following the escalation of country-wide Covid-19 infections during the first and second wave of the pandemic. Such lockdowns encompassed the nationwide closing of most of the recreational and retail sector, educational institutions, and public spaces, as well as serious restrictions on the mobility and social behavior of individuals (Bundesregierung 2022). This sharp increase in economic and social restrictions is readily reflected within the behavior of the OCSI, as the index displays a pronounced increase at these two particular dates.

Following these initial spikes, the OCSI exhibits a stepwise decreasing pattern in subsequent weeks, which reflects the gradual easing of containment policies that occurred after the economy's temporary shutdown. This easing process encompassed the successive reopening of workspaces, the lifting of restrictions on social gatherings as well as a general switch from the nationwide application of restrictions to a more local focus (Bundesregierung 2022). According to Figure 3, this reopening process took longer after the second lockdown than after the first, as the OCSI's decay following the 51st week of 2020 is significantly slower than the one that followed the 12th week of 2020. Such difference between the first and the second lockdown is due to the fact that infections were much harder to get under control during the second wave of the pandemic, as can be seen in the respective duration of both waves in Figure 3. Accordingly, confinement measures remained in place for longer during the second than during the first lockdown.

Based on these empirical properties of containment policies in Germany, we introduce positive shocks $\{a_t^{\mu,c}, a_t^{\mu,n}, a_t^{\mu,l}\}$ into the model's simulation during the 12th week of 2020 and another one during the 51st week of 2020 to reflect the sharp increase in economic and social restrictions during these periods. Additionally, we set the initial value of ρ_t to 0.92 and, following the second lockdown, we further accompany the set of shocks $\{a_t^{\mu,c}, a_t^{\mu,n}, a_t^{\mu,l}\}$ in the 51st week of 2020 by an additional shock a_t^{ρ} that changes the value of ρ_t to 0.94 [7].

The model easily includes containment dynamics around the 12th and 51st weeks of 2020, but dealing with the OCSI's subtle behavior during the 45th week of 2020 requires additional analysis, a topic that can be explored in future works.

After displaying a local minimum during previous weeks, the index reaches an intermediate plateau in week 45, before ultimately increasing further towards its aforementioned, global maximum in week 51. This somewhat inconspicuous pattern reflects the circumstance that, in response to the second wave of Covid-19, the German government initially explored an intermediate solution before implementing a second, full lockdown. Similar to the first lockdown in March 2020, the government also ordered the closure of much of the recreational sector and imposed restrictions on social gatherings during week 45 of 2020. However, in contrast to March 2020, it initially allowed educational institutions and the majority of the retail sector to remain open to partially avoid the economic and social fallout of a renewed total confine-

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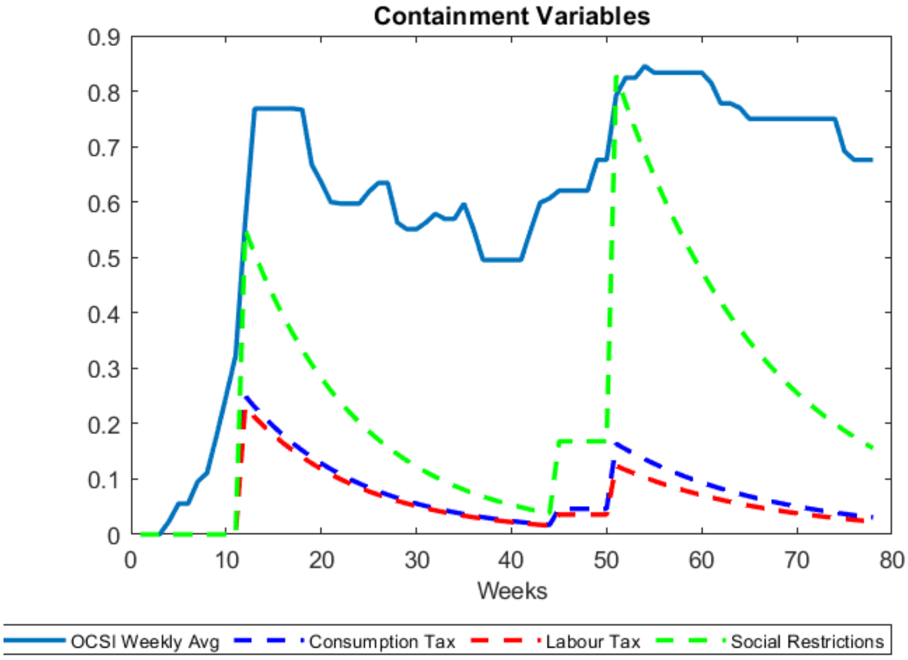
[7] Estimating a simple AR-l model on sub-samples of the OCSI Index that correspond to the respective lockdown periods, we find that both of these parameter values are within a 2 standard error band of the respective parameter estimates.

ment (Bundesregierung 2022). Nevertheless, as it can be seen in Figure 3, this intermediate strategy was not enough to sufficiently reduce infections, and the government was ultimately forced to adopt a second, full lockdown in week 51.

Thus, to reflect the economic and social restrictions associated with this initial lockdown light approach of the German government, we introduce a last set of shocks $\{a_t^{\mu,c},\ a_t^{\mu,n}, a_t^{\mu,l}\}$, as well as an additional shock a_t^{ρ} into the model's simulation during week 45 of 2020. The main difference in this particular instance is that we set a_t^{ρ} such that between weeks 45 and 51, it holds that $\rho_t = 1$. This calibration of a_t^{ρ} accounts for the fact that in the time interval between weeks 45 and 51, restrictions were not eventually eased again, but remained in place until their further aggravation in week 51.

The patterns of the containment variables μ_t^c, μ_t^n, and μ_t^l that result from the entire shock calibration outlined above are summarized in Figure 4. As we can see, consumption, labor, and social restrictions approximate the major empirical trends of the OCSI. Hence, they are consistent with the central empirical features of containment policies in Germany between 2020:Q1 and 2021:Q2.

Figure 4: Model Containment Dynamics



5.2 SIR Parameters

In order to calibrate the epidemiological parameter that governs the weekly probability of dying from a Covid-19 infection (π_d), we rely on the weekly reported data on Covid-19 cases and deaths in Germany provided by RKI (2022*c*) and RKI (2022*e*).

Based on these data, we compute an initial, age-weighted average of Covid-19 case fatality rates in the German population aged 10 to 69 between 2020:Q1 and 2021:Q2, which results in a value of about 0.0055. Then, the average case fatality rate is adjusted by a factor of $\frac{1}{1.8}$ in order to transform it into an *infection fatality* rate. The adjustment factor of $\frac{1}{1.8}$ is taken from the RKI's publication on the share of under-reported Covid-19 cases in Germany (RKI 2022*d*) and represents an estimate of the ratio of detected to undetected Covid-19 cases based on serological samples. It is important to adjust the crude case fatality rate by this factor because pure case fatality rates are calculated as the ratio between the number of individuals that died from Covid-19 and the amount of actually detected infections. Since deaths are usually observed more accurately than infections, crude case fatality rates tend to significantly overstate the actual probability of dying from a Covid-19 infection.

The final *infection fatality* rate for Germany that results from adjusting the initial case fatality rate by the number of undetected infections is approximately equal to $0.0055/1.8 \approx 0.003$. With this numerical value for the infection fatality rate at hand, we follow Eichenbaum et al. (2020) by assuming that the average time to either recover or die from Covid-19 is about two weeks. From this assumption, it follows that π_d equals $\frac{7}{14}0.003 = 0.0015$ and that $\pi_d + \pi_r = 0.5$, which in turn determines the weekly probability of recovering from Covid-19 as $\pi_r = 0.5 - 0.0015 = 0.4985$.[8]

Following Eichenbaum et al. (2020), we calibrate the remaining epidemiological parameters π_1, π_2, and π_3 so that the relative shares of infections that result from consumption, labor, and social activities in the model population are consistent with the respective estimates of these shares obtained from the epidemiological data. However, as there is no direct source that reports Covid-19 infections in Germany based on economic activity, we obtain such benchmark estimates by matching and aggregating a broad range of micro-level data. We use the dataset of (RKI 2022*b*) which reports the weekly amount of Covid-19 infections in Germany that can be attributed to

