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Asymmetries in Education Spread – "Equilibrium" versus "Social Optimum"

by

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Date: June 2024

Abstract: We try to explain main empirical regularities of the distribution of higher education attainment across regions by using a theoretical framework inspired by Uzawa (1965)'s neoclassical growth model and Lucas (1988)'s view of positive externalities of education. We rationalize the strong correlation between educational attainment and regional accessibility, the relative importance of public universities in less accessible areas and the smaller regional variation in schooling rates shown by the public universities as compared with private establishments.

Keywords: Higher Education, University, Accessibility, Market Potential, Endogenous Technical Progress

JEL Codes: I20, O40, R10

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1. Introduction

It is common sense that post-compulsory schooling rates vary considerably not only across countries, but also across regions within the same country. Furthermore, this kind of geographical asymmetry is mainly fuelled by private schooling decisions while the maximization of social welfare would imply a more even spatial distribution of schooling rates.

In what follows, we consider the spatial distribution of universities or college (ISCED 5-8) attainment rates, as in many countries secondary education is now compulsory.

The idea that educational spread is eased by high population density appears very early in economic thought (see for instance, Von Thünen ,1826, pp. 293-294). This line of reasoning was developed in spatial models such as Salop (1979) and Helsley and Strange (1990), where most costs borne by schools are fixed and students support significant travel costs between residence and the nearest school. In this context, a rise in population density eases the operation of a school network on two different grounds.

First, if the number of schools remains constant, more youngsters "share" the existing fixed assets, namely professors, buildings and laboratories, thereby diminishing the cost per student. Second, if the rise in population allows the number of schools to increase, the average distance between a student's residence and the closest college is reduced so that the match between schools and students improves.

Nevertheless, the consideration of fixed costs and transport costs in a spatial economy fails to account for two important issues of the geographical dispersion in schooling rates. On the one hand, the regional schooling rate seems to be related not only with local population density but also with the overall accessibility or "market potential" of the region, a concept which additionally encompasses the centrality level within a transport network (see Harris, 1954). On the other hand, spatial competition models do not deliver a clear explanation of the gap between the equilibrium spatial distribution of schooling, which is driven by individual decisions, and the socially optimal one as might be commanded by a central planner.

In this paper, we model schooling decisions made by workers engaged in a constant returns to scale productive process. In addition to enhancing his own ability (as in Uzawa, 1965), a worker's decision to complete college has a direct impact on aggregate productivity in line with Lucas (1988).

In this paper, we first try to explain the fact that the productivity enhancing effect of schooling is related with the accessibility of the training worker within the spatial economy, i.e., with his specific ability to communicate and interact with his fellow workers. Furthermore, we assess the influence of worker accessibility on educational efficiency under two distinct regimes, namely a market equilibrium decentralized setting and a social optimum planning context.

2. Motivation

In Table 1, we record data on higher education (ISCED 5-8) attainment, the share of university and college students in publicly owned institutions and population density across NUTSII Portuguese regions in year 2021. We add the *Area Metropolitana do Porto* to the set of regions despite it is not a NUTII region and withdraw its data from the *Norte* region.

Região (NUTII)	(1)	(2)	(3)	(4)
Norte (without Area Metropolitana do Porto)	97	14.7	92.4	13.6
Area Metropolitana do Porto	857	21.0	62.9	13.2
Centro	72	18.1	96.4	17.4
Oeste e Vale do Tejo	89	15.5	90.0	14.0
Grande Lisboa	1489	29.0	75.7	22.0
Península de Setúbal	501	20.5	77.8	15.9
Alentejo	17	14.4	100	14.4
Algarve	94	17.3	90.0	15.6
Região Autónoma dos Açores	103	14.7	100	14.7
Região Autónoma da Madeira	315	16.5	75.0	12.4

Table 1

Meaning of variables:

(1) Population density: unity people per Km^2. Source: PORDATA, with data compiled from DGT/MAAC – MCT, INE.

(2) Higher education schooling rate: Share of resident population older than 15 with a complete tertiary education (ISCED 5-8) degree according to 2021 Census. Source: PORDATA-INE.

(3) Share of tertiary education students enrolled in publicly owned universities and colleges in 2021. Source PORDATA-INE.

(4) Public higher education schooling rate = $\frac{(2)\times(3)}{100}$.

In Figure 1, we plot a scatter diagram of points $(\text{population density, higher education attainment})$ with a line curve fitted by OLS. This plot shows that educational attainment increases with population density.

