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The Dynamics of Exchange Traded Funds: a geometrical and topological approach

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Abstract

Using a metric related to the returns correlation, a method is applied to the reconstruction of an economic space from Exchange-Traded Funds (ETFs) data. In the past, the same method was used in a geometrical analysis of times series of stock returns implying that the most of the systematic information of that market is contained in a space of small dimension. Here we have worked with ten years of daily returns of 85 ETF securities and the same dimensional reduction was obtained. Having a metric defined in the space of ETF securities, a topological approach is used to define a complete network of ETFs and its corresponding Minimum Spanning Tree (MST). An outstanding separation of the two main classes of securities over the MST is uncovered. The dimensional reduction as well as the uncovered pattern in the topological structure, they both emerge from the data itself rather than from any modelling resolution.

Keywords: Dimensional Reduction, Sthocastic Geometry, Market Networks, Financial Markets, ETFs, Financial Crises

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1 Introduction

Capital markets allow to allocate capital stocks, exchanging money from savers to entrepreneurs, optimizing the capital allocation and therefore the outcome to society. It is accomplished by different players, with multiple roles such as: retail investors, professional investors, pension and endowment funds and insurance companies. These players share the same space, with banks and governments, the ultimate liquidity providers.

Traditional economic and financial theories, steeped in axiomatic methodologies, often fall short in elucidating the market's stylized facts. Consider, for instance, the Efficient Market Hypothesis (EMH), based on the fundamental assumption that market prices incorporate the entirety of available information. Furthermore, this hypothesis hinges on the assumption that market participants are inherently rational, supporting their stock transactions on complete information. These foundational assumptions are so deeply ingrained that any endeavor to prognosticate such prices is rendered futile and unlikely to yield success.

On the other hand, if market prices are expected to follow a stochastic pattern, where gains and losses offset each other, it would be difficult to discern any structure beyond a randomly generated one. Nevertheless, the idea of a random structure is challenged by empirical evidence, exemplified by some stylized facts, which uncover the presence of patterns, both in time and space.

To address stylized facts of financial markets researchers have adopted alternative approaches in recent decades [9] [10], [14], [3], [7], [6]. In the past, a geometric analysis of series of returns has been preformed. In so doing, rather than viewing market prices as a purely stochastic process, the Stochastic Geometry Technique (SGT) [10] allowed for uncovering structures and patterns in stock markets [2], challenging the assumption of complete randomness. It has allowed to explore dependencies among financial assets, banking institutions, and countries [13], in the global financial landscape.

In the present paper, the Stochastic Geometry Technique (SGT) [10] is applied to explore the geometrical relations among a representative set of market components, based on their historical returns. Instead of the set of stocks used in the past, now a geometric analysis is performed on a large set of Exchange-Traded Funds (ETFs).

Since the early 2000s, ETFs have become one of the most popular investment vehicles in the globe, boasting billions of dollars in assets under management. Compared to single stocks and indexes, ETFs offer several advantages in this context. Firstly, those types of assets are investable, which means that researchers can directly analyze the behavior of investors and their impact on the market dynamics. Secondly, ETFs provide diversified exposure to different asset classes, which helps to reduce the idiosyncratic risk associated with individual stocks.

Reference [5] highlights the advantages of using ETFs as investable assets in constructing portfolios. ETFs provide diversified exposure to different asset classes, reduce idiosyncratic risk, and offer transparency in data access and analysis. The authors present a global portfolio encompassing 11 asset classes, including equities, bonds, loans, real estate, private equity, and cash, within 87 securities. Such a portfolio offers a more accurate representation of the global capital stock, accommodating both financial and non-financial assets. This approach provides valuable insights for investors seeking optimal asset allocation strategies in the global financial markets.

Often promoted as cheaper and better than mutual funds, ETFs offer low-cost diversification, trading, and arbitrage options for investors. Typically tracking a particular index, sector, or asset class, such as equity (large cap, small cap, emergent), fixed-income (high grade, high yield), commodities (metals, agricultural), and even volatility, it has been widely adopted by agents as a proxy for asset classes in portfolio construction.

Geometric analysis may be complemented with a topological one. In so doing, we are able to address existing relations in the global markets and to further explore the nature of financial systems.

The globalization of economies has led to a significant increase in the interdependence of countries within the global financial system. The repercussions of major financial crises, such as the late-2000s financial crisis, underscore the critical role of understanding financial systems as interconnected networks of countries with cross-border financial linkages [12]. As economies and financial markets become more intertwined, the influence of market movements and economic news from one country is almost instantaneously transmitted to others through various information channels, such as professional information providers, media, and social media. This high degree of connectedness makes the global financial system remarkably sensitive to changes in any of its components and therefore suitable to be described as a network of financial agents.

Scagliarini et al. (2020) [11] further affirm the interconnected nature of financial markets, highlighting that economic globalization has facilitated strong links among the equity markets of different countries. The performance and log-return dynamics of market indices from various regions exhibit significant correlations due to these interconnections. This connectedness fosters a constant flux of information between market indices, amplifying the potential for synergistic information transfer within the global financial system.

1.1 Research Questions

In this paper, it was undertaken an exploration of the connectedness of an ETF market, employing a geometric and a topological analysis through the application of the SGT [2]. By incorporating some insights from the Global Market Portfolio (GMP) as proposed by reference [5], we address the following research questions.

1. Does the ETF market bring new dimensions to the low-dimensional geometric space earlier obtained using market stocks?

2. Can the induction of a network of ETF securities help to identify important patterns in the ETF market structure?

3. Does such a topological approach contribute to the understanding of the dynamics driving the ETF financial system?

Answering these questions could not only bring a contribution to the field of financial network analysis but also offer a first step in guiding investors and policymakers in optimizing their investment strategies in the global financial landscape.

The remaining of the paper is organized as follows: In Section 2, we provide the presentation of the data. Section 3 is dedicated to the presentation of the methodology employed in this study. Section 4 presents and discuss the results obtained so far. Finally, Section 5, concludes and outlines future work.

2 Data

In our data selection process, we have gathered raw data from Exchange-Traded Funds (ETFs) chosen to encompass the wide array of asset classes from Bloomberg. The Appendix comprises a detailed description of each asset class, comprising their respective performances and characteristics.

2.1 Data outline

A total of 85 securities, representing 11 distinct asset classes, were gathered, covering the time range from January 1, 2013, to December 31, 2022. In cases where any specific ETF data was unavailable, the alternative source was the index that the ETF is designed to replicate. In a few instances, close substitutes were employed to ensure that each security was faithfully represented as it would be within the GMP proposal [5].

Class	**# Securities**	**Cumulative Return**
Equity	47	0.279212
Public Debt(Gov)	13	-0.151828
Money Market	4	-0.305978
Non-Securitized Loans	3	-0.293062
Non-Financial Bonds	5	-0.199814
Financial Bonds	3	-0.246523
Securitized Loans	2	-0.202297
Private Equity	1	1.120536
Real Estate	1	0.050192
Land	2	0.593620
Cash	4	-0.194691

Table 1: Eleven asset classes comprising 85 securities

Table 1 provides an overview of the securities used in our research. These securities were selected from a range of ETFs to represent various asset classes, following the method outlined in reference [5]. Notably, the class `Equity` emerges as the dominant asset class with 47 securities, while `Private Equity` shows the highest Cumulative Return of 112%. Cumulative Return is computed as $\left(\prod_{t=1}^{N}\left(\frac{P_{i,t}}{P_{i,0}}\right)\right)^{252} - 1$, where $P_{i,t}$ is the security price at time t and 252 stands for a year of 252 trading days. A detailed description of the securities within each class can be found in the Appendix.