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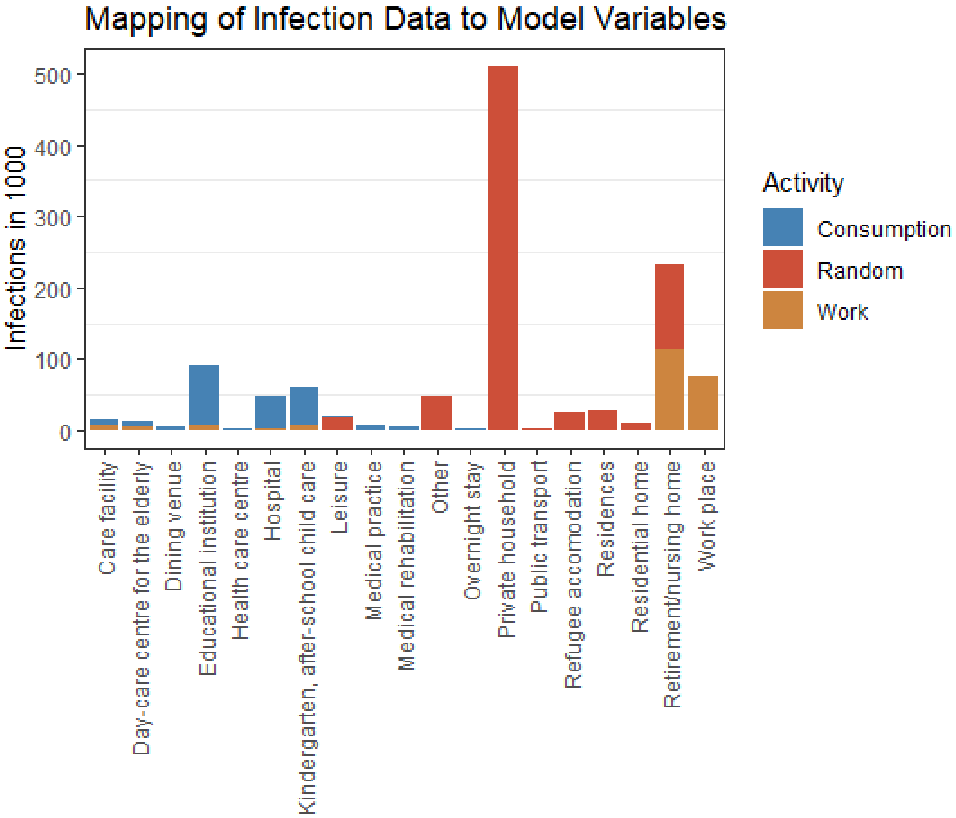
[8] It is important to note that these parameter values are based on Covid-19 data for the working-age population only. The reason for using this particular subset of the population is that in the model, the representative household is implicitly assumed to only consist of working-aged individuals.

outbreaks in 19 distinct settings. These settings refer to a broad range of common locations where transmission of Covid-19 was likely to occur, such as educational institutions, private homes, public transport, etc.

Based on this data, we compute the total amount of infections that have occurred in each of these locations over the course of the pandemic. Then we match such time-aggregated data for every location with a variety of sources that contain information regarding the number of people that visit each location for either consumption, work, or unrelated purposes. As an example, we match the total amount of infections that resulted from outbreaks in hospitals with data on the average German patient-to-staff ratio (Destatis 2022*b*, 2022*c*). Such a ratio carries information regarding the number of people that visit hospitals for either consumption- or work-related purposes, as we assume that by going to the hospital, patients are consuming a service, whereas staff members are there to supply labor. According to this logic, we further match the total number of infections in overnight accommodations with customer-to-staff ratios (Destatis 2022*a*, 2022*f*), the number of infections in retirement homes with resident-to-staff ratios (Destatis 2022*d*, 2022*e*), etc. We follow this approach for all locations except public transport and leisure, for which there are no such ratios available. Hence, for these particular locations, we resort to ratios related to time-use instead, for example on the time that individuals spend in public transport to travel to work, relative to the time they spend commuting to consumption-related activities (Eurostat 2021).

Subsequently, after having established such matchings, we disaggregate the total amount of infections in a particular location based on the respective ratios. Since according to the patient-to-staff ratio in German hospitals approximately 95% of all individuals in these locations follow a consumption purpose, with the remaining 5% being there for work, we assign 95% of all infections in hospitals as related to consumption and 5% as related to labor. A visual summary of the results of this disaggregation procedure for every location can be found in Figure 5.

Figure 5: Calibration of Infection Risk by Activity



Finally, having disaggregated the number of infections in every individual location into consumption, labor, and social interactions, we simply sum the number of infections for each of these categories across all locations. Following this, we divide the total number of infections related to consumption, labor, or other interactions that result from this procedure by the total number of infections across all locations and activities to obtain the final estimates for aggregate infection shares that can be attributed to either of the three activities. The final estimates that result from this approach are that consumption and labor activities each account for about 18% of total infections in Germany, whereas random social interactions account for the remaining 64%. This result is very close to the findings of Eichenbaum et al. (2020) for the US economy, which establish infection shares to be approximately 17%,17%, and 66%, respectively.

In line with Eichenbaum et al. (2020), we map those final estimation results for the conditional transmission probabilities for Covid-19 in Germany into the model parameters by imposing that π_1, π_2, and π_3 should be such that at the beginning of the pandemic 18% of Covid-19 transmissions are due to consumption, 18% due to labor and 64% due to random interactions. Hence, we set:

$$\frac{\pi_1 (C^*)^2}{\pi_1 (C^*)^2 + \pi_2 (N^*)^2 + \pi_3} = 0.18, \tag{24}$$

$$\frac{\pi_2 (N^*)^2}{\pi_1 (C^*)^2 + \pi_2 (N^*)^2 + \pi_3} = 0.18, \tag{25}$$

$$\frac{\pi_3}{\pi_1 (C^*)^2 + \pi_2 (N^*)^2 + \pi_3} = 0.64, \tag{26}$$

where C^* and N^* denote aggregate consumption and working hours in the pre-infection steady state.

In addition, we further impose that the values of π_1, π_2, and π_3 have to be such that, without containment policies, about 15% of the model's population becomes infected within the first 9 months of the pandemic. This additional assumption is necessary, since π_1, π_2, and π_3 not only govern the relative contribution of consumption, labor, and social interactions to the development of the pandemic, but also the overall speed by which the pandemic spreads among the model's population. Since there are multiple solutions to equations (24) to (26), which all reflect a different degree of the virus's overall contagiousness, the calibration of π_1, π_2, and π_3 requires an additional benchmark related to the transmissibility of Covid-19 to pin down a unique set of parameter values. To this end, Eichenbaum et al. (2020) rely on a hypothetical scenario which imposes that until a postulated end of the pandemic, eventually 60% of the total population should become infected. In contrast to this approach, we rely on a slightly different benchmark by observing that Sweden, the only industrialized country in the world that did not impose strict lockdowns during the first wave of the pandemic, reported an estimated Covid-19 seroprevalence of 15% after the first 9 months of the pandemic (Rostami et al. 2021). Assuming that the transmission of Covid-19 in Sweden is governed by conditions that are comparable to those in Germany, this finding provides a good benchmark for the baseline calibration of the virus's infectiousness in Germany. The parameter values of π_1, π_2, and π_3 that result from their calibration with regard to both the conditional and absolute transmission of Covid-19 in Germany are $\pi_1 = 2428 \times 10^{-7}$, $\pi_2 = 23706 \times 10^{-4}$, and $\pi_3 = 0.4252$.

5.3 Economic Parameters

Based on Hinterlang et al. (2021), we calibrate the labor share in the aggregate production function α to be equal to a standard value of $\frac{2}{3}$ and the weekly depreciation rate of capital δ as $\frac{0.025}{13}$. The Calvo parameter ξ is equal to 0.9808, implying that the average price duration of intermediate-good firms is approximately one year, which is consistent with Alvarez et al.'s (2006) observations regarding the average price duration of European firms. Similarly, for the calibration of the markup parameter γ, we follow the estimates of Christopoulou & Vermeulen (2012)in which the average markup in Germany is equal to 1.33.

Moreover, noting that in the model's pre-infection steady-state it holds that $\frac{1}{\beta} = \frac{1+R^{b*}}{\pi^*}$, we calibrate the discount factor β such that it reflects the average annualized real rate of return of the leading German stock index, the DAX, between 1993 and 2019. This yields an annual beta, β_{annual}, and, since the model is set at a weekly frequency, we compute the final value of beta as $\beta_{annual}^{1/52}$. This leads to a weekly discount factor β of approximately 0.9994.

Additionally, due to the low-inflation environment that existed before the onset of Covid-19, we set the weekly gross-inflation rate in the pre-epidemic steady-state π^* equal to one. Furthermore, following Eichenbaum et al. (2020), we calibrate aggregate income and working hours in the pre-epidemic steady-state, Y^* and N^*, according to their average, weekly values just prior to Covid-19. To this end, we compute the real values of GDP and total hours worked per working-age person of the last quarter of 2019 based on data from the OECD Economic Outlook (OECD 2021) and break them down into their weekly counterparts by dividing them by 13. This procedure results in weekly real per working-age population values for output and hours worked of $Y^* = 1162$ and $N^* = 22.46$.[9] The steady-state values of income and hours worked implicitly define the values of the aggregate productivity parameter (A) and the labor scaling parameter (θ) to be equal to 2.99793 and 0.00165, respectively. This result follows from the pre-Covid steady-state equations for Y^* and N^*.