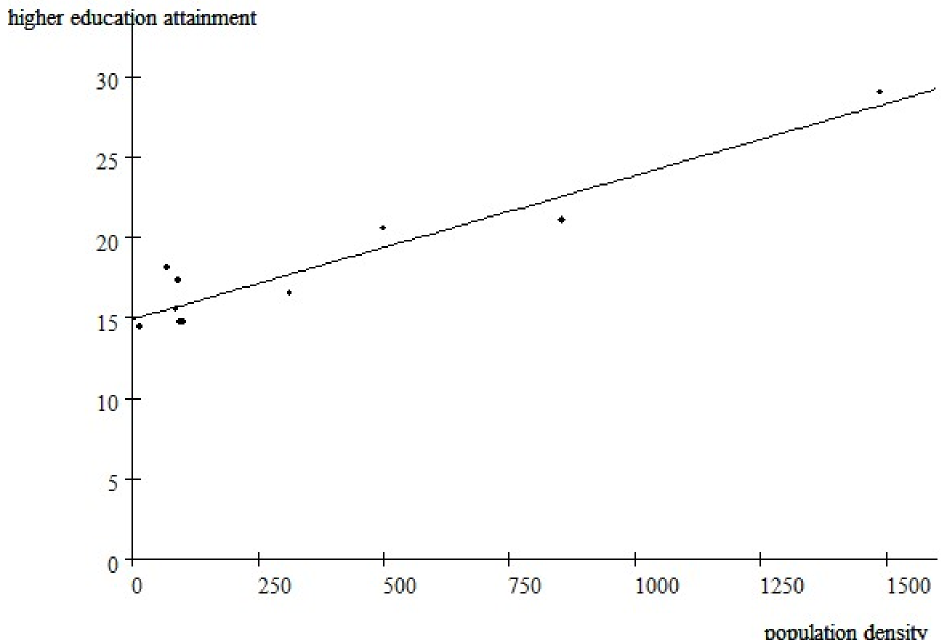

Figure 1: population density and higher education attainment and across regions

In Figure 2, we plot a scatter diagram of points (population density, share of college students enrolled in publicly owned universities) with two lines estimated by OLS. The thin line takes in account all observations and shows a decreasing pattern although relatively tenuous. However, the region *Grande Lisboa* is an outlier. Since it hosts the country's capital, it shows a disproportionately high share of students enrolled by public universities. If we exclude this outlier, the fitted thick OLS line shows a much clearer decreasing pattern.

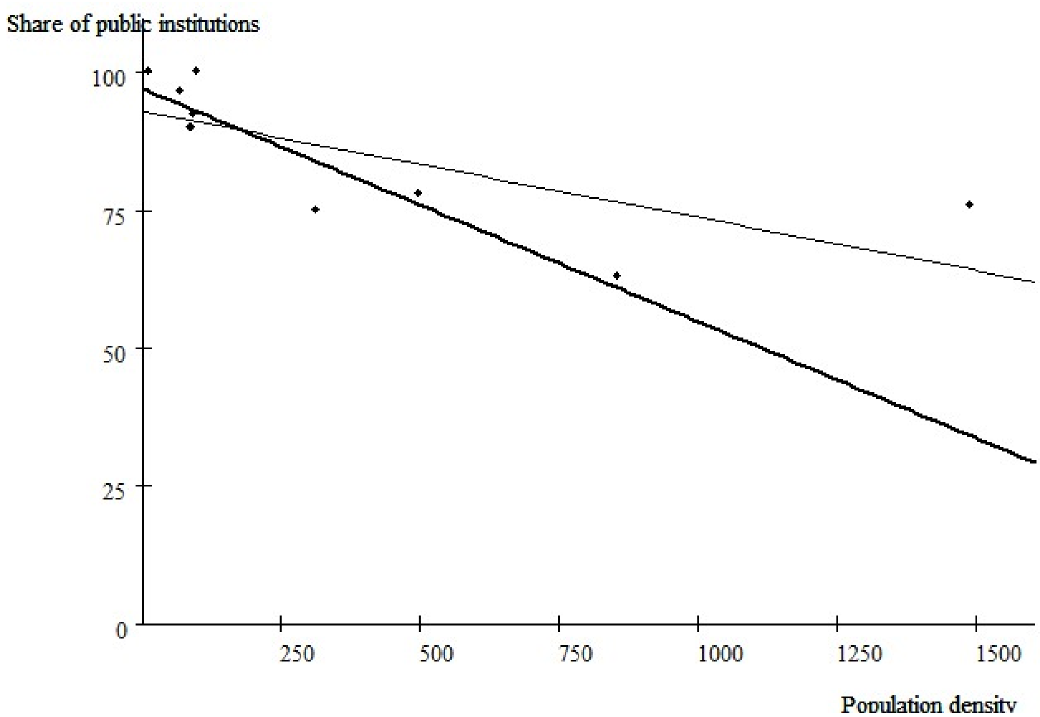

Figure 2: Population density and share of students enrolled in publicly owned universities and colleges

Hence, we try to explain two main empirical regularities. First, the share of people endowed with a university degree is positively correlated with population density. Second, the share of students enrolled by publicly owned universities and colleges seems to decline with population density, except for the region that hosts the country's capital. The latter feature suggests that the trade-off between the "equilibrium" and the "social optimum" tertiary schooling patterns appears to be more stringent in sparsely populated areas.