Of the 85 securities, 68 are already priced in US dollars, 8 in British pounds, 6 in Euros and 1 in Canadian dollars, 1 in Japanese Yen, and 1 in Hong Kong dollars. Nonetheless, all prices were adjusted to be reflected in dollars, so the portfolio is USD hedged. With that transformation, we also reduce the dimension of the problem, removing any currency effect.

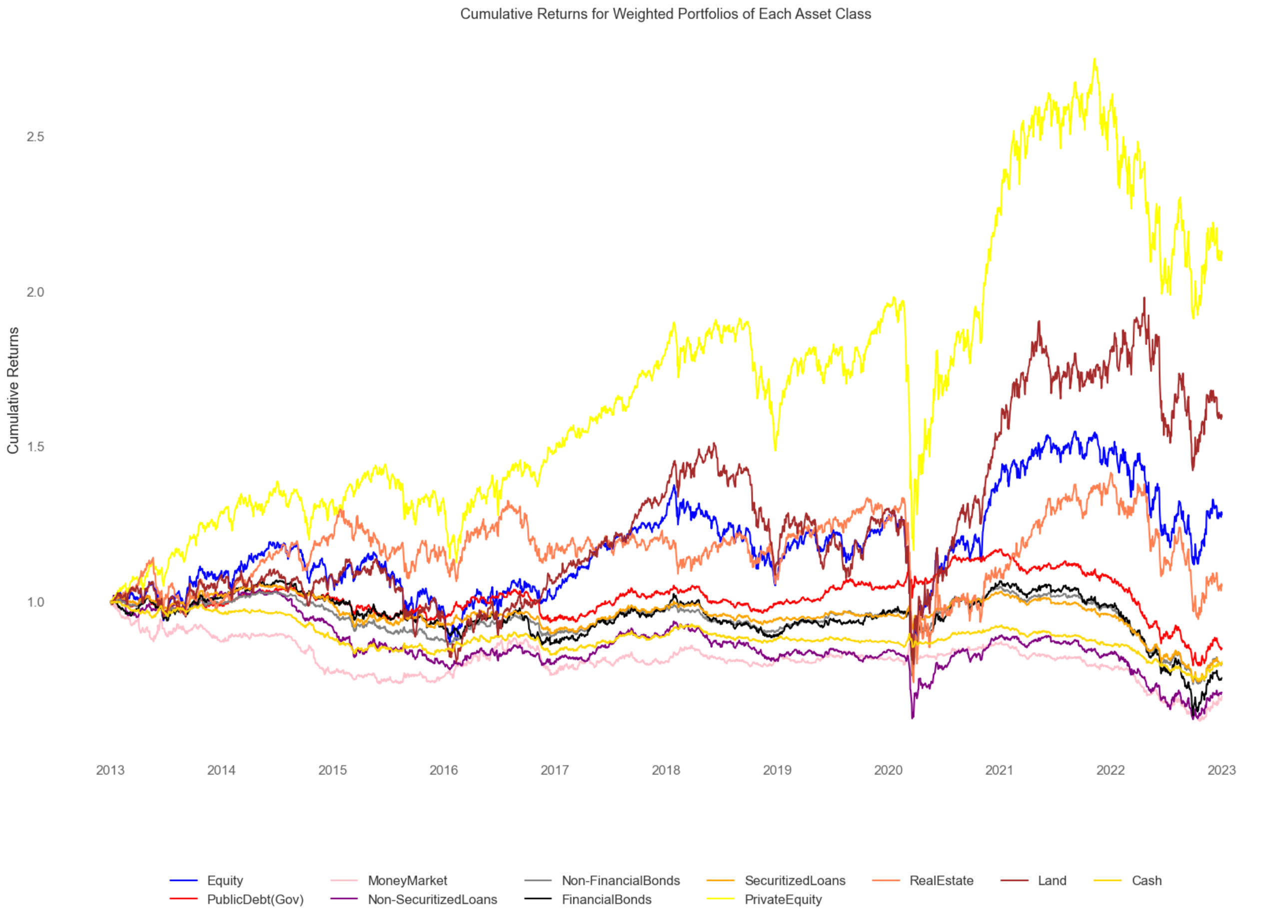

Figure 1: The evolution of the cumulative one-day returns for each asset class

The plot presented in Fig.1 shows the dynamics of various asset classes represented by their cumulative returns . While individual securities may exhibit variations in volatility, the overarching observation is the co-movement of assets within the same class. This shared behavior underscores the influence of broader market trends, economic conditions, and external factors on these asset classes. A remarkable example of such a shared behaviour is the impact of the Covid Crisis in 2020, where various asset classes demonstrated correlated reactions, reinforcing the idea that market dynamics shows interconnected performance patterns across different classes of assets.

3 Methodology

The Stochastic Geometry Technique (SGT)

According to reference [2], the idea can be stated simply by the following steps:

1. Choose a representative set of n ETFs and collect their historical return data over a specific time period.

2. From the correlation between each pair of securities, compute the distance matrix among the n ETFs daily-returns. This transforms the problem into an embedding problem, wherein the goal is to determine the smallest manifold that encompasses the given set. The computed distances between ETFs, derived from their return fluctuations, encompass both systematic and unsystematic contributions. Hence, to extract factor information from the market, it is necessary to separate these two effects. We employ the following SGT.

   The correlation matrix is computed based on the logarithmic price returns of the selected ETFs. Let r_{ij} represent the logarithmic price returns of ETFs i and j over a given time window of t observations. The correlation coefficient C_{ij} is then calculated as:

   $$C_{ij} = \frac{\sum_{k=1}^{n}(r_{ik} - \bar{r}_i)(r_{jk} - \bar{r}_j)}{\sqrt{\sum_{k=1}^{n}(r_{ik} - \bar{r}_i)^2}\sqrt{\sum_{k=1}^{n}(r_{jk} - \bar{r}_j)^2}} \tag{1}$$

   where $\bar{r}_i$ and $\bar{r}_j$ are the means of the logarithmic price returns for ETFs i and j, respectively.

3. From the distance matrix, compute the coordinates of the n ETFs in a Euclidean space with a dimension smaller than n.

   From correlation matrix, we construct a matrix of distances, denoted as D_{ij}.

   $$D_{ij} = \sqrt{2(1 - C_{ij})} \tag{2}$$

   D_{ij} represents the distances between the returns $\vec{l}_i$ and $\vec{l}_j$ of ETFs i and j, respectively. These distances are calculated along the time window of t observations.

   Having obtained the distance matrix for the set of n asset classes, their coordinates in R^{N-1} can be defined. Each ETF is represented by a set $\{\vec{x}_i\}$ of points in R^{N-1}. We calculate the center of mass $\vec{R}$ and the center of mass coordinates $\vec{y}_k = \vec{x}_k - \vec{R}$.

4. Apply the standard analysis of reducing the coordinates to the center of mass and determine the eigenvectors of the inertial tensor.

5. Repeat the same procedure on surrogate data, which involves independent time permutations for each ETF.

Compute the covariance distance matrix, denoted as $T_{ij} = \sum_k y_i(k)y_j(k)$. This matrix is diagonalized to obtain the set of eigenvalues and normalized eigenvectors $\{\lambda_i, \vec{e}_i\}$. The eigenvectors $\vec{e}_i$ define the characteristic directions of this geometric space. The coordinates $z_i(k)$ along these directions are obtained by projection: $z_i(k) = \vec{y}(k) \cdot \vec{e}_i$.