Finally, we calibrate the pre-infection government consumption to output ratio (η^*) according to the fraction between government final consumption expenditures and GDP in the last quarter of 2019 (OECD 2021) and set the values of the Taylor-rule parameters equal to those of Eichenbaum et al. (2020), with $r_\pi = 1.5$ and $r_x = 0.5$. A comprehensive summary of all the aforementioned economic, as well as epidemiological parameter calibrations,

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[9] We adjust GDP and hours worked by the working-age population and the price level in this process since all economic variables in the model are implicitly assumed to be on this scale.

Table 2: Parameter Calibration

Parameter	Value	Description
α	2/3	Labor share
γ	1.33	Markup
δ	0.025/13	Weekly depreciation rate
ξ	0.9808	Calvo price stickiness
β	0.994	Weekly discount factor
r_π	1.5	Taylor rule coefficient for inflation
r_x	0.5	Taylor rule coefficient for output Gap
Y^*	1162	Steady-state GDP
N^*	22.46	Steady-state Labor Supply
η^*	0.21	Steady-state gov. consumption share
π^*	1	Steady-state gross inflation
$\breve{p}^*$	1	Steady-state price dispersion
A	2.99793	Productivity parameter
θ	0.00165	Labor-scaling parameter
π_d	$(1/1.8) \times 7 \times 0.0055/14$	Weekly probability of dying
π_r	$7/14 - \pi_d$	Weekly probability of recovering
π_1	$2.428x10^{-7}$	Transmission share consumption
π_2	$2.3706x10^{-4}$	Transmission share labor
π_3	0.4252	Transmission share random
RorD	0.15	Share of Infected/Dead after 9 months
ϵ	0.0003	Initial Seed of Infections

can be found in table 2.

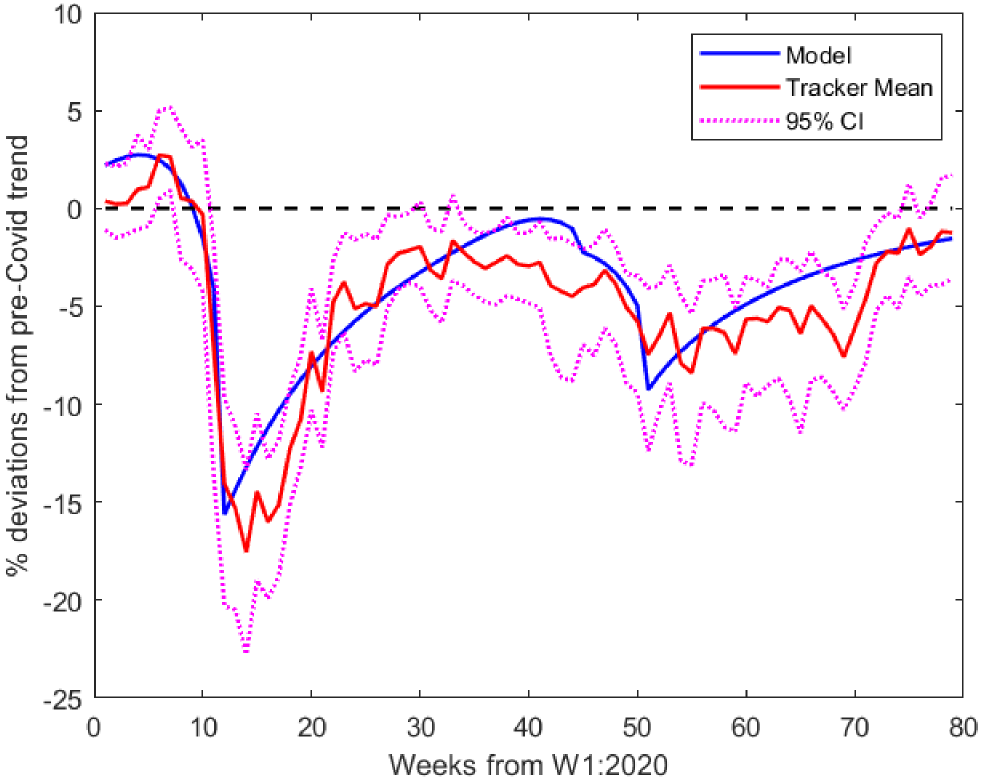
6 Lockdowns and the severity of the German Covid recession

Using our structural model and the lockdown shocks, we perform simulations at both weekly and quarterly frequencies.

6.1 Weekly Dynamics

To evaluate the model's ability in replicating high-frequency output dynamics, we compare the model-implied weekly growth rates of GDP, expressed as percentage deviations from the pre-pandemic steady-state, to the OECD's tracker of weekly GDP in Germany (Woloszko 2020). Essentially, the OECD tracker estimates weekly output growth relative to its pre-Covid trend based on Google Trends search data. Therefore, it also describes the development of GDP relative to its pre-pandemic state, which facilitates clear comparability between the OECD tracker and the model-generated data.

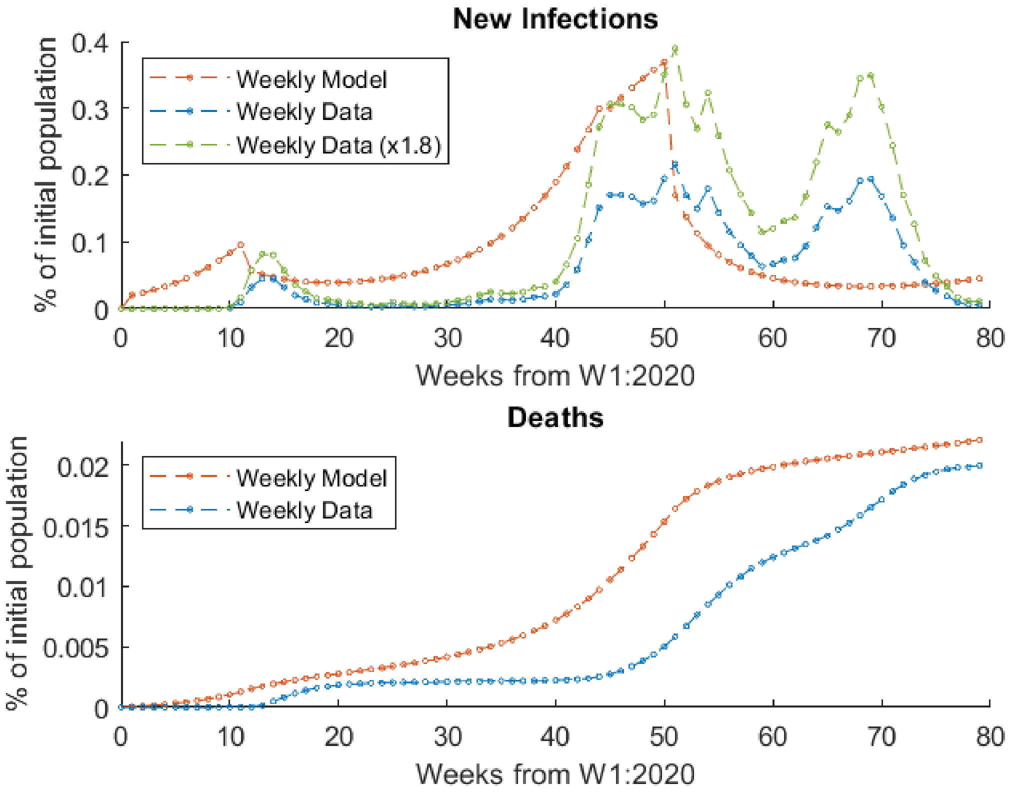
Figure 6: Model implied weekly GDP growth vs OECD Weekly Tracker



According to Figure 6, the model-generated weekly output growth provides a very good approximation to the corresponding estimates reported by the OECD tracker. In almost all instances between the first quarter of 2020 and the second quarter of 2021, the model produces GDP growth values that are within the 95% confidence range of the OECD tracker's estimates.

Additionally, Figure 7 presents the comparison between the model's output and observed data regarding the epidemiological dynamics of Covid in Germany. To this end, it presents the percentages of the model's pre-Covid working-age population that cumulatively dies from the disease or is newly infected within a given week to those percentages reported in the epidemiological data in RKI(2022*c*, 2022*e*).

Figure 7: Deaths and Infections



Similarly, Figure 7 also reports a good performance of the model along its epidemiological dimension. Taking into account that actual infections were under-reported by at least a factor of 1.8 (RKI 2022*d*), the model produces plausible values of weekly new infections. Furthermore, it generates multiple waves which, in their respective timing, closely match those observed in the data. Finally, it also replicates the share of the German working-age population that has cumulatively died from Covid-19 until the end of 2021:Q2. The model's only discernible shortcoming is that it misses the renewed peak of infections occurring towards the end of the second wave.