From the relationships plotted in Figures 1 and 2, we might infer that the regional variation in tertiary education attainment appears to be weaker if we only consider students enrolled by public universities than if we take in account the whole higher education system.

In column (4) of Table 1, we compute a schooling rate concerning only public universities and colleges, which we define for each region as the product of overall tertiary schooling rate times the share of students enrolled in the public higher education system. Then, we calculate coefficients of variation across regions of overall university schooling rate and of the rate in the public subsystem only. We find that the former rate shows a much higher variation coefficient than the latter, the coefficient values being 0.246 and 0.180, respectively.

3. A schooling decision model with endogenous aggregate productivity

We now present the model of a spatial economy where the efficiency of a training decision made by a worker relates with the accessibility of his location, which constrains his ability to communicate or interact with other workers in the region.

We regard the acquisition of skills by means of education as depending on two main factors. First, it implies the investment of a share s of his non-leisure time, so that the individual might dedicate only a proportion $1-s$ to productive labour. Second, this “investment” is more efficient the closer the individual is in relation to his fellow workers. This connection stems from the fact that learning is a group process, where workers obtain skills by engaging in face-to-face contacts with colleagues (see Benabou, 1993). Such contacts are likelier across nearby learners and are clearly hindered by high travel costs.

Before introducing a formal model, we need to be more precise about what we mean as the “accessibility” of a worker’s location. Let us assume that workers are continuously distributed along a closed interval whose length is the unit measure of distance. The density of workers in point r is given by $b(r)$, which we presuppose to be positive everywhere.

Following Fujita and Ogawa (1982), we model the accessibility of point r as its “market potential” $F(r)$, which is defined as the integral of workers’ density in $[0,1]$, spatially discounted by the distance of each location to the individual site in r, i.e., by,

$$F(r) = \int_0^1 b(y) e^{-\tau|r-y|} dy \tag{2}$$

where τ is the unit communication cost. The specification of $F(r)$ means that each worker is more likely to interact with nearby individuals than with far away ones.

Even though the “market potential”concept is sound in theoretical terms, it is difficult to handle both analytically and empirically. Hence, we will deal with two polar cases of $F(r)$, which correspond to extreme values of the unit communication cost τ.

First, we will presuppose that the unit communication cost τ is positive but close to zero. In this case, the “market potential” might be computed by using the linear approximation $1-\tau|r-y|$ of the spatial decay term $e^{-\tau|r-y|}$ around $\tau=0$, so that $F(r)$ becomes,

$$F(r) \approx \int_0^1 b(y)\left(1-\tau|r-y|\right)dy \tag{3}$$

Hence, we may write,

$$F(r) \approx 1-\tau T(r) \tag{4}$$

where we define

$$T(r) \equiv \int_0^1 b(y)|r-y|dy \tag{5}$$

as the *total transaction distance* that an individual in r must travel over to interact with the same probability with every other worker or – which is equivalent – to interact with each colleague once per unit of time. Then, $\tau T(r)$ stands for the total communication cost that an individual worker must support to become trained. Consequently, $\tau T(r)$ is an inverse measure of accessibility and, since it is linear, it might be easily handled analytically, so that we will use this specification in the theoretical model of this paper. By doing so, we follow the line of reasoning by Ogawa and Fujita (1980).

It is clear that $T(r)$ is minimized by the *median* of the spatial distribution of workers. To show this we calculate the first and second derivatives of (5), namely

$$T'(r) = \int_0^r b(y)\,dy - \int_r^1 b(y)\,dy$$
and (6)
$$T''(r) = 2b(r) > 0$$

However, the total transaction distance $T(r)$ is hard to compute empirically. Hence, we will refer to the other polar case where τ is arbitrarily high. In this case, $F(r)$ in (2) is mainly determined by interactions with neighbouring workers, so that we may approximate the "market potential" as follows,

$$F(r) \approx \int_{r-\varepsilon}^{r} b(y)\,dy + \int_{r}^{r+\varepsilon} b(y)\,dy \tag{7}$$

With ε positive and small. Hence, $F(r)$ might be approximated by,

$$F(r) \approx 2\varepsilon b(r) \tag{8}$$

The first and second derivatives of $F(r)$ have the same sign of those concerning the population density in point r. If we presuppose that $b(r)$ is quasiconcave, then the point of maximal accessibility is just its *mode*, i.e., the location r^* that satisfies the conditions of maximum,

$$\begin{aligned} b'(r^*) &\leq 0 \text{ if } r^* = 0 \\ b'(r^*) &= 0 \text{ if } 0 < r^* < 1 \\ b'(r^*) &\geq 0 \text{ if } r^* = 1 \end{aligned} \tag{9}$$

This argument allows us to use the population density of a region as a proxy of its accessibility, as we did in the data that motivated this paper.