6. Compare the eigenvalues obtained in step 4 with those from step 5. Identify the directions in which the eigenvalues significantly differ as the characteristic dimensions of the market. This approach aims to identify the empirically constructed variables that drive the market, and the number of surviving eigenvalues represents the effective dimension of this economic space. To analyze the significance of the eigenvalues, they are compared with those obtained from surrogate data, including random and time-permuted data. The characteristic directions correspond to the eigenvalues λ_i that significantly differ from those obtained from the surrogate data. These eigenvalues define a subspace V_d of dimension d that contains the systemic information related to the interbank market structure. Once the number of characteristic dimensions d is defined, we denote the restriction of the asset classes k to the subspace V_d as $\vec{z}(k, d)$. Additionally, we define $d(d)_{ij}$ as the distances between ETFs returns i and j restricted to this low-dimensional space.

4 Results

Fig.2 shows the decrease of the top-30 eigenvalues of both surrogate and actual data, now computed for data set of 85 ETF securities. By examining the decrease of the largest 30 eigenvalues, we are informed that, as in the mutiple studies using stocks ([1],[2],[8],[12] and [13]) the number of relevant dimensions shaping the market structure is six.

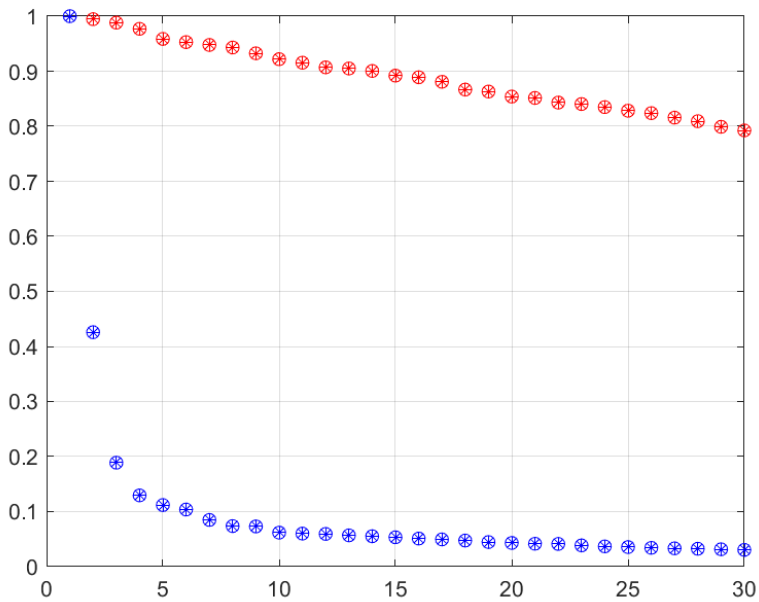

Figure 2: The 30 largest eigenvalues of actual data *versus* the 30 largest eigenvalues of surrogate data

The most relevant characteristic directions are those that correspond to the eigenvalues which are clearly different from those obtained from surrogate or random data. They define a subspace that carries the (systematic) information related to the market correlation structure.

Time-permuted data were generated by permuting each ETF security (one-day return data) randomly in time. As each security is independently permuted, time correlations among securities disappear while the resulting surrogate data preserve the mean and the variance that characterize actual data.

Looking at the decrease of the eigenvalues of both surrogate and actual data, we observe that a significant amount of information is concentrated in the top six eigenvalues. This observation indicates that a small number of factors or dimensions play a crucial role in shaping market dynamics. From the 7th eigenvalue to the 30th, their decrease is very similar for both actual and surrogate data.

It was empirically found in other papers that stock markets of different sizes, ranging from 70 to 424 stocks, across different time windows and also from different market indexes (S&P500 and Dow Jones), can be described by six effective dimensions as discussed in references [10], [14] and [2]. The six-dimensional spaces define the reduced subspaces which carry the systematic information related to the correlation structures of the markets. Indeed, the six effective dimensions capture the structure of the deterministic correlations and economic trends that are driving the market, whereas the remainder of the market space may be considered as being generated by random fluctuations.

4.1 Geometrical perspective

Since the systematic information governing market dynamics can be effectively represented in a lower-dimensional subspace the first three dimensions appear to be crucial in capturing the underlying structure of deterministic correlations and economic trends that drive the market dynamics.

Furthermore, by comparing the decay of actual eigenvalues with those obtained from surrogate data, we have observed that the initial six dimensions account for significant patterns and trends in the market dynamics. In contrast, eigenvalues smaller than the 6th one exhibit similar behavior in both the actual data and the surrogate data. The three-dimensional plot in Fig.3 shows the manifold obtained from applying the SGT to the set of 85 securities and taking the time series of the whole ten-years time period. There, one sees that the securities belonging to the Equity class (in blue) occupy close positions in the three-dimensional space. Differently, securities belonging to the other ten classes seem to occupy unrelated positions in that reduced space.

$$V(t) = \sqrt[3]{\prod_{i=1}^{3} \lambda_i(t)} \tag{3}$$

The corresponding Market Space Volume ($V(t)$) is now computed for the first three dimen-