One plausible explanation for this particular limitation is that, according to RKI (2022*a*), the Covid mutation AlphaB.1.1 (commonly known as the

British variant) became the dominant strain in Germany by the 9th week of 2021. Since this mutation was significantly more lethal than the original virus, this circumstance is likely to have caused the renewed increase in cases shortly after the first peak of the second wave. Such increase in cases, however, was met with the beginning of the Covid-19 vaccination campaign in Germany, which ultimately reduced the virus's infectiousness again by providing at least partial immunization and breaking infection chains. Hence, the second peak of the second wave was likely the result of a back-and-forth between positive and negative shocks to the virus's transmissibility, which is difficult to reconcile within the model's simple SIR framework.

However, despite this shortcoming in replicating infection dynamics towards the end of the observational period, the model is still capable of explaining fluctuations in weekly GDP and epidemiological variables with substantial accuracy. Consequently, it rationalizes that output fluctuations in Germany during Covid-19 were driven by a combination of the mechanisms described below.

As the model's response to the general lockdown in the 12th week of 2020 closely replicates the initial, sharp decline of weekly OECD tracker data in Figure 6, it implies that the sharp drop in aggregate output at the onset of the pandemic is primarily driven by the consumption and labor supply constraints resulting from containment policies. Following this initial, drastic decline in economic activity, the model rationalizes that the subsequent, V-shaped recovery in output is the result of a combination of two related factors. First, economic and social restrictions associated with containment policies are lifted. In the model, this corresponds to the geometric decay in variables μ_t^c,μ_t^n, and μ_t^l after the initial shock. Second, as Figure 7 illustrates, containment policies lead to a significant decline in infections. This, in turn, increases the desire of susceptibles to consume and supply labor, as health risks associated with these activities are reduced. Hence, the relaxation of consumption and labor constraints is complemented by an actual willingness of individuals to engage in these activities. Working in conjunction, these two mechanisms appear to drive the fast recovery of output until around week 30 of 2020. Then, given the upturn in economic and social activity, infections start to rapidly increase as well. This, according to the model, prevents a full recovery of output towards its pre-pandemic levels, as with rising infections, susceptible individuals start to cut back on consumption and labor supply again. Moreover, in response to accelerating Covid infections and deaths, a second set of lockdown measures is implemented in weeks 45 and 51, which explains the renewed declines in output in these instances.

Following the sharp drop in GDP in week 51, the model implies that its subsequent recovery is again the result of the phasing-out process of containment policies. However, as after the second lockdown containment policies were more persistent, such recovery process is slower compared to after the first lockdown. Moreover, Figure 6 also shows that, while model-generated GDP growth still falls within the 95% confidence range of the OECD tracker, it persistently lies above its mean estimates and close to the upper confidence bound. This tendency is the result of the model's shortcoming in replicating the renewed spike of infections towards the end of the second wave, as by understating the number of infections, it misses the negative contribution of voluntary containment to weekly output dynamics.

6.2 Quarterly Dynamics

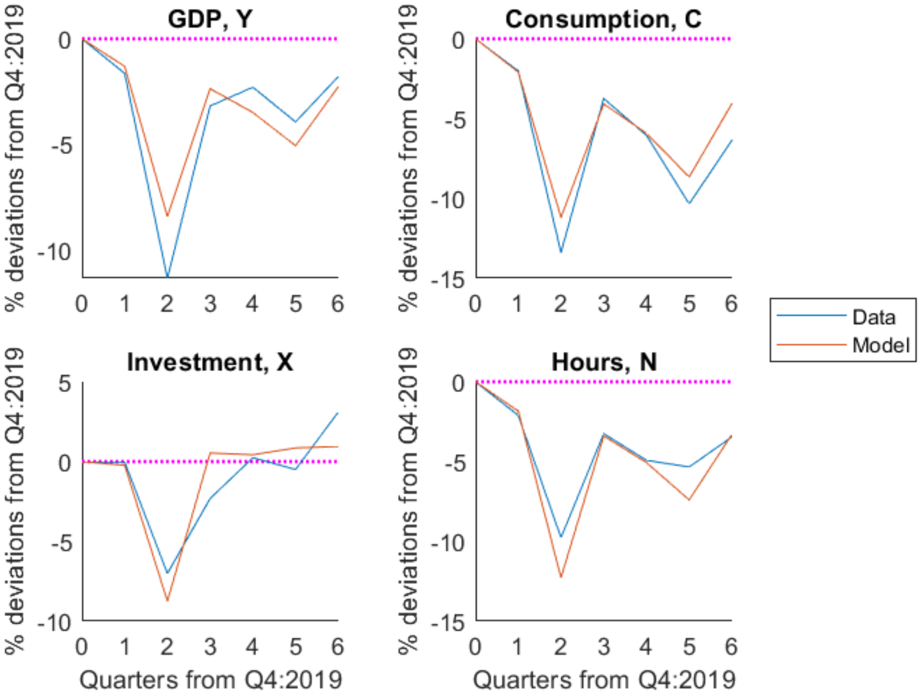
In addition to evaluating the model's ability to replicate weekly output fluctuations, Figure 8 presents the output dynamics on a quarterly frequency. Moreover, it also assesses the model's ability to rationalize quarterly consumption, labor, and investment patterns. To this end, it compares the model implied quarterly growth rates for German GDP, investment, consumption, and hours worked relative to 2019:Q4, to those based on data from the OECD's Economic Outlook (OECD 2021).

As previously illustrated, the weekly OECD tracker data is based on estimates which are surrounded by a potentially high degree of uncertainty. Hence, comparing the model-generated data also with quarterly GDP values based on National Accounts provides a robustness check of the model's explanatory abilities. Furthermore, since self-mitigation and containment policies are assumed to translate themselves into output fluctuations primarily through their consumption and labor-supply effects, it is crucial that the model provides an appropriate description of these variables as well. Ideally, we would also like to analyze the model's capabilities of explaining consumption and working hours fluctuations on a high-frequency level. However, to the best of our knowledge, there are no available indicators of weekly consumption and hours worked in Germany.

According to the upper-left panel of Figure 8, the model also accounts for the output quarterly dynamics, besides slightly understating GDP growth in 2020:Q2 and overstating it in 2020:Q4. Hence, comparing the model-generated data to empirical observations on quarterly GDP corroborates the conclusion that output fluctuations in Germany were primarily driven by the self-mitigation of households and containment policies.

This interpretation of the German Covid-19 recession is further supported by the upper-right and lower-right panels of Figure 8. The upper-right panel demonstrates that model-generated consumption patterns follow very closely those observed in the data. A similar observation can be made for hours

Figure 8: Model implied vs Data-implied Quarterly Growth Rates



worked in the lower-right panel, albeit a slight tendency of the model to overstate declines in working hours. These results support the model's central assumption that both the endogenous responses of households to the pandemic and containment policies translate themselves into output fluctuations primarily via their consumption and labor-supply effects.

Finally, the lower-left panel of Figure 8 demonstrates that, while overstating the recovery of investment in 2020:Q3 to some degree and understating it in 2021:Q2, the model also provides a reasonable description of quarterly investment dynamics. As within the model's theoretical framework, investment dynamics are primarily driven by the combination of negative supply and demand shocks resulting from the consumption and labor-supply effects of self-mitigation and containment policies, this observation further highlights the importance of these channels for explaining macroeconomic fluctuations during the German Covid-19 crisis. Moreover, since investment constitutes an integral part of aggregate output, it further supports their role in explaining output dynamics as well.

6.3 Counterfactual Analysis: Evaluating the lockdowns

Having concluded that containment policies are responsible for large output losses in Germany during Covid-19, we further conduct a counterfactual analysis to quantify those losses relative to a no-lockdown scenario. The results of such analysis are displayed in Figures 9 and 10.

Figure 9 shows that even in the absence of containment policies, the model implies a sizeable decline in consumption, hours worked, investment, and GDP. The underlying reason for this result is displayed in Figure 10. It illustrates that without binding containment policies, infections and deaths rise to a significantly higher level than with containment policies. In response to this higher level of infections, susceptibles reduce their consumption and labor supply more drastically, as the health risks associated with those activities are far more pronounced. Thus, even though individuals are not forced to drastically cut back on consumption and labor supply through business closures and stay-at-home orders, they eventually would do so voluntarily in response to escalating epidemiological dynamics, triggering a significant recession nonetheless.