In the case of a bell-shaped density $b(r)$, median and mode are coincident when the population density is symmetric, while they differ but are related in the general case (see Figure 3, drawn from Greenhut, Norman and Hung, 1987).

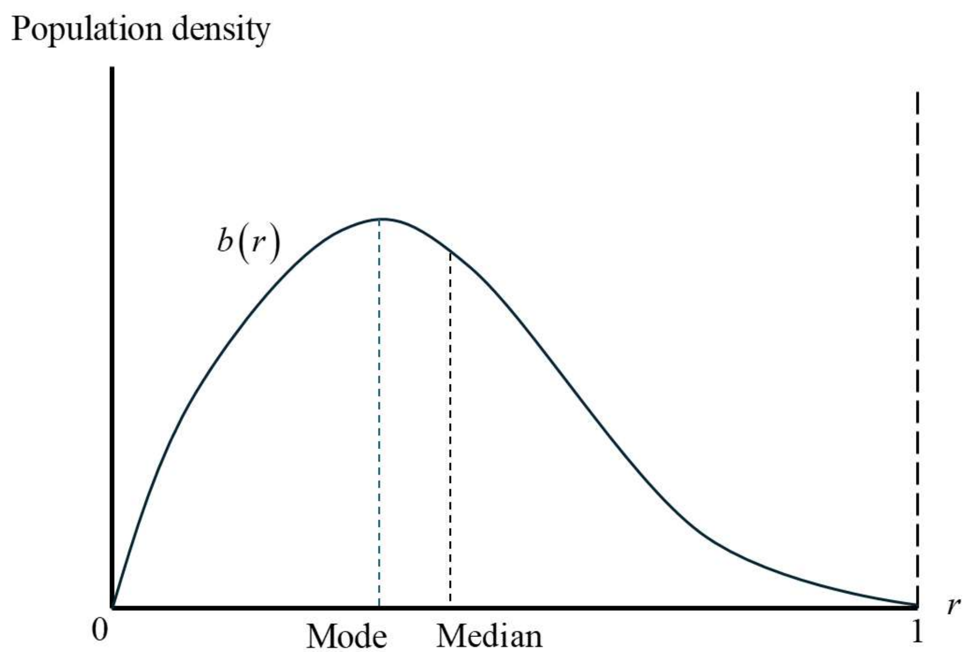

Figure 3: Median and Mode of a quasiconcave population distribution

3.1. The model main assumptions

Workers are uniformly distributed with density L over the interval $[0,1]$. In each point r, L workers produce Y units of a composite consumer good with labour under constant returns to scale according to the following production function,

$$Y(r) = Ah(r)\left[1 - s(r)\right]L \tag{10}$$

where we define the following variables,

- $A > 0$ is an aggregate productivity term, which reflects the technical knowledge available in the economy and it concerns every worker in the economy. In addition, we define,
- $h(r) > 0$ is the skill level of workers living in point r.
- $s(r) \in (0,1)$ is the schooling rate, i.e., the proportion of non-leisure time that workers in point r dedicate to training.

Dividing $Y(r)$ by the number of local workers L, we obtain the output per worker in location r, $y(r)$.

$$y(r) \equiv \frac{Y(r)}{L} = Ah(r)\left[1 - s(r)\right] \tag{11}$$

Skill accumulation between time periods 0 and 1 in location r follows a process that is like the one featured by Uzawa (1965) and Lucas (1988), namely,

$$\frac{h_1(r) - h_0}{h_0} = \delta(r)s(r) \tag{12}$$

For simplicity, we presuppose that , in (12), the skill level in the previous period h_0 is the same in every point in space and $\delta(r) > 0$ stands for the efficiency of training in location r. Since workers must travel to learn and meet each other, $\delta(r)$ is assumed to be a strictly decreasing function of total transaction distance $T(r)$ as was defined in (5). Hence, we have,

$$T(r) = \int_0^1 L\left|r - y\right| dy \quad \text{or}$$

$$T(r) = \frac{L}{2}\left(2r^2 - 2r + 1\right) \tag{13}$$

Dividing $T(r)$ by the number of workers in the economy, we obtain the average distance $t(r)$ that a worker must travel to meet another worker.

$$t(r) \equiv \frac{T(r)}{L} = \frac{1}{2}\left(2r^2 - 2r + 1\right) \tag{14}$$

We plot $t(r)$ over space $[0,1]$ in Figure 4.