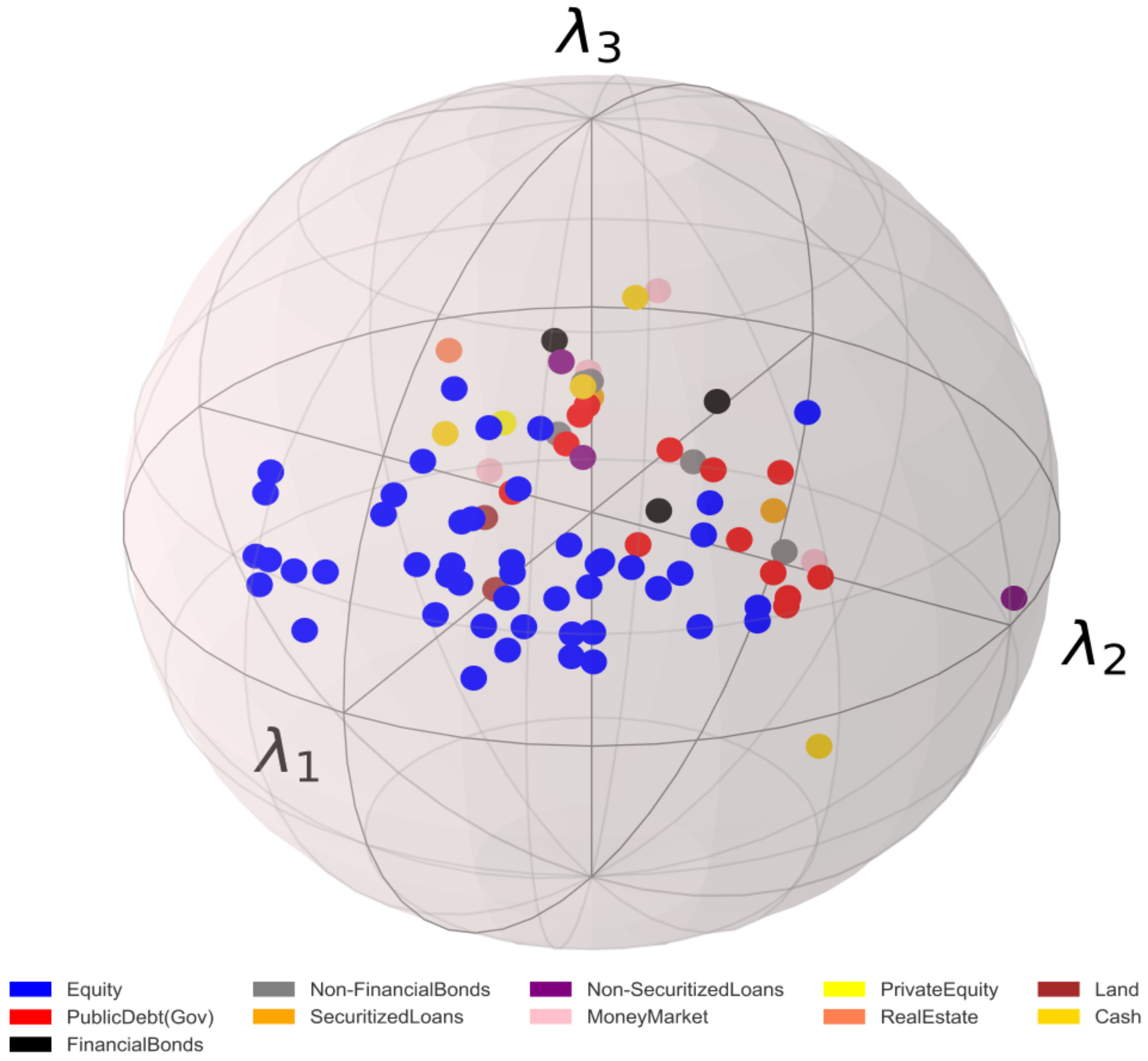

Figure 3: The three-dimensional manifold of 85 ETF securities

sions, represented by the largest three eigenvalues $(\lambda_i(t))$.The three-dimensional plot in Fig.3 has a space volume of $V(t) = 72.16$. Performing the same calculation for a 3-dimensional subspace constructed from the surrogate data, the space volume shows a much larger manifold, where $V(t) = 195.16$. Such an increased volume of a subspace constructed from the surrogate data is due to the lost of synchronization of the market fluctuations. Because each security is independently permuted, time correlations among securities disappear. On the contrary, when the volume is computed for actual data and using the first three dimensions, the synchronization of the market fluctuations leads to a contraction of volume in that reduced subspace.

4.2 The topological perspective

Now, the focus is shifted to a topological perspective. This approach allows for constructing a complete network from the distances between each pair of ETF securities.

In so doing, the market entities (e.g., securities) are represented as nodes, and the links between each pair of nodes is defined by the inverse of their corresponding distance.

In the complete network presented in Fig.4, the highlighted links correspond to those con-

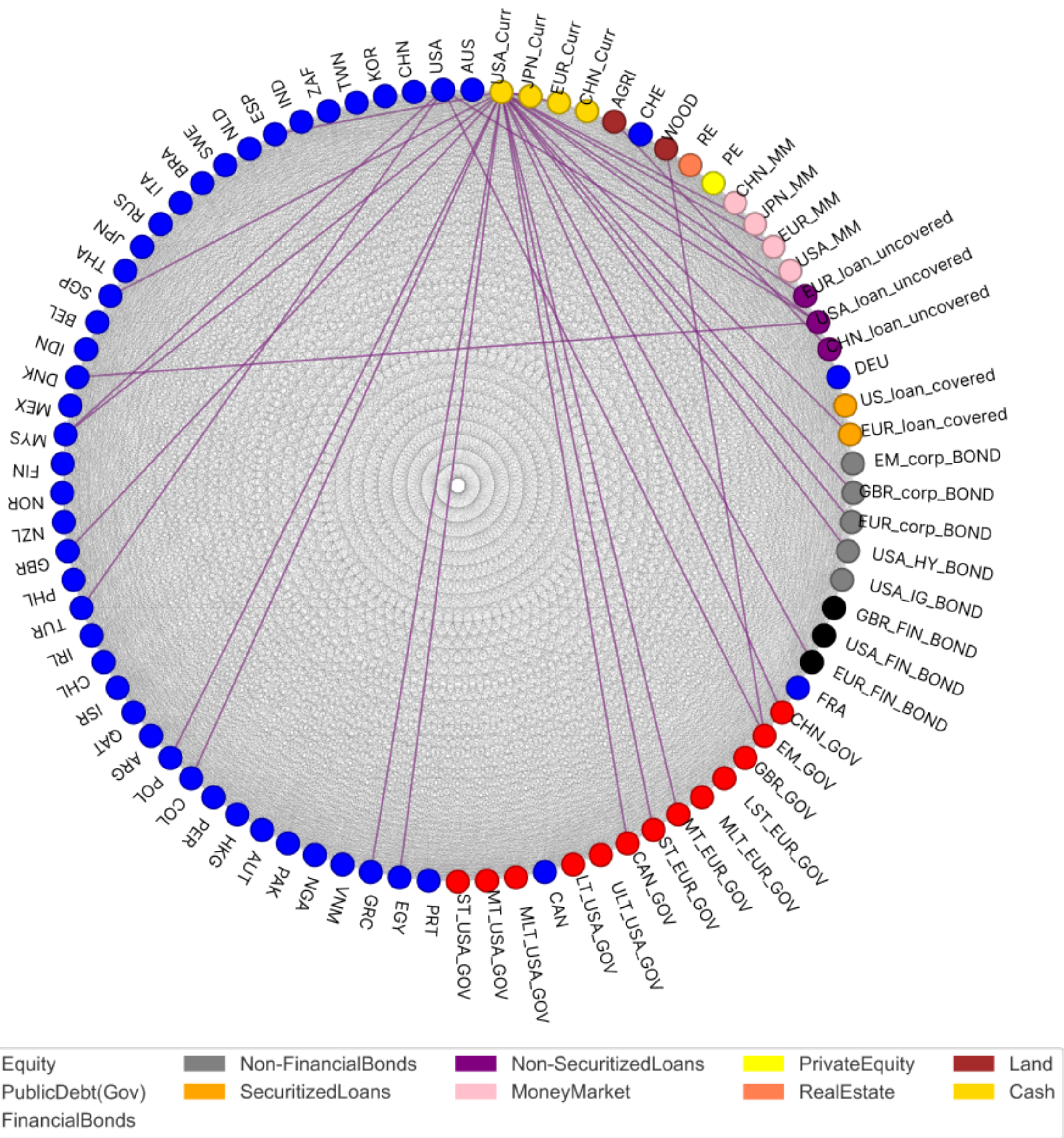

Figure 4: Complete network of 85 securities

necting nodes at large distances. It shows that the largest distances (weaker links) seem to characterize links relating non-equity entities, i.e. nodes outside the class of equities. There are few highlighted links connecting equities and when they happen to occur, the connected pair of nodes in the large majority of cases, include a non-entity node.

Once the complete network of securities is defined, one is able to extract its Minimum Spanning Tree (MST). The densely-connected nature of the complete network presented in Fig.4 does not help to characterize its topological structures. The large number of links make the extraction of the truly relevant connections forming the network a challenging problem. One first step in the direction of extracting relevant information from a given network is to obtain its corresponding MST.

4.3 Minimum Spanning Tree

In the construction of a Minimum spanning tree (MST) by the *nearest neighbor* method, one takes the ETF securities as the nodes (n_i) of a weighted and connected graph. From the nxn distance matrix $D_{i,j}$, a hierarchical clustering is performed using the *nearest neighbor* method. Initially n clusters corresponding to the n Author Keywords are considered. Then, at each step, two clusters c_i and c_j are clumped into a single cluster if

$$d\{c_i, c_j\} = \min\{d\{c_i, c_j\}\}$$

with the distance between clusters being defined by

$$d\{c_i, c_j\} = \min\{d_{pq}\} \text{ with } p \in c_i \text{ and } q \in c_j$$

This process is continued until there is a single cluster. This clustering process is also known as the *single link method*, this being the method by which one obtains the MST of a graph [4]. In a connected graph, the MST is a tree of $n-1$ edges that minimizes the sum of the edge distances. In a network with n nodes, the hierarchical clustering process takes $n-1$ steps to be completed, and uses, at each step, a particular distance $d_{i,j} \in D$ to clump two clusters into a single one.

In so doing, the MST is a tree that spans all nodes in a connected graph while minimizing the total sum of edge weights. This approach, applied to market networks, enables to explore the underlying structure of the market and extract meaningful insights from a sparser network representation.