This recession, however, lags behind the one that resulted from actual containment policies (see Figure 9) since the peak declines of GDP and other macroeconomic aggregates occur in 2020:Q4 instead of 2020:Q2, implying that individuals are somewhat slower to react to epidemiological developments than policymakers. Moreover, whereas Figure 9 shows that a no-lockdown solution eventually leads to a pronounced recession as well, it also

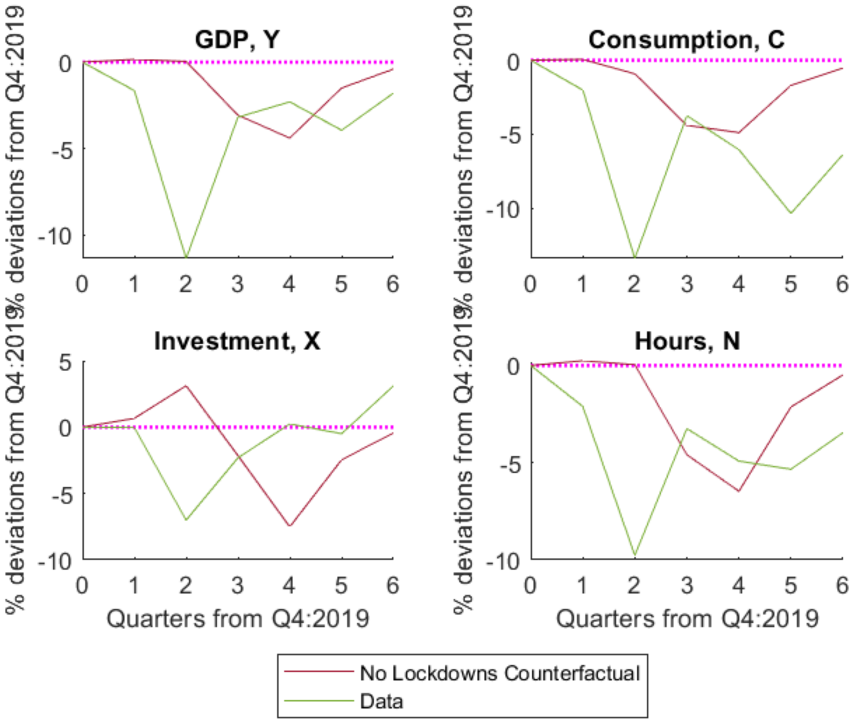


Figure 9: Quarterly Model implied Growth Rates without Lockdowns vs. actual Growth Rates

shows that the respective declines in aggregate output, consumption, and hours worked still remain significantly below those that have resulted under the realized containment policies.

Therefore, the counterfactual analysis implies that containment policies in Germany substantially aggravated economic losses during the pandemic, relative to a no-intervention scenario. Nevertheless, such policies also helped to substantially reduce the toll of infections and deaths, as shown in Figure 10. A natural question that arises from these opposing developments of GDP and deaths between the simulated laissez-faire and the actual containment approach taken by the German government is which policy would have been preferable ex-post. To answer this question, we employ a simple accounting framework based on the Value-of-Life (VOL henceforth) estimates for Germany reported in Sprengler (2004).

According to Sprengler (2004) the nominal VOL in Germany in 2004 was about 1.65 million euros, which translates into a value of 1.9 million euros in 2015 prices[10], using the GDP deflator for adjustment. Based on

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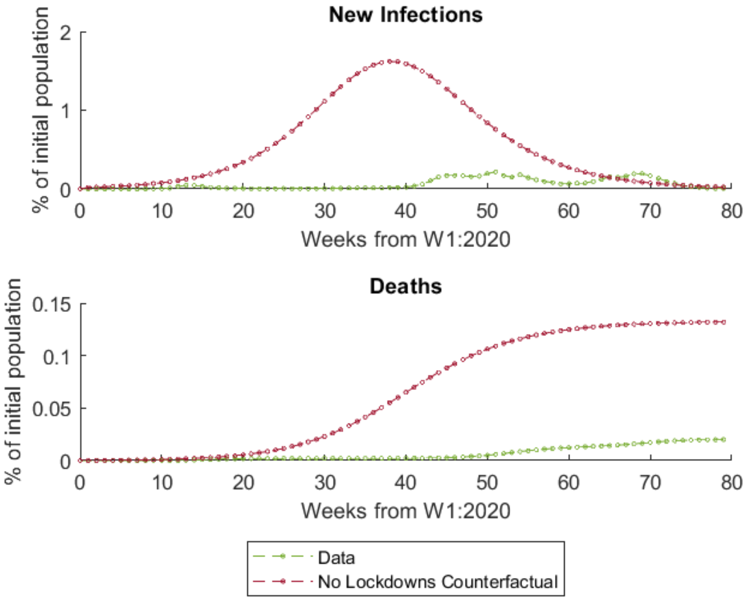
[10] Which is the unit of measurement for GDP in our analysis

Figure 10: Model implied Deaths and Infections: Lockdown vs No-Lockdown



this VOL estimate, we compute the monetary benefit of the reduced death toll under containment policies, compared to the laissez-faire approach, by multiplying it with the difference in total deaths between the laissez-faire approach and actual deaths reported in the data from 2020:Q1 to 2021:Q2. According to the model, the number of deaths[11] in the laissez-faire case would have exceeded those in reality by about 70000. Hence, the monetary benefit of containment policies in terms of preventing potential deaths amounts to roughly 132 Billion Euros ($70000 * 1.9M$ Euros).

On the other hand, computing the cumulative difference between the model-implied levels of GDP under the laissez-faire scenario and actual GDP between 2020:Q1 and 2021:Q2 implies that total output losses under a no-lockdown rule would have potentially been 121 Billion Euros below those that actually materialized. Hence, containment policies can be seen as having created excess GDP losses of 121 Billion Euros.

Comparing potential excess losses In GDP to the monetary benefit of saving lives under containment policies reveals that the net-benefit of these policies, compared to the laissez-faire solution, amounts to about 11 Billion Euros. Therefore, even though containment policies were indeed subject to a strong trade-off between saving lives and saving livelihoods (Kaplan et al. 2020), they appear to have been the more favourable policy option compared

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[11] In the working age population

to a no-lockdown solution.

Of course, this result strongly hinges on the models assumption that in the absence of containment policies individuals have a strong incentive to engage in voluntary containment, as this creates a lower bound to the economic fallout that could have potentially been avoided by adopting a laissez-faire policy, which drives the positive net-effect when health benefits and GDP losses are being compared. However, the experience of Sweden, which even in the absence of strict containment policies in 2020 experienced a peak decline in quarterly GDP of around 8.8% (relative to 2019:Q4), supports this assumption[12].

6.4 Limitations and Additional Specifications

Although providing a theoretical framework that accounts for crucial features of the Covid-19 crisis, such as the explicit connection between epidemiological and macroeconomic outcomes and the effects of lockdowns, our model still features a variety of abstractions that demand some further discussion. First, it abstracts entirely from the fiscal policy measures adopted by the German government in order to cushion the effects of the pandemic. Due to their unprecedented size, those measures might also have shaped German output dynamics in a non-trivial manner, rendering their omission a potentially weakening simplification.

Nevertheless, Hinterlang et al. (2021) show that the differences between the trajectory of German output with and without fiscal policy interventions is estimated to be relatively small. The authors provide a comprehensive analysis of Germany's fiscal package employing a sectoral, large-scale DSGE model. This result provides some comfort that the potential inclusion of fiscal interventions into our model would not have altered its fundamental conclusions.

Second, the model assumes a closed economy framework, which stands in contrast to Germany's traditionally export-reliant economy. Nevertheless, as the BCA analysis has shown, the government wedge, which is essentially a measure of frictions associated with government spending and trade dynamics, is of relatively low importance for explaining output variations. Furthermore, Bonadio et al. (2020) demonstrate, using a global-network model, that the significant output losses in Germany towards the beginning of the pandemic were caused predominantly by the effects of domestic lockdowns. Hence, while open-economy channels certainly contributed to the develop-

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[12] In comparison, quarterly GDP in Germany decreased by about 11.5% (relative to 2019:Q4) during the peak of the crisis (OECD 2021).

ment of GDP in Germany during the pandemic, they appear to have been dominated by the effects of domestic shutdowns and mobility restrictions. Thus, for the sake of simplicity, we relied on a leaner structure based on the evidence pointing toward a more domestically-oriented episode.

Finally, our analysis also abstracts from any type of financial frictions which might have amplified Covid-19 associated supply shocks or might have been a source of macroeconomic disturbances themselves. However, according to Eichenbaum et al. (2020), while exhibiting a spike in response to the pandemic, financial stress in Germany was relatively low and short-lived compared to previous crises. Hence, we assume that their conclusion also holds for the German Covid-19 recession, abstracting also from financial frictions, which seems to be an admissible first-order simplification.

7 Conclusion

In this paper, we investigate which mechanisms can explain the observed fluctuations of output during the German Covid-19 crisis. The labor wedge is the main driver of output during the episode. With a macro-epidemiological model, we identify the following mechanism: (i) susceptible agents adjust consumption and labor supply in response to the level of infections (generating a labor wedge). This is the pandemic influencing the economy. Also, (ii) the transmission of the disease is influenced by consumption activities, labor, and social interactions (the feedback channel, from the economy to the pandemic). By adding lockdowns to the model, we have a framework in which output fluctuations are driven by a time-varying labor wedge, which is the result of the self-containment behavior by households and containment policies by the German government.