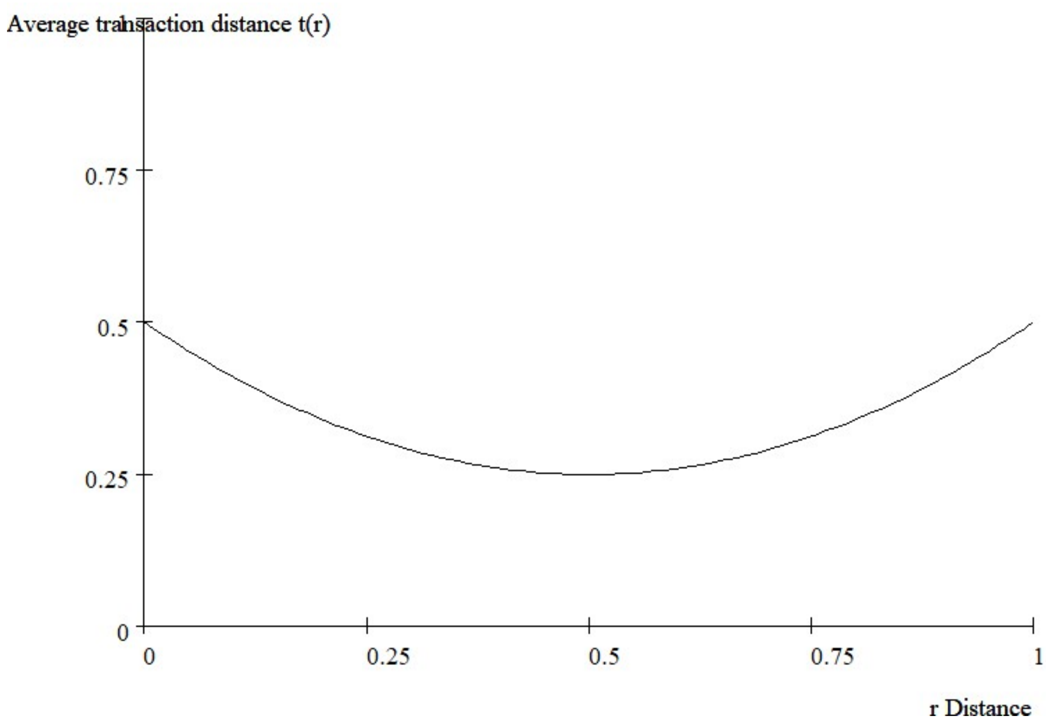

Figure 4: Average transaction distance $t(r)$

We assume that the efficiency of training a worker in point r is a linear strictly decreasing function of $t(r)$, i.e.,

$$\delta(r) = 2\left[1 - t(r)\right] \tag{15}$$

Figure 4 shows that educational efficiency is maximal in the central point and declines as we move to more peripheral locations.

3.2. Schooling in equilibrium

We now deal with the case where workers in each location r make individual decisions on their schooling level $s(r)$ to maximize output per worker $y(r)$. For simplicity, we assume that $h_0 = 1$ in (12) and define $h(r) \equiv h_1(r)$, so that skills in the current time period are,

$$h(r) = 1 + \delta(r)s(r) \tag{16}$$

Given our assumptions, it should be noticed that in (16) $h(r)$ stands both for the level of skills in point r and for their growth rate across time periods.

By substituting $t(r)$ from (14) in (15) and then $\delta(r)$ in (16), the skill level in point r becomes,

$$h(r) = 1 + s(r) + 2rs(r)(1 - r) \tag{17}$$

By substituting (17) in (11), we may write the output per worker in location r as a function of r and $s(r)$, i.e.,

$$y(r,s) = A(1-s(r))\left[1+s(r)+2rs(r)(1-r)\right]$$

In what follows, we wish to determine the optimal schooling level for an individual living in point r. In what follows, $s(r)$ will be replaced by s while keeping the same meaning, so that $y(r,s)$ will be written in alternative simply as,

$$y(r,s) = A(1-s)\left[1+s+2rs(1-r)\right] \tag{18}$$

We maximize the output per worker function (18) in relation to s in each point r. For this purpose, we compute the first and second partial derivatives of y in relation to s.

$$\frac{\partial y}{\partial s} = 2A\left[r(1-r)(1-2s)-s\right] \tag{19}$$

$$\frac{\partial^2 y}{\partial s^2} = 2A\left(2r^2-r-1\right) \tag{20}$$

It might be easily concluded that,

$$\frac{\partial^2 y}{\partial s^2} < 0 \quad \text{for} \quad 0 \le r \le 1$$

so that $y(r,s)$ is a strictly concave function of s. Hence, the necessary conditions of a local maximum are also necessary and sufficient conditions of a maximum. If the maximum value of s is interior, then the maximum condition is $\frac{\partial y}{\partial s} = 0$. We might solve the latter condition to yield,

$$s_1(r) = \frac{r(1-r)}{1+2r(1-r)} \tag{21}$$

which is indeed an interior value of s for every r such that $0 < r < 1$. We plot in Figure 5 the curve $s_1(r)$ of schooling rates that maximize output per worker in each location.