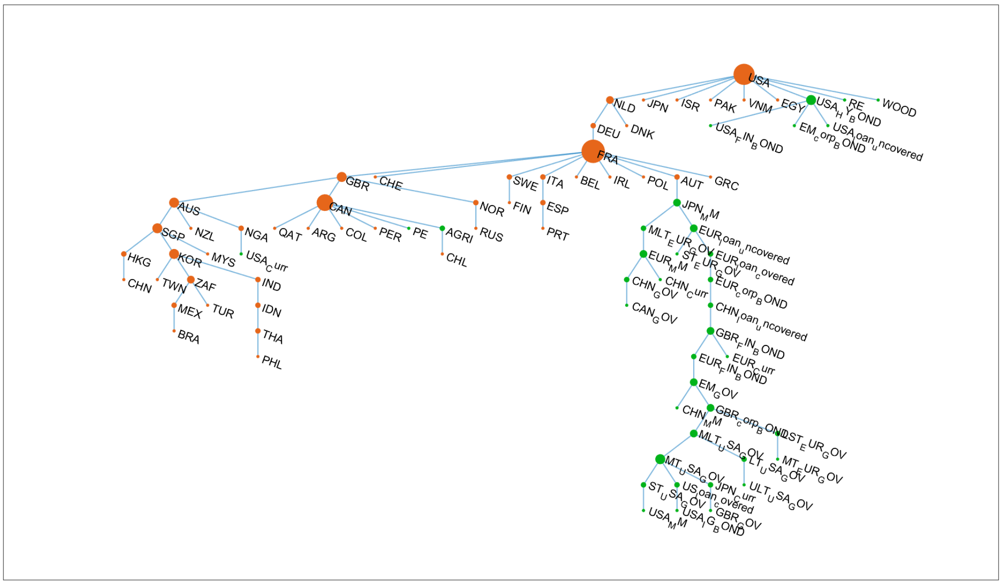

Figure 5: MST of the network with 85 securities

The MST depicted in Fig.5 illustrates the interplay of securities within the ETF financial landscape. There, the size of the nodes are defined by their degrees. Larger nodes represent those with a high number of links.

The MST structure can be understood by considering several economic and financial factors, which can be related to clusters i.e., to sets of nodes occupying close positions on the branches of the tree. The MST in Fig.5 highlights an important cluster comprising all the nodes belonging to the Equity class, where France (FRA) and USA work as outstanding hubs. Having FRA as a root, a first hierarchically structured set of securities is organized. It gives place to a branch (in green) rooted in the JPN_MM node, which comprises just non-equity securities. On the other hand, another hierarchically structured set is rooted in FRA (those with larger sizes, in red) and comprises just equities. Such an outstanding separation of the two main classes of securities over the MST highlights the utility of this topological approach where the observed pattern emerges from the data itself.

In the extreme right of Fig.5, the USA node is linked to some European securities like, Germany (DEU), France (FRA) and UK (GBR) and Canada (CAN). Those links may be accounted by the fact that the involved countries are affected by the same geopolitical events, since the trading and economic relations are highly interconnected between bigger economies. Geopolitical events can create temporary but significant links between securities, reflecting the market's reaction to international affairs. The monetary policies of entities like the central banks are key influences that affect the behaviour of ETF securities globally.

Also, interestingly is the observation of a great deal of securities of large Emerging Markets clustered on the left side of the MST. There, the *BRICS* (BRA, CHN, IND) and the Four Asian Tigers (HKD, SGP, KOR) are placed along with other similar economies as South Africa (ZAF), Mexico (MEX) and New Zealand (NZL). Yet another cluster is rooted in Canada (CAN), where some highly dependent on commodities markets stay closed on the tree: big oil producers like Russia (RUS), Norway (NOR), Qatar (QAT), Nigeria (NGA), and Argentina (ARG), Colombia (COL), Peru (PER) and Chile (CHL) also highly commoditized economies. The place of the AGRI node highlights the dependence on commodities. The interaction among commodities, especially those securities tied to oil prices, form distinctive patterns uncovered by the MST.

5 Concluding remarks

In the terms of the research questions raised at the beginning of this paper, and according to the results presented in the last section, we conclude that:

1. The geometric analysis of the ETF time series does not imply the need for new dimensions besides the six dimensions obtained in the past to represent stock markets like the stocks of the SP&500 or Dow Jones. Therefore, it implies that the systematic information of the ETF market is also contained in a dimensional reduced space.

2. The Market Space Volume ($V(t)$) of the dimensional reduced space computed for ten years of both actual and surrogate data shows a huge increase in the later case. Such an increased volume of a subspace constructed from the surrogate data is due to the lost of synchronization of the ETF market fluctuations.

3. The complete network of ETF securities shows that most of the long-distant nodes (weak linked nodes) correspond to links relating non-equity entities.

4. The computation of the MST from the complete network of ETF securities shows an outstanding separation of the two main classes of securities over the MST. In so doing, it highlights the utility of the topological approach where the observed pattern emerges from the data itself.

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A Appendix

As the following two tables show, Equity is the largest asset class with 47 securities. Private Equity exhibits the highest return in the sample, with an annual return of 7.53% and a cumulative return of 112.05%. Land class features the highest annualized volatility at 18.92%, while Cash shows the lowest volatility at 4.72%.