By comparing the simulation results of our model with high-frequency estimates of GDP growth and major epidemiological data we conclude that self-mitigation and containment policies can explain practically all fluctuations in German output between the first quarter of 2020 and the second quarter of 2021. Among these two mechanisms, containment policies are found to be responsible for large, non-linear output variations, while self-mitigation plays a complementary role.

The model's simulation results based on quarterly National Accounts data further support this interpretation of the German Covid-19 recession. On a quarterly frequency, the model retains its ability to explain output dynamics. Additionally, it also provides an accurate description of consumption, working hours, and investment patterns, supporting its notion that self-containment and containment policies translated themselves into aggregate-output effects primarily via their impact on consumption demand and labor supply.

Finally, a counterfactual analysis further confirms that while containment policies helped avoid both direct losses in human life and indirect ones due to the stressful effect of more infections on the National Health Service, they contributed to large losses in aggregate output. Nevertheless, due to the self-containment behavior of households in the absence of such policies, a recession would have occurred even under a laissez-faire response to the pandemic.

By highlighting the role of containment policies and self-mitigation in explaining output fluctuations in Germany, our paper exposes the atypical nature of the Covid-19 crisis in Germany and its translation into macroeconomic outcomes. This follows as both containment policies and epidemiological dynamics constitute uncharted territory in the analysis of German business cycles prior to Covid-19. We conclude that the self-mitigation by households plays a complementary role in the presence of lockdowns and a potentially central role in their absence.

Given our results that lockdowns play an important role in explaining significant output losses in Germany, but also in preventing substantial losses in human life, a potential path for future research is to extend our work by deriving theoretically optimal-containment policies in the sense of Eichenbaum et al. (2021). Comparing such optimal policies to those implemented in reality would provide an interesting evaluation of Germany's Covid-19 response.

Another promising direction for future research is to introduce a greater deal of heterogeneity into our analytical framework. Extending the model with regional heterogeneity in the sense of Acharya et al. (2020) could provide valuable insights into the differential effects of containment policies across Germany's federal states. Similarly, introducing household heterogeneity in the form of a two-agent framework, such as in Eichenbaum et al. (2022*b*), might contribute to the understanding of the distributional effects of lockdowns in Germany.

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A Appendix: Business Cycle Accounting Data and Sources

For the BCA analysis, we obtain data on quarterly GDP, investment, government consumption, imports, exports, hours worked per employee and total employment from the OECD Economic Outlook Database (OECD 2021). From the same database, we further obtain yearly data on Germany's working-age population (Age 15-64) and private consumption of durable goods. Finally, we also retrieve yearly observations on consumption tax revenues (expressed in % of GDP) from the OECD tax statistics database (OECD 2022).

As the series of durable goods consumption, population and consumption tax revenues are only available on a yearly frequency, we interpolate them using the Denton-Cholette method(Denton 1971). Furthermore, we express all nominal variables in real terms by dividing with the GDP Deflator (which has also been obtained from OECD (2021)) and compute total hours worked as the product of hours worked per employee and total employment.

Following these initial adjustments of the data, we correct GDP for the consumption of durable goods and taxes on consumption in the following way. First, we compute the stock of durable consumption goods in each period according to the simple law of motion:

$$C_t^{D,s} = (1-\delta)C_{t-1}^{D,s} + C_t^{D,f} \tag{27}$$

where $C_t^{D,s}$ denotes the stock of durable consumption goods, $C_t^{D,f}$ the flow and δ the depreciation rate, which we assume to be 25% annually. The flow values of durable consumption goods that we feed into this equation are the interpolated quarterly values of durable consumption obtained previously. As the initial stock value, we set the first quarterly flow value multiplied by a factor of 16, which represents the approximate average ratio of flow to stock in durable consumption goods.

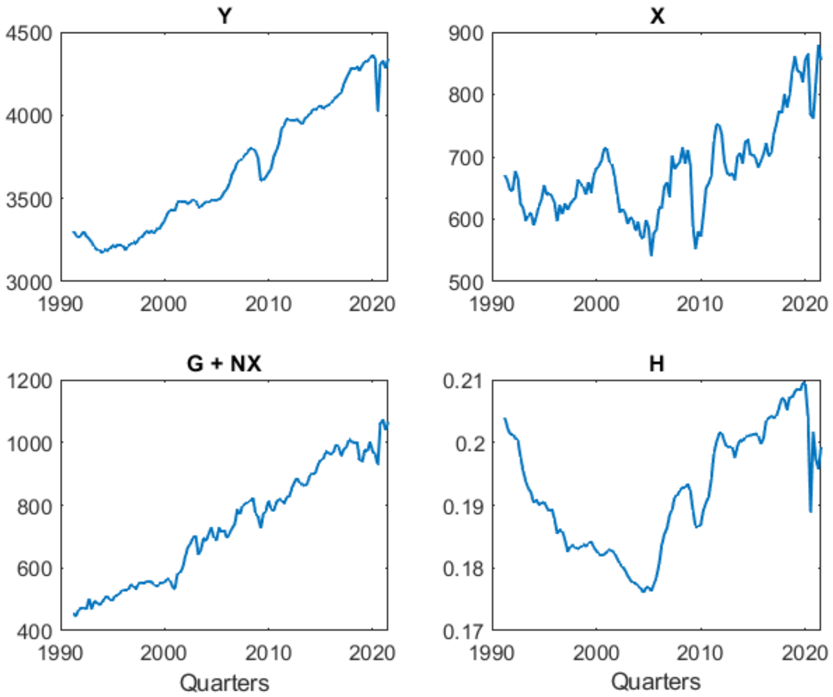
Based on the stock of durables, we derive the value of services related to the maintenance of these goods, which we assume to be 1% of the total stock value each period. Furthermore, we calculate the total value of stock depreciation each period as $\delta C_t^{D,s}$. Lastly, to adjust for consumption taxes as well, we compute the levels of consumption tax revenues by simplify multiplying the revenues expressed in % of GDP by the corresponding level of GDP in each period.

Having computed all these intermediate series related to durable consumption goods and taxes, we calculate an adjusted GDP series according to the following final equation:

$$Y_t^{adj} = Y_t + 0.01C_t^{D,s} + \delta C_t^{D,s} - \tau_t^C * Y_t \tag{28}$$

where Y_t and Y_t^{adj} denote the unadjusted and adjusted values of GDP, respectively, and τ_t^C the tax on private consumption in % of GDP.

Figure A.1: Business Cycle Accounting Data



As all variables of the prototype economy in the BCA framework are implicitly assumed to be in real per-capita terms, we further divide all remaining variables by the quarterly working-age population values. Then, we define four final time series to be used in the estimation procedure, which are adjusted real GDP (Y), total hours worked (H), real Investment (X) and government spending plus net exports ($G + NX$), as depicted in Figure A.1. The total length of our data sample spans from 1991:Q1 to 2021:Q2. For the estimation procedure of the wedge-time series and the respective simulations for each individual wedge and different combinations of wedges we use the "BCAppIt" application developed by Brinca et al. (2020).

B Appendix: Model

Lagrangian and solution of the Household's Problem

The Lagrangian function is defined as:

$$\begin{aligned}
L \quad = \quad & \sum_{t=0}^{\infty} \beta^t \Big\{ s_t\big(log(c_t^s) - \frac{\theta}{2}(n_t^s)^2\big) + \ i_t\big(log(c_t^i) - \frac{\theta}{2}(n_t^i)^2\big) + r_t\big(log(c_t^r) - \frac{\theta}{2}(n_t^r)^2\big) \\
& -\lambda_t^b\Big[(1+\mu_t^c)P_t(s_t c_t^s + i_t c_t^i + r_t c_t^r) + P_t(k_{t+1} - (1-\sigma)k_t) + \psi_t + B_{t+1} \\
& \qquad -(1-\mu_t^n)W_t(s_t n_t^s + i_t n_t^i + r_t n_t^r) - R_t^k k_t - R_{t-1}^b B_t - \phi_t\Big] \\
& -\lambda_t^\tau\Big[\tau_t - \pi_1 s_t c_t^s(I_t C_t^I) - \pi_2 s_t n_t^s(I_t N_t^I) - \pi_3 s_t I_t(1-\mu_t^l)\Big] \\
& -\lambda_t^s\Big[s_{t+1} - s_t + \tau_t\Big] \\
& -\lambda_t^i\Big[i_{t+1} - i_t - \tau_t + (\pi_r + \pi_d)i_t\Big] \\
& -\lambda_t^r\Big[r_{t+1} - r_t - \pi_r i_t\Big]\Big\}
\end{aligned}$$