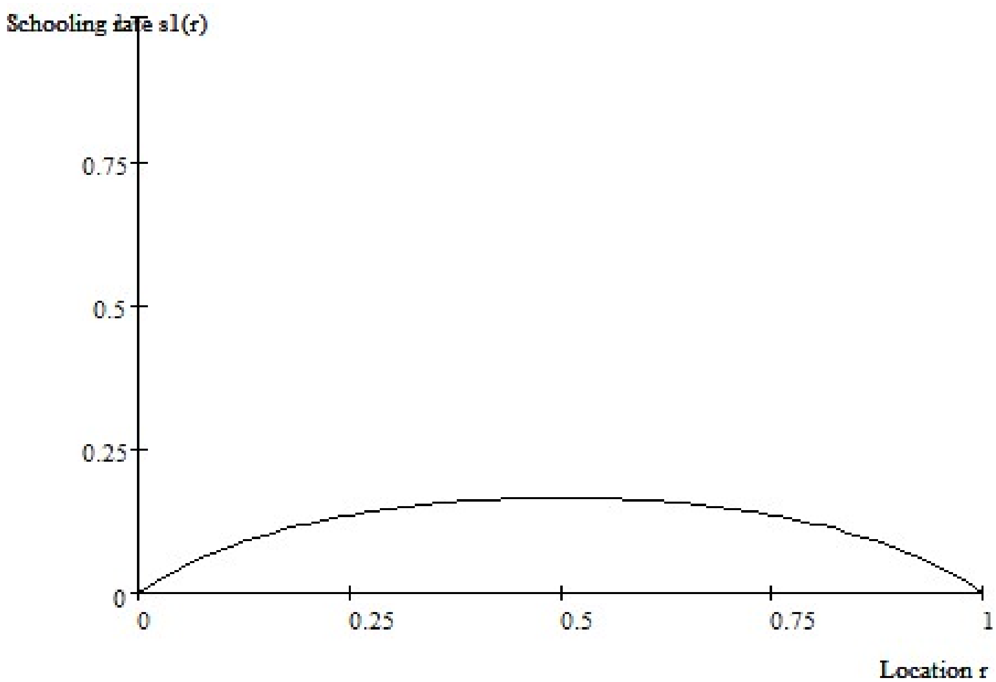

Figure 5: Individually optimal schooling rate $s_1(r)$

3.3. Socially optimal schooling rate

We now presuppose that a central planner sets the schooling rate $s(r)$ in each location. While doing so, he takes in account the positive externalities that a rise in $s(r)$ has upon the productivity of every worker.

By following here endogenous growth theory, particularly Lucas (1988), the aggregate productivity term A will be modelled as an increasing function of the average schooling level in the economy, i.e., the central planner assumes that,

$$A = \theta h_a^{\gamma} \quad \text{with } \theta \text{ and } \gamma \text{ positive} \tag{22}$$

where h_a is the average schooling level in the economy, as defined by,

$$h_a \equiv \frac{\int_0^1 Lh(r)dr}{\int_0^1 Ldr} = \int_0^1 h(r)dr \tag{23}$$

By substituting the aggregate productivity term A from (22) in the output per worker function (11), the latter becomes,

$$y(r,s) = \theta h_a^{\gamma} h(r)(1-s) \tag{24}$$

For simplicity, we will set $\theta = \gamma = 1$, so that output per work in point r when local inhabitants have a schooling rate s is,

$$y(r,s) = h_a h(r)(1-s)$$

or,

$$y(r,s) = \left(\int_0^1 h(r)\,dr\right) h(r)(1-s) \tag{25}$$

The central planner sets a schooling rate curve $s(r)$ that maximizes aggregate output Y given by the functional,

$$Y = \int_0^1 \left[\int_0^1 h(r)\,dr\right] h(r)(1-s)\,dr \tag{26}$$

We recall that that the skill level of a worker living in r is given by $h(r)$ from (17) as,

$$h(r) = 1 + s + 2rs(1-r) \tag{27}$$

which is not directly influenced by the schooling decision made by a worker in a different location $r' \neq r$. Consequently, the maximization of the functional Y in (26) may be decomposed into a continuum of problems of maximization of output per worker in each location r in relation to schooling rate s, i.e.,

$$\max_{0 \le s \le 1} y(r,s) = \left(\int_0^1 h(r)\,dr\right) h(r)(1-s) \tag{28}$$