Super Class	Asset Class	Simple Ticker	GSV Equivalent	Bloomberg Ticker	Bloomberg Name	Start Date	Exchange	NAV	Currency	NAV USD (July-23)	Index tracked	Bloomberg Name index	Start Date
Equity	Equity	USA	iShares Russell 3000	IWV US Equity	ISHARES RUSSELL 3000 ETF	26.05.2000	US	260.2909	USD	260.29	RU30INTR Index	Russell 3000 TR Idx	29.12.1978
Equity	Equity	CHN	iShares MSCI China	MCHI US Equity	ISHARES MSCI CHINA ETF	31.03.2011	US	45.5122	USD	45.51	NDEUCHF Index	MSCI Daily TR Net China	31.12.1998
Equity	Equity	JPN	iShares MSCI Japan	EWJ US Equity	ISHARES MSCI JAPAN ETF	18.03.1996	US	62.2084	USD	62.21	NDDUJN Index	MSCI Daily TR Net Japan	31.12.1969
Equity	Equity	GBR	iShares MSCI UK	EWU US Equity	ISHARES MSCI UNITED KINGDOM	18.03.1996	US	33.255	USD	33.26	NDDUUK Index	MSCI Daily TR Net UK USD	31.12.1969
Equity	Equity	HKG	iShares MSCI Hong Kong	EWH US Equity	ISHARES MSCI HONG KONG ETF	18.03.1996	US	19.3221	USD	19.32	NU730026 Index	NU730026	30.05.2008
Equity	Equity	CAN	iShares MSCI Canada	EWC US Equity	ISHARES MSCI CANADA ETF	18.03.1996	US	35.8029	USD	35.8	M1CXBLK Index	CA CUSTOM C NETR USD	07.06.2017
Equity	Equity	FRA	iShares MSCI France	EWQ US Equity	ISHARES MSCI FRANCE ETF	18.03.1996	US	39.4024	USD	39.4	NDDUFR Index	MSCI Daily TR Net France	31.12.1969
Equity	Equity	DEU	iShares MSCI Germany	EWG US Equity	ISHARES MSCI GERMANY ETF	18.03.1996	US	29.09	USD	29.09	NDDUGR Index	MSCI Daily TR Net German	31.12.1969
Equity	Equity	CHE	iShares MSCI Switzerland	EWL US Equity	ISHARES MSCI SWITZERLAND ETF	18.03.1996	US	47.3732	USD	47.37	M1CH2550 Index	MSCI Swiss 25-50 Nt$	30.11.1998
Equity	Equity	AUS	iShares MSCI Australia	EWA US Equity	ISHARES MSCI AUSTRALIA ETF	18.03.1996	US	23.1611	USD	23.16	NDDUAS Index	MSCI Daily TR Net Austra	31.12.1998
Equity	Equity	KOR	iShares MSCI South Korea	EWY US Equity	ISHARES MSCI SOUTH KOREA ETF	12.05.2000	US	66.32	USD	66.32	M1KR2550 Index	MSCI Korea 25-50 Nt $	30.11.1998
Equity	Equity	TWN	iShares MSCI Taiwan	EWT US Equity	ISHARES MSCI TAIWAN ETF	23.06.2000	US	47.81	USD	47.81	M1CXBLZ Index	MSCI TW 25/50 C NR USD	20.06.2000
Equity	Equity	ZAF	iShares MSCI South Africa	EZA US Equity	ISHARES MSCI SOUTH AFRICA ETF	07.02.2003	US	42.6801	USD	42.68	M1CXBAC Index	ZA 25-50 NETR USD	07.06.2017
Equity	Equity	IND	iShares MSCI India	INDA US Equity	ISHARES MSCI INDIA ETF	03.02.2012	US	44.2343	USD	44.23	NDEUSIA Index	MSCI Emerging Markets In	31.12.1998
Equity	Equity	ESP	iShares MSCI Spain	EWP US Equity	ISHARES MSCI SPAIN ETF	18.03.1996	US	29.486	USD	29.49	M1CXBLRP Index	MSCI ES 25-50 NR USD	31.12.1998
Equity	Equity	NLD	iShares MSCI Netherlands	EWN US Equity	ISHARES MSCI NETHERLANDS ETF	18.03.1996	US	44.1515	USD	44.15	M1CXNIC Index	NL IMI 25-50 Cust NR USD	12.03.1996
Equity	Equity	SWE	iShares MSCI Sweden	EWD US Equity	ISHARES MSCI SWEDEN ETF	18.03.1996	US	35.7875	USD	35.79	M1CXBLV Index	MSCI SE 25/50 C NR USD	29.03.1996
Equity	Equity	BRA	iShares MSCI Brazil	EWZ US Equity	ISHARES MSCI BRAZIL ETF	14.07.2000	US	32.9958	USD	33	M1CXBLRN Index	MSCI BR 25-50 NR USD	10.07.2000
Equity	Equity	ITA	iShares MSCI Italy	EWI US Equity	ISHARES MSCI ITALY ETF	18.03.1996	US	32.9828	USD	32.98	M1CXBLRM Index	MSCI ITALY 25-50 NR USD	31.12.1998
Equity	Equity	RUS	iShares MSCI Russia	ERUS US Equity	ISHARES MSCI RUSSIA ETF	10.11.2010	US	0.0699	USD	0.07	MSEURU$N Index	MSCI Russia 25-50 Index	31.08.2009
Equity	Equity	THA	iShares MSCI Thailand	THD US Equity	ISHARES MSCI THAILAND ETF	28.03.2008	US	68.4396	USD	68.44	M1CXTH5I Index	MSCI THAILAND IMI	26.03.2008
Equity	Equity	SGP	iShares MSCI Singapore	EWS US Equity	ISHARES MSCI SINGAPORE ETF	18.03.1996	US	19.2771	USD	19.28	M1CXBLY Index	MSCI SG 25/50 C NR USD	29.03.1996
Equity	Equity	BEL	iShares MSCI Belgium	EWK US Equity	ISHARES MSCI BELGIUM ETF	18.03.1996	US	19.1634	USD	19.16	M1BE5IM Index	MSCI Belgium I 25-50NETR	31.12.1969
Equity	Equity	IDN	iShares MSCI Indonesia	EIDO US Equity	ISHARES MSCI INDONESIA ETF	07.05.2010	US	23.534	USD	23.53	NU724020 Index	INDESA IMI 25-50 NETRUSD	31.05.1994
Equity	Equity	DNK	iShares MSCI Denmark	EDEN US Equity	ISHARES MSCI DENMARK ETF	26.01.2012	US	109.3157	USD	109.32	M1DK5IM Index	MSCI Denmark I 25-50Nt	28.11.2011
Equity	Equity	MEX	iShares MSCI Mexico	EWW US Equity	ISHARES MSCI MEXICO ETF	18.03.1996	US	62.9126	USD	62.91	M1CXBLRL Index	MSCI MX IMI 25-50 NR USD	31.12.1998
Equity	Equity	MYS	iShares MSCI Malaysia	EWM US Equity	ISHARES MSCI MALAYSIA ETF	18.03.1996	US	20.8871	USD	20.89	NDDUMAF Index	MSCI Daily TR Net Malays	31.12.1998
Equity	Equity	FIN	iShares MSCI Finland	EFNL US Equity	ISHARES MSCI FINLAND ETF	26.01.2012	US	35.4074	USD	35.41	M1FI5IM Index	MSCI Finland I 25-50Nt	28.11.2011
Equity	Equity	NOR	iShares MSCI Norway	ENOR US Equity	ISHARES MSCI NORWAY ETF	24.01.2012	US	23.7502	USD	23.75	M1NO5IM Index	MSCI Norway I 25-50 Nt	28.11.2011
Equity	Equity	NZL	iShares MSCI New Zealand	ENZL US Equity	ISHARES MSCI NEW ZEALAND ETF	02.09.2010	US	48.9452	USD	48.95	M1NZ5IM Index	MSCI N Zealand IM2550Nt$	03.12.2012
Equity	Equity	PHL	iShares MSCI Philippines	EPHE US Equity	ISHARES MSCI PHILIPPINES ETF	29.09.2010	US	26.9826	USD	26.98	NU710465 Index	NU710465	30.11.1998
Equity	Equity	TUR	iShares MSCI Turkey	TUR US Equity	ISHARES MSCI TURKEY ETF	28.03.2008	US	31.3103	USD	31.31	NU724019 Index	TR IMI 25-50 NETR USD	31.05.1994