The FOCs with respect to c_t^s, c_t^i, c_t^r are:

$$\frac{1}{c_t^s} = \tilde{\lambda}_t^b(1+\mu_t^c) - \lambda_t^\tau \pi_1(I_t C_t^I) \tag{29}$$

$$\frac{1}{c_t^i} = \tilde{\lambda}_t^b(1+\mu_t^c) \tag{30}$$

$$\frac{1}{c_t^r} = \tilde{\lambda}_t^b(1+\mu_t^c) \tag{31}$$

The FOCs for n_t^s, n_t^i, n_t^r are:

$$\theta n_t^s = \tilde{\lambda}_t^b(1-\mu_t^n)w_t + \lambda_t^\tau \pi_2(I_t N_t^I) \tag{32}$$

$$\theta n_t^i = \tilde{\lambda}_t^b(1-\mu_t^n)w_t \tag{33}$$

$$\theta n_t^r = \tilde{\lambda}_t^b(1-\mu_t^n)w_t \tag{34}$$

Additionally, the FOCs with respect to the household's health (s_t, i_t, r_t, τ_t) are:

$$\begin{aligned}
0 = {} & log(c_{t+1}^s) - \frac{\theta}{2}(n_{t+1}^s)^2 + \tilde{\lambda}_{t+1}^b\big((1-\mu_{t+1}^n)w_{t+1}n_{t+1}^s - (1+\mu_{t+1}^c)c_{t+1}^s\big) \\
& +\lambda_{t+1}^\tau\big(\pi_1 c_{t+1}^s(I_{t+1}C_{t+1}^I) + \pi_2 n_{t+1}^s(I_{t+1}N_{t+1}^I) + \pi_3 I_{t+1}(1-\mu_{t+1}^l)\big) \\
& -\frac{\lambda_t^s}{\beta} + \lambda_{t+1}^s
\end{aligned} \tag{35}$$

$$\begin{aligned} 0 = log(c^i_{t+1}) - \frac{\theta}{2}(n^i_{t+1})^2 + \tilde{\lambda}^b_{t+1}\big((1-\mu^n_t)n^i_{t+1}w_{t+1} - (1+\mu^c_{t+1})c^i_{t+1}\big) \\ -\frac{\lambda^i_t}{\beta} + \lambda^i_{t+1}(1-\pi_r-\pi_d) + \lambda^r_{t+1}\pi_r \end{aligned} \tag{36}$$

$$\begin{aligned} 0 = log(c^r_{t+1}) - \frac{\theta}{2}(n^r_{t+1})^2 + \tilde{\lambda}^b_{t+1}\big((1-\mu^n_{t+1})n^r_{t+1}w_{t+1} - (1+\mu^c_{t+1})c^r_{t+1}\big) \\ -\frac{\lambda^r_t}{\beta} + \lambda^r_{t+1} \end{aligned} \tag{37}$$

$$\lambda^i_t = \lambda^\tau_t + \lambda^s_t \tag{38}$$

Whereas the conditions regarding Capital accumulation and Bonds are:

$$\tilde{\lambda}^b_t = \beta\tilde{\lambda}^b_{t+1}(r^k_{t+1} + 1 - \delta) \tag{39}$$

$$\tilde{\lambda}^b_t = \tilde{\lambda}^b_{t+1}\beta\frac{R^b_t}{\pi_{t+1}} \tag{40}$$

Supply Side

The supply side of the model consists of a single, perfectly competitive firm producing a final good, and a continuum of monopolistically competitive producers of intermediate-goods. The final good firm only takes intermediate-goods as inputs to produce its single good output, whereas intermediate-good producers use capital and labour in their production process.

In contrast to intermediate-good producers having a certain degree of market power in setting the respective prices of their goods, they face perfect competition in the factor markets. Furthermore, intermediate-good firms cannot freely adjust their prices each period as they are subjected to probabilistic price setting frictions in the sense of Calvo (1983).

Final Good Firm

The final good firm aggregates intermediate goods into one, final good according to the technology defined as:

$$Y_t \quad = \quad \Big(\int_0^1 Y_{i,t}^{\frac{1}{\gamma}}\ di\Big)^{\gamma} \tag{41}$$

The optimization problem of the final goods firm is to maximize its profits by choosing the optimal amount of every intermediate-good i, given its price .The solution to this problem leads to the following conditional input demands for every intermediate-good i:

$$Y_{i,t} = \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}Y_t \tag{42}$$

where P_t refers to the aggregate price level in the economy and $P_{i,t}$ to the prices of intermediate producers. The parameter γ denotes the mark-up of intermediate-good firms.

Intermediate Good Firms

Intermediate-good firms optimize their behaviour along two dimensions. First, they have to choose their optimal demands for the input factors labour N_t and capital K_t, given their desired level of production. As factor markets are perfectly competitive, all intermediate-good firms face the same factor prices W_t and R_t^K in this decision process.

Then, having monopoly power while simultaneously facing sticky prices in the sense of Calvo (1983), intermediate producers have to choose the desired prices for their respective goods, given the risk of being stuck with these prices for several periods.

The optimal input demands of each firm i are determined by solving the following cost minimization problem:

$$\min_{N_{i,t},K_{i,t}} L = -W_t N_{i,t} - R_t^k K_{i,t} + \lambda_{i,t}\Big(AK_{i,t}^{1-\alpha}N_{i,t}^{\alpha} - \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}} Y_t\Big)$$

where $AK_{i,t}^{1-\alpha}N_{i,t}^{\alpha} = AF(N_{i,t}, K_{i,t})$ denotes the Cobb-Douglas production function that is common to every intermediate-good firm. The solution to the minimization problem leads to the following conditional input demand and marginal cost (mc_t) functions , which are equal across *all* intermediate firms, as every firm faces the same production technology and the same factor prices.

$$w_t = mc_t A\alpha\Big(\frac{N_t}{K_t}\Big)^{\alpha-1} \tag{43}$$

$$r_t^k = mc_t A(1-\alpha)\Big(\frac{N_t}{K_t}\Big)^{\alpha} \tag{44}$$

$$mc_t = \frac{w_t^{\alpha}(r_t^k)^{1-\alpha}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{45}$$

Having determined their conditional input demands, intermediate-goods producers set their final prices by maximizing the flow of all current and future, real profits:

$$\frac{\pi_{i,t}}{P_t} = \frac{P_{i,t}}{P_t}Y_{i,t} - (w_t N_{i,t} + r_t^k K_{i,t}) = \frac{P_{i,t}}{P_t}Y_{i,t} - mc_t Y_{i,t} = \frac{P_{i,t}}{P_t}\Big(\frac{P_{i,t}}{P_t}\Big)^{-\eta} Y_t - mc_t\Big(\frac{P_{i,t}}{P_t}\Big)^{-\eta} Y_t \tag{46}$$

subject to the constraint that in each period they are allowed to change their price with probability $1-\xi$ or remain stuck with the previous period's price with probability ξ. The formal maximization problem corresponding to this probabilistic price setting framework is defined as:

$$\max_{P_{i,t}} \sum_{j=0}^{\infty} (\xi\beta)^j \frac{\tilde{\lambda}^b_{t+j}}{\tilde{\lambda}^b_t} \left(\left(\frac{P_{i,t}}{P_{t+j}}\right)^{(1-\eta)} Y_{t+j} - mc_{t+j} \left(\frac{P_{i,t}}{P_{t+j}}\right)^{-\eta} Y_{t+j} \right)$$

where $\eta = \frac{\gamma}{\gamma-1}$ and firms are assumed to discount each periods profits by the psychological discount factor, β, the probability of still being stuck with period t's price j periods ahead, as well as the stochastic discount factor $\frac{\tilde{\lambda}^b_{t+j}}{\tilde{\lambda}^b_t}$. The corresponding FOC to this maximization problem is:

$$\begin{aligned} (1-\eta)(P^*_{i,t})^{-\eta} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}^b_{t+j} P^{\eta-1}_{t+j} Y_{t+j} & \\ + \eta P^{-\eta-1}_{i,t} \sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}^b_{t+j} P^{\eta}_{t+j} Y_{t+j} mc_{t+j} & \overset{!}{=} 0 \\ \iff P^*_{i,t} = \underbrace{\left(\frac{-\eta}{1-\eta}\right)}_{=\gamma} \frac{\sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}^b_{t+j} P^{\eta}_{t+j} Y_{t+j} mc_{t+j}}{\sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}^b_{t+j} P^{\eta-1}_{t+j} Y_{t+j}} & = P^*_t \end{aligned}$$

In this first-order condition, none of the variables which define the optimal price of firm i that is allowed to reset its price in period t are specific to such firm. Therefore, all firms that *are allowed* to reset their price in period t choose the same price.