By inserting (27) in (28) and simplifying, we obtain an equivalent expression,

$$\max_{0\le s\le 1} y(r,s) = (1-s)\left[s\left(2r-2r^2+1\right)+1\right]\left(\frac{4}{3}s+1\right) \tag{29}$$

To solve each problem (29) for a different location r, we compute its first and second derivatives in relation to s.

$$\frac{\partial y}{\partial s} = s^2\left(8r^2-8r-4\right)+s\left(\frac{4}{3}r-\frac{4}{3}r^2-2\right)+\left(2r-2r^2+\frac{4}{3}\right) \tag{30}$$

$$\frac{\partial^2 y}{\partial s^2} = s\left(16r^2-16r-8\right)+\left(\frac{4}{3}r-\frac{4}{3}r^2-2\right) \tag{31}$$

It may be easily checked that $\frac{\partial^2 y}{\partial s^2} < 0$ for values of s and r within the domain $[0,1]$, so that y is a strictly concave function of s within this region.

Furthermore, the first order condition has a positive solution $s_2(r)$, namely,

$$s_2(r) = \frac{2r(1-r)+\sqrt{156r-8r^2-296r^3+148r^4+57}-3}{12\left[2r(1-r)+1\right]} \tag{32}$$

It may be checked easily that for any feasible value of r, $s_2(r)$ given by (32) is an interior point of $[0,1]$. Hence, $s_2(r)$ is indeed the socially optimal schedule of schooling rates.

In Figure 6, we plot the equilibrium schooling rates schedule $s_1(r)$, given in (21), and the socially optimal one $s_2(r)$, expressed by (32), where the latter is represented by a thick curve.

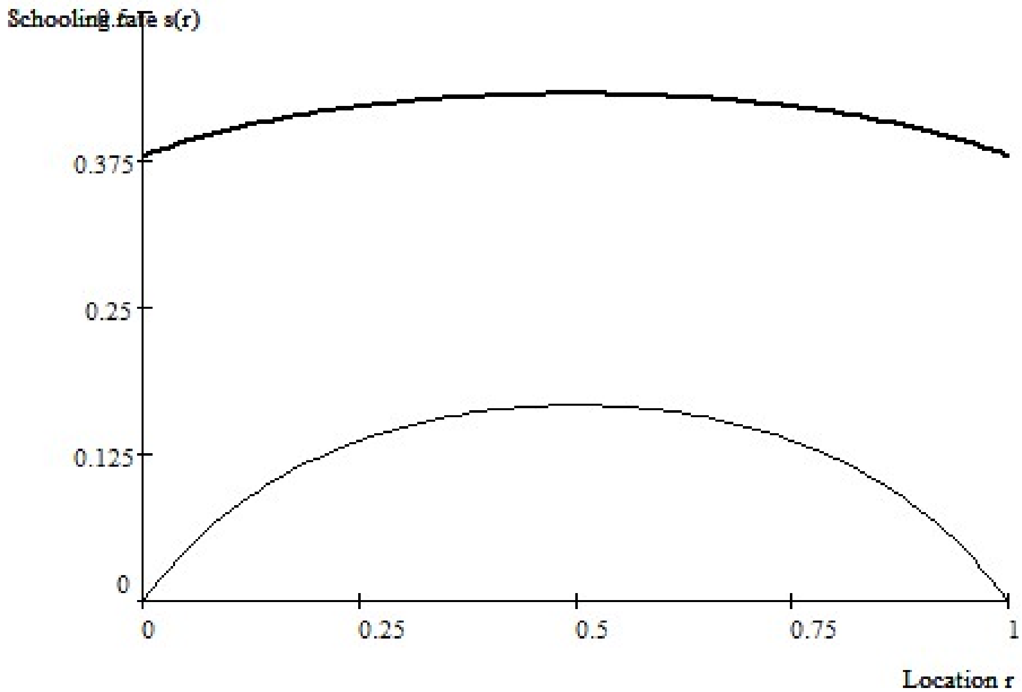

Figure 6: Spatial schooling rates schedules

The two curves in Figure 6 are similar in the sense that they are both concave and they reach a maximum at the central point $r = \frac{1}{2}$. However, they show a few qualitative differences.

First, $s_2(r)$ lies consistently above $s_1(r)$. For each location, the socially optimal schooling rate lies above the equilibrium one. Second the difference in schooling rates between equilibrium and social optimum is considerably greater for peripheral locations than for central ones. Finally, the equilibrium schedule shows a much higher spatial variation in schooling than the socially optimal curve.

To show that the gap between equilibrium and social optimum schooling decreases with accessibility, we compute for each point r the difference $s_2(r) - s_1(r)$ and plot it in Figure 7.