Equity	Equity	IRL	iShares MSCI Ireland	EIRL US Equity	ISHARES MSCI IRELAND ETF	07.05.2010	US	59.558	USD	59.56	M1AIECP Index	MSCI ALL IRELAND CP NRU	25.11.2008
Equity	Equity	CHL	iShares MSCI Chile	ECH US Equity	ISHARES MSCI CHILE ETF	16.11.2007	US	31.032	USD	31.03	M1CL5IM Index	MSCI Chile IMI 25-50 Nt$	30.11.1998
Equity	Equity	ISR	iShares MSCI Israel	EIS US Equity	ISHARES MSCI ISRAEL ETF	28.03.2008	US	57.4468	USD	57.45	MISCNU Index	MSCI Israel Capped Index	06.07.2007
Equity	Equity	QAT	iShares MSCI Qatar	QAT US Equity	ISHARES MSCI QATAR ETF	01.05.2014	US	18.2301	USD	18.23	M1QLCE Index	All Qatar Capped NETR $	25.11.2008
Equity	Equity	ARG	Global X MSCI Argentina	ARGT US Equity	GLOBAL X MSCI ARGENTINA ETF	03.03.2011	US	47.42	USD	47.42	M1AAR5R Index	MSCI Al Argentina25-50Nt	30.11.2010
Equity	Equity	POL	iShares MSCI Poland	EPOL US Equity	ISHARES MSCI POLAND ETF	26.05.2010	US	20.6615	USD	20.66	M1CXPL5I Index	MSCI POLAND IMI	25.03.2010
Equity	Equity	COL	iShares MSCI Colombia	GXG US Equity	GLOBAL X MSCI COLOMBIA ETF	09.02.2009	US	22.62	USD	22.62	M1CXGXB Index	MSCI A CO S 25/50 NR USD	30.11.2009
Equity	Equity	PER	iShares MSCI Peru	EPU US Equity	ISHARES MSCI PERU AND GLOBAL	22.06.2009	US	32.9335	USD	32.93	M1PECAPD Index	MSCI ALL PERU CAPPED NR	24.05.2010
Equity	Equity	AUT	iShares MSCI Austria	EWO US Equity	ISHARES MSCI AUSTRIA ETF	18.03.1996	US	21.156	USD	21.16	M1AT5IM Index	MSCI Austria IMI25-50Nt$	31.12.1969
Equity	Equity	PAK	Global X MSCI Pakistan	PAK US Equity	GLOBAL X MSCI PAKISTAN ETF	23.04.2015	US	13.29	USD	13.29	MXPK Index	MSCI PAKISTAN	31.12.1992
Equity	Equity	NGA	Global X MSCI Nigeria	NGE US Equity	GLOBAL X MSCI NIGERIA ETF	03.04.2013	US	10.36	USD	10.36	M1ANI5R Index	MSCI Al Nigeria 25-50 Nt	30.11.2010
Equity	Equity	VNM	Market Vectors Vietnam	VNM US Equity	VANECK VIETNAM ETF	14.08.2009	US	14.1943	USD	14.19	MVVNMTR Index	MVIS Vietnam (TRN)	15.12.2006
Equity	Equity	GRC	Global X MSCI Greece	GREK US Equity	GLOBAL X MSCI GREECE ETF	08.12.2011	US	39.46	USD	39.46	M1CXGXA Index	MSCI ALL GR 25-50 NR USD	14.01.2016
Equity	Equity	EGY	Market Vectorsgypt	EGPT US Equity	VANECK EGYPT INDEX ETF	18.02.2010	US	19.2708	USD	19.27	MVEGPTTR Index	MVIS Egypt (TRN)	29.12.2005
Equity	Equity	PRT	Global X FTSE Portugal 20	PGAL US Equity	GLOBAL X MSCI PORTUGAL ETF	13.11.2013	US	10.76	USD	10.76	MXPT Index	MSCI PORTUGAL	02.01.1995
[HTML]EFEFEF Bonds	Public Debt (Gov)	ST_USA_GOV	iShares 1-3 Yr Treasury ETF	SHY US Equity	ISHARES 1-3 YEAR TREASURY BO	26.07.2002	US	81.084	USD	81.08	IDCOT1B4 Index	USTsy 1-3 4PM	31.12.2004
[HTML]EFEFEF Bonds	Public Debt (Gov)	MT_USA_GOV	iShares 3-7 Yr Treasury ETF	IEI US Equity	ISHARES 3-7 YEAR TREASURY BO	11.01.2007	US	115.3785	USD	115.38	IDCOT3B4 Index	USTsy 3-7 4PM	31.12.2004
[HTML]EFEFEF Bonds	Public Debt (Gov)	MLST_USA_GOV	iShares 7-10 Yr Treasury ETF	IEF US Equity	ISHARES 7-10 YEAR TREASURY B	26.07.2002	US	96.3987	USD	96.4	IDCOT7B4 Index	USTsy 7-10 4PM	31.12.2004
[HTML]EFEFEF Bonds	Public Debt (Gov)	LST_USA_GOV	iShares 10-20 Yr Treasury ETF	TLH US Equity	ISHARES 10-20 YEAR TREASURY	11.01.2007	US	109.5989	USD	109.6	IDCOT10B Index	USTsy 10-20 4PM	31.12.2005
[HTML]EFEFEF Bonds	Public Debt (Gov)	ULST_USA_GOV	iShares +20 Yr Treasury ETF	TLT US Equity	ISHARES 20+ YEAR TREASURY BO	26.07.2002	US	101.6501	USD	101.65	IDCOT324 Index	USTsy 20+ 4PM	31.12.2004
[HTML]EFEFEF Bonds	Public Debt (Gov)	CAN_GOV	iShares Canada Gov Bond ETF	XGB CN Equity	ISHARES CORE CANADIANGOVERNM	06.11.2006	CN	18.8296	CAD	14.24	CBAGUTR Index	Universe All Govt	30.12.2005
[HTML]EFEFEF Bonds	Public Debt (Gov)	ST_EUR_GOV	iShares Euro Gov 1-3 Yr ETF	IBGS LN Equity	ISHARES EUR GOVT 1-3YR	05.06.2006	LN	119.3254	GBP	153.38	BCEX6T Index	BCEX6T	31.12.1998
[HTML]EFEFEF Bonds	Public Debt (Gov)	MT_EUR_GOV	iShares Euro Gov 3-5 Yr ETF	IBGX LN Equity	ISHARES EURO GOVT 3-5Y	30.10.2006	LN	136.1048	GBP	174.95	BCEX2T Index	5 Y Term TR	31.12.1998
[HTML]EFEFEF Bonds	Public Debt (Gov)	MLST_EUR_GOV	iShares Euro Gov 7-10 Yr ETF	IBGM NA Equity	ISHARES EURO GOVT 7-10YR	24.04.2007	NA	184.6163	EUR	205.37	BCEX4T Index	10 Y Term TR	31.12.1998
[HTML]EFEFEF Bonds	Public Debt (Gov)	LST_EUR_GOV	iShares Euro Gov 15-30 Yr ETF	IBGL LN Equity	ISHARES EURO GOVT 15-30YR	30.10.2006	LN	154.8907	GBP	199.1	BCEX1T Index	30 Y Term TR	31.12.1998
[HTML]EFEFEF Bonds	Public Debt (Gov)	GBR_GOV	Vanguard UK Gov Bond ETF	VGOV LN Equity	VANG GBPGVBD GBPD	22.05.2012	LN	16.63588	GBP	21.38	L24FTRGU Index	Gilts Float Adjusted	31.07.2009
[HTML]EFEFEF Bonds	Public Debt (Gov)	EM_GOV	iShares EM Gov Bond ETF	EMGB LN Equity	VANECK JPM EM LOC CUR BD ETF	14.06.2017	LN	58.828	GBP	75.62	GBIEMCOR Index	J.P. Morgan Government Bond In	31.12.2007
[HTML]EFEFEF Bonds	Public Debt (Gov)	CHN_GOV	CSOP China 5-Year Treasury	CBON US Equity	VANECK CHINA BOND ETF	11.11.2014	US	22.0993	USD	22.1	SPBCNGBT Index	S&P China Government Bill Index	29.12.2006
[HTML]EFEFEF Bonds	Financial Bonds	EUR_FIN_BOND	iShares Bond Euro Financials ETF	IS3B GY Equity	ISHARES EURO CORP BOND FINCL	29.07.2013	GY	95.5671	EUR	106.31	LEEFTREU Index	Financials	30.06.1998
[HTML]EFEFEF Bonds	Financial Bonds	USA_FIN_BOND	iShares iBoxx Investment Grade ETF	SPHY US Equity	SPDR PORTFOLIO HIGH YIELD BO	19.06.2012	US	22.8987	USD	22.9	H0A0 Index	US High Yield	31.08.1986
[HTML]EFEFEF Bonds	Financial Bonds	GBR_FIN_BOND	iShares UK Corporate £ Bond ETF	SLXX LN Equity	ISHARES CORE GBP CORP	29.03.2004	LN	117.9784	GBP	151.65	IBXXGLAT Index	GBP liq Corp Large Cap	31.12.2007