Another way of representing the FOC above is by the term $P^*_t = \frac{k^f_t}{f_t}$, which can be obtained by replacing the infinite sums in the fraction with the recursive auxiliary variables:

$$k^f_t = \gamma mc_t \tilde{\lambda}^b_t Y_t P^{\eta}_t + \beta\xi k^f_{t+1}$$

which by forward substituion recovers the expression

$$\sum_{j=0}^{\infty} (\xi\beta)^j \tilde{\lambda}^b_{t+j} P^{\eta}_{t+j} Y_{t+j} mc_{t+j}$$

and

$$f_t = P^{\eta-1}_t Y_t \tilde{\lambda}^b_t + \beta\xi f_{t+1}$$

which by forward substituion recovers the expression

$$\sum_{j=0}^{\infty}(\xi\beta)^j \tilde{\lambda}_{t+j}^b P_{t+j}^{\eta-1} Y_{t+j}$$

Defining the optimal price $P_{i,t}^*$ through such recursive variables is, apart from simplifying notation, necessary to solve the model using Dynare, as Dynare cannot handle expressions composed of infinite sums.

In order to formulate the model in terms of gross inflation rates $\pi_t = \frac{P_t}{P_{t-1}}$ rather than in absolute price levels, I further define the auxiliary variables above in "real" terms according to:

$$K_t^f = \frac{k_t^f}{P_t^\eta} = \gamma mc_t \tilde{\lambda}_t^b Y_t + \beta\xi \frac{k_{t+1}^f}{P_t^\eta} \frac{P_{t+1}^\eta}{P_{t+1}^\eta} = \gamma mc_t \tilde{\lambda}_t^b Y_t + \beta\xi K_{t+1}^f \pi_{t+1}^{\frac{\gamma}{\gamma-1}} \tag{47}$$

and

$$F_t = \frac{f_t}{P_t^{\eta-1}} = Y_t \tilde{\lambda}_t^b + \beta\xi \frac{f_{t+1}}{P_t^{\eta-1}} \frac{P_{t+1}^{\eta-1}}{P_{t+1}^{\eta-1}} = Y_t \tilde{\lambda}_t^b + \beta\xi F_{t+1} \pi_{t+1}^{\frac{1}{\gamma-1}} \tag{48}$$

With these modified auxiliary variables, the optimal price to which firms reset their prices in period t, given that they are allowed to, can be defined as:

$$P_t^* = \frac{k_t^f}{f_t} = \frac{P_t^\eta}{P_t^{\eta-1}} \frac{K_t^f}{F_t} = P_t \frac{K_t^f}{F_t}$$

which after dividing through by $1/P_{t-1}$ leads to the following expression:

$$\pi_t^* = \pi_t \frac{K_t^f}{F_t} \tag{49}$$

Hence, according to equation eq:reset price inflation, inflation in the optimal price level of firms that are allowed to reset their prices in period t is proportional to the overall level of inflation and the expressions K_t^f and F_t.

Equilibrium and Aggregation

Due to the presence of a continuum of intermediate-goods producers, which can all set their individual prices, the aggregate price level in the economy is defined by a price index. This index is defined through the equation for aggregate, nominal output:

$$\begin{aligned}
P_tY_t &= \int_0^1 P_{i,t}Y_{i,t}\ di \\
\iff P_tY_t &= \int_0^1 P_{i,t}\Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}Y_t\ di \\
\iff P_t^{\frac{1}{1-\gamma}} &= \int_0^1 P_{i,t}^{\frac{1}{1-\gamma}}\ di
\end{aligned}$$

which, due to the calvo assumption and the fact that all firms reset their price to the same value, if given the opportunity, can be rewritten as

$$\begin{aligned}
P_t^{\frac{1}{1-\gamma}} &= \int_0^{1-\xi} (P_{i,t}^*)^{\frac{1}{1-\gamma}}\ di + \int_{1-\xi}^1 P_{i,t-1}^{\frac{1}{1-\gamma}}\ di \\
\iff P_t^{\frac{1}{1-\gamma}} &= (1-\xi)(P_t^*)^{\frac{1}{1-\gamma}} + \xi P_{t-1}^{\frac{1}{1-\gamma}} \\
\iff \pi_t^{\frac{1}{1-\gamma}} &= (1-\xi)(\pi_t^*)^{\frac{1}{1-\gamma}} + \xi \qquad (50)
\end{aligned}$$

To derive an aggregate production function for the economy based on the output of intermediate-goods- and the final good firm, it can be used that, in equilibrium, the supply of each intermediate-good i has to equal its demand and aggregate capital demand has to equal aggregate capital supply :

$$\begin{aligned}
AK_{i,t}^{1-\alpha}N_{i,t}^{\alpha} &= \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}Y_t \\
\iff A\Big(\frac{N_t^{\alpha}}{K_t^{\alpha}}\Big)K_{i,t} &= \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}Y_t \\
\iff A\Big(\frac{N_t^{\alpha}}{K_t^{\alpha}}\Big)\int_0^1 K_{i,t}\ di &= \int_0^1 \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}Y_t\ di \\
\iff AK_t^{1-\alpha}N_t^{\alpha} &= Y_tv_t \\
\iff Y_t &= \frac{AK_t^{1-\alpha}N_t^{\alpha}}{v_t} \qquad (51)
\end{aligned}$$

In the derivation above, it was used that the capital to labour ratio is equal across all intermediate firms. This relationship can be observed by taking the fraction $\frac{w_t}{r_t^k}$ composed of equations 43 and 44 and noting that none of the variables in this fraction depend on variables specific to firm i. Furthermore, the derivation above also implicitly defined the variable:

$$v_t = \int_0^1 \Big(\frac{P_{i,t}}{P_t}\Big)^{\frac{-\gamma}{\gamma-1}}\ di$$

which is a measure for the aggregate price dispersion in the economy. Applying once more the properties of the calvo pricing assumption, this variable can also be written in a recursive form and in terms of inflation rather than in price levels:

$$v_t = \int_0^{1-\xi} \left(\frac{P_{i,t}^*}{P_t}\right)^{\frac{-\gamma}{\gamma-1}} di + \int_{1-\xi}^1 \left(\frac{P_{i,t-1}}{P_t}\right)^{\frac{-\gamma}{\gamma-1}} di$$

$$\iff v_t = \int_0^{1-\xi} \left(\frac{P_{i,t}^*}{P_{t-1}}\right)^{\frac{-\gamma}{\gamma-1}} \left(\frac{P_{t-1}}{P_t}\right)^{\frac{-\gamma}{\gamma-1}} di + \int_{1-\xi}^1 \left(\frac{P_{i,t-1}}{P_{t-1}}\right)^{\frac{-\gamma}{\gamma-1}} \left(\frac{P_{t-1}}{P_t}\right)^{\frac{-\gamma}{\gamma-1}} di$$

$$\iff v_t = (1-\xi)(\pi_t^*)^{\frac{-\gamma}{\gamma-1}} \pi_t^{\frac{\gamma}{\gamma-1}} + \xi \pi_t^{\frac{\gamma}{\gamma-1}} v_{t-1} \tag{52}$$

Finally, for a general Equilibrium solution to hold, it further has to be satisfied that the consumption and labour supply of all household members equals aggregate consumption demand and labour supply, so

$$C_t = s_t c_t^s + i_t c_t^i + r_t c_t^r \quad and \quad N_t = s_t n_t^s + i_t n_t^i + r_t n_t^r$$

and that aggregate labour and capital supply equal aggregate labour and capital demand

$$\int_0^1 N_{i,t}\, di = N_t \quad , \quad \int_0^1 K_{i,t}\, di = K_t$$

Additionally, it has to be satisfied that $B_t = 0$ for Bond markets to clear and SIR variables on the household level have to be equal to aggregate SIR variables

$$s_t = S_t, \quad i_t = I_t, \quad r_t = R_t, \quad d_t = D_t$$

Combining these general equilibrium conditions with the budget constraints of the household:

$$(1+\mu_t^c)P_t(s_t c_t^s + i_t c_t^i + r_t c_t^r) + P_t x_t + \psi_t + B_{t+1} =$$
$$(1-\mu_t^n)W_t(s_t n_t^s + i_t n_t^i + r_t n_t^r) + R_t^k k_t + R_{t-1}B_t + \phi_t$$

and the government:

$$\psi_t + \mu_t^c P_t C_t + \mu_t^n W_t N_t = P_t G$$

it follows that:

$$P_t C_t + P_t X_t + P_t G = W_t N_t + R_t^k K_t + \phi_t$$

Since ϕ_t denotes the total profits of the monopolistically competitive firms, it is, using (46), defined as:

$$\phi_t = \int_0^1 \pi_{i,t} \; di = \int_0^1 P_{i,t} Y_{i,t} \; di - \int_0^1 W_t N_{i,t} + R_t^k K_{i,t} \; di$$

which, imposing equilibrium in the markets for labour and capital, is equal to:

$$\phi_t = P_t Y_t - W_t N_t - R_t^k K_t$$

Plugging this into the constraint above and dividing by the price level leads to the standard aggregate resource constraint:

$$C_t + X_t + G = Y_t \tag{53}$$