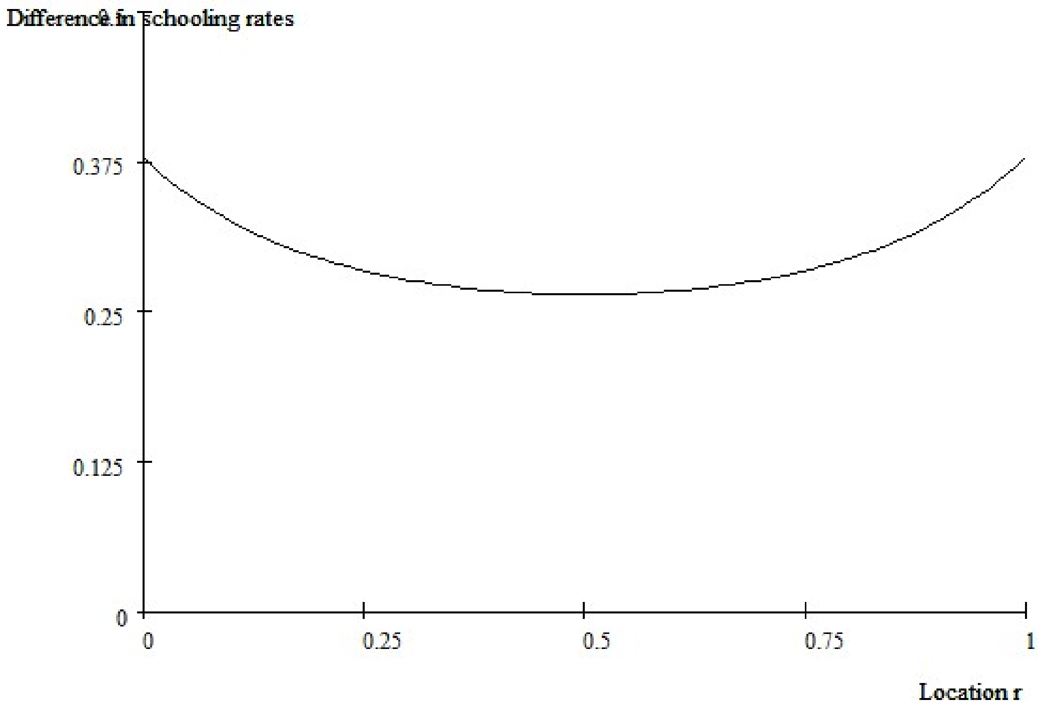

Figure 7: Difference between schooling rates $s_2(r) - s_1(r)$

4. Concluding remarks

In this paper, we tried to explain reasonably the following stylized facts concerning the variation in higher education attainment across Portuguese regions. First, schooling rates seem strongly correlated with regional population density. Second, the share of students in the publicly owned college subsystem is inversely correlated with demographic density, except for *Grande Lisboa* region, which hosts the capital and has a traditionally strong concentration of public universities and colleges. Lastly, from the latter facts a corollary follows that the regional variation in schooling in the public college subsystem is significantly lower than overall variation, private institutions included.

To easily explain these facts, we extended the human capital accumulation model by Uzawa (1965) and Lucas (1988) to encompass a geographical economy where workers locate in points endowed with different accessibility levels. While in the data differential accessibility is measured by regional population density, the theoretical model presupposes a uniform distribution of population over a bounded, so that the ability of a worker to interact socially is determined by his centrality. Hence, we implicitly assume that in reality “local density” and “centrality” are highly correlated despite not being fully coincident.

An increasing share of time dedicated to training raises both individual skills and aggregate productivity. As in Lucas (1988), the average worker skill level in the economy operates as a “collective input” that directly enhances aggregate productivity. This kind of positive externality of education determines that socially optimal schooling always exceeds the private “equilibrium” level.

We have firstly found that both the privately and collectively output maximizing schooling rates increase with workers’ accessibility. Since training a worker implies not only to refrain from productive work, but also to communicate with fellow workers, its skill generating efficiency rises with individual accessibility.

Second, the gap between individual and collective optimal schooling level seems to diminish with workers’ accessibility. This indicates public authorities should achieve a positive discrimination while supplying tertiary education by favouring less dense and accessible regions. However, this effort should not lead to fully equalize university attainment across regions, as spatial variation related with differential accessibility remains socially optimal. In addition to public universities, core regions might continue to benefit from private institutions.

In this paper, we featured a competitive economy where productive activities – both consumer good production and education – take place under constant returns to scale. In reality, activities in a spatial setting operate under increasing returns as they bear significant fixed costs and distance related costs. We purport to generalize our framework in future research to account for these crucial aspects.

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