[HTML]EFEFEF Bonds	Non-Financial Bonds	USA_IG_BOND	iShares US iBoxx Investment Grade Corp ETF	LQD US Equity	ISHARES IBOXX INVESTMENT GRA	26.07.2002	US	107.6365	USD	107.64	IBOXIG Index	Liquid Investment Grade TR	31.12.1998
[HTML]EFEFEF Bonds	Non-Financial Bonds	USA_HY_BOND	iShares US iBoxx HY Corp ETF	HYG US Equity	ISHARES IBOXX HIGH YLD CORP	11.04.2007	US	75.1518	USD	75.15	IBOXHY Index	Liquid High Yield TR	31.12.1998
[HTML]EFEFEF Bonds	Non-Financial Bonds	EUR_corp_BOND	iShares Bond Euro ex-Financial ETF	EEXF LN Equity	ISHARES EURO CORP EX-FINCL	30.09.2009	LN	89.6275	GBP	115.21	LECFTREU Index	Corp Ex Finance	31.12.1998
[HTML]EFEFEF Bonds	Non-Financial Bonds	GBR_corp_BOND	iShares Corporate Bond UK ex-Financials ETF	IBCX LN Equity	ISHARES EURO CORP LARGE CAP	17.03.2003	LN	119.5044	EUR	132.94	IBXXELAT Index	EUR liq Corp Large Cap	31.12.2007
[HTML]EFEFEF Bonds	Non-Financial Bonds	EM_corp_BOND	iShares $ Emerging Markets Corporate ETF	CEMB US Equity	ISHARES JP MORGAN EM CORPORA	19.04.2012	US	43.2366	USD	43.24	JBCDCORE Index	JP EM CORP BOND Index	NA
Securitized/Non-Securitized loans	Securitized Loans	EUR_loan_covered	iShares Euro Covered Bond ETF	ICOV IM Equity	ISHARES EURO COVERED BOND	10.03.2009	IM	134.8245	EUR	149.98	IYGB Index	IBOXX (EUR) COVERED TR	01.07.2003
Securitized/Non-Securitized loans	Securitized Loans	US_loan_covered	iShares MBS ETF	MBB US Equity	ISHARES MBS ETF	16.03.2007	US	93.2009	USD	93.2	LUMSTRUU Index	U.S. MBS	30.01.1976
Securitized/Non-Securitized loans	Non-Securitized Loans	CHN_loan_uncovered	PowerShares Chinese Yuan Dim Sum Bond Portfolio ETF	CNYB LN Equity	ISHARES CHINA CNY BOND USD-D	26.10.2020	LN	3.9529	GBP	5.08	SPBCNCPT Index	S&P China Corporate Bond Index	29.12.2006
Securitized/Non-Securitized loans	Non-Securitized Loans	USA_loan_uncovered	iShares iBoxx US High Yield Corporate Bond ETF	BKLN US Equity	INVESCO SENIOR LOAN ETF	03.03.2011	US	21.1539	USD	21.15	SPBDLL Index	SPLSTAUSLevgLn 100	28.12.2001
Securitized/Non-Securitized loans	Non-Securitized Loans	EUR_loan_uncovered	iShares Euro High Yield Corporate Bond ETF	IHYG LN Equity	ISHARES EURO HY CORP	06.09.2010	LN	90.6224	EUR	100.81	IBOXXMJA Index	EUR Liquid High Yield TR	31.12.2005
[HTML]EFEFEF Cash	Money Market	USA_MM	iShares 1–3 Yr Treasury ETF	SPTS US Equity	SPDR PORT SHRT TRM TRSRY	01.12.2011	US	28.801	USD	28.8	LT01TRUU Index	1-3 Yr	31.01.1992
[HTML]EFEFEF Cash	Money Market	EUR_MM	iShares Euro Gov 1–3 Yr ETF	MTA FP Equity	LYXOR EURO GOV BOND 1-3Y	28.09.2005	FP	118.2327	EUR	131.52	I35206EU Index	Euro Treasury 50bn 1-3 Year	30.07.2010
[HTML]EFEFEF Cash	Money Market	JPN_MM	Japan Currency ETF	PFLJPYL LX Equity	PICTET-SHRT TRM MMKT JPY-I	06.07.2007	LX	100207.26	JPY	707.06	SBWMJY1L Index	1Mo Japan Yen	01.01.1998
[HTML]EFEFEF Cash	Money Market	CHN_MM	CSOP China Ultra Short-Term Bond ETF	3122 HK Equity	CSOP RMB MONEY MARKET ET-HKD	20.01.2015	HK	176.217	HKD	22.54	CFETSGB Index	CFETS Short Term Govt	29.06.2009
Alternatives	Private equity	PE	SPDR MSCI World Small Cap UCITS ETF	WDSC LN Equity	SPDR WORLD SMALL CAP	25.11.2013	LN	95.5192	USD	95.52	NCUDWI Index	MSCI Daily TR Net Small	31.12.1998
Alternatives	Real Estate	RE	iShares Global REIT ETF	REET US Equity	ISHARES GLOBAL REIT ETF	10.07.2014	US	23.9388	USD	23.94	RNXG Index	FTSE RNXG INDEX	31.10.2008
Alternatives	Land	WOOD	iShares Global Timber and Forestry ETF	WOOD US Equity	ISHARES GLOBAL TIMBER & FORE	25.06.2008	US	73.1572	USD	73.16	SPGTTFN Index	S&P Gb Tb&Frstry Net TR	15.11.2002
Alternatives	Land	AGRI	iShares Global Agriculture Index ETF	VEGI US Equity	ISHARES MSCI AGRICULTURE PRO	02.02.2012	US	41.6855	USD	41.69	M1WDSGPI Index	MSCI ACWI SelAgProd I Nt	31.08.2001
[HTML]EFEFEF Cash	Cash	CHN_Curr	CNY	CYB US Equity	WISDOMTREE CHINESE YUAN STRA	14.05.2008	US	24.4898	USD	24.49	CYB US Equity	WISDOMTREE CHINESE YUAN STRA	NA
[HTML]EFEFEF Cash	Cash	EUR_Curr	EUR	FXE US Equity	INVESCO CURRENCYSHARES EURO	12.12.2005	US	102.6549	USD	102.65	FXE US Equity	INVESCO CURRENCYSHARES EURO	NA
[HTML]EFEFEF Cash	Cash	JPN_Curr	JPY	FXY US Equity	INVESCO CURRENCYSHARES JAPAN	13.02.2007	US	65.5902	USD	65.59	FXY US Equity	INVESCO CURRENCYSHARES JAPAN	NA
[HTML]EFEFEF Cash	Cash	USA_Curr	USD	UUP US Equity	INVESCO DB US DOLLAR INDEX B	20.02.2007	US	27.967	USD	27.97	USDUPX Index	DEUTSCHE BANK USD LONG	22.11.2006

Table 2: Securities Table based on GMP

Class	# Securities	Avg.Daily Return	Volatility	Annual Returns	Cum.Returns
Equity	47	0.000152	0.169434	0.024079	0.279212
Public Debt(Gov)	13	-0.000054	0.067060	-0.015786	-0.151828
Money Market	4	-0.000128	0.076519	-0.034677	-0.305978
Non-Securitized Loans	3	-0.000116	0.093115	-0.032956	-0.293062
Non-Financial Bonds	5	-0.000077	0.065236	-0.021309	-0.199814
Financial Bonds	3	-0.000091	0.093921	-0.026980	-0.246523
Securitized Loans	2	-0.000080	0.055765	-0.021603	-0.202297
Private Equity	1	0.000348	0.173382	0.075333	1.120536
Real Estate	1	0.000083	0.178044	0.004743	0.050192
Land	2	0.000250	0.189189	0.046058	0.593620
Cash	4	-0.000079	0.047199	-0.020705	-0.194691

Table 3: Eleven asset classes comprising 85 securities

Table 3 provides a concise summary of the quantity of securities within each asset class, along with key performance metrics including average daily returns, volatility, annual returns, and cumulative returns for the entire data period.