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Technological waves, income functions and surplus-value of equipment: a new approach

João Ferreira do Amaral[1]

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Abstract

The purpose of this paper is to study the relations between the concept of technical progress of a certain type (technological wave with technical progress embodied in innovative capital) and the concept of surplus-value of the stock of equipment. For that purpose we define an income function instead of a production function.

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Keywords: economic growth; digital revolution; technological progress; innovation

JEL codes : E10, E11,E22,N10,O30

Introduction

The difficulties that studies of economic growth faced in dealing with the determination of the value of the stock of capital was the trigger for intense discussions in the fifties and sixties of the 20th century on the meaning of having a number that represents the aggregate value of all the equipment goods that exist in a given economy.

Although the critics of such an aggregation won the debate the consequences on growth theory were scarce. It is not easy to dispense with a number for the value of

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the stock of capital. However this doesn't explain the paucity of results of growth theory mainly in what concerns the introduction of technical progress.

Two aspects are especially important in explaining this paucity: the use in growth theory of assumed production functions that aren't real production functions and the neglect of the increased value of the stock of capital that is a consequence of combining more or less efficiently the equipment goods that the firms employs in its production plan.

The ideas of two of the above mentioned critics of growth theory influenced the present paper: Joan Robinson (1903-1983) in what concerns the concept of production function (Robinson, 1971 and 1973) and Wilfred Salter (1929-1963) about obsolescence of equipment (Salter, 1969).

The present paper combines ideas of the critics that were not sufficiently considered by subsequent growth theory with some of present ideas on technical progress. The intention of the paper is to study the impact of technological waves (Amaral, 2022) on growth using an income function that replaces the usual production function. We distinguish between stock of capital and stock of equipment both at the macro and micro levels and consider explicitly what we call the surplus of equipment that results from the combination of equipment goods in the production plan.

In section 1 we introduce the difference between stock of capital and stock of equipment. This isn't a new distinction (it goes back to the sixties, Tinbergen et al), 1962, chapter 2, 2.2) but it is the basis for the definition of the surplus-value of equipment.

In section 2 we develop an aggregate model using an income function instead of a production function and establish the relations between growth of the stock of equipment, surplus-value of this stock and increase of the proportion of innovative capital on total capital.

In section 3 we focus on some of the results of the model, namely the determination of the value of the rate of growth of the stock of equipment and its relation with the maximum of income that can be generated in the economy. Additionally we suggest the use of a new indicator or efficiency for qualifying the path of growth and we discuss the relation between conjuncture and long term growth.

1.The stock of capital and the stock of equipment: two principles

We start by enunciating as a Principle an obvious truth but frequently a forgotten one.

Principle 1. Equipment goods don´t produce goods and services (to simplify, goods). Goods are produced by firms and also by the State.

To keep the analysis simple we eliminate the productive activity of the State so that production is done by firms (including individual firms and public firms)

Assume that there are *n* goods produced in the economy and *m* firms that produce those goods.

For each firm i $(i = 1,\ldots m)$ we associate a vector $\mathbf{y}^i$ of n components such that each component y^i_j $(j=1,\ldots n)$ represents the value added of the production of good j by the firm i[2].Obviously, most of the components of $\mathbf{y}^i$ are equal to 0, so that the total of the value added generated by the productive activity of the firm i is $va^i = \sum_j y^i_j$ $(j = 1,\ldots n)$.

For the value added of the economy (that is GDP[3]) we have

$$GDP \equiv Y = \sum_i \sum_j y^i_j \quad (i = 1,2\ldots m;\ j=1,\ldots.n)$$

In what concerns productive factors we simplify again the analysis by considering that at both the macro and micro levels the only limitative factor is capital.

Each firm i at the beginning of each year has equipment such that its value is represented by the vector $\mathbf{k}^i$ such that each component k^i_j is the stock of the equipment good j (we ignore inventories).

Obviously most of the components of $\mathbf{k}^i$ are equal to 0 because most of the goods of the economy are not equipment goods and not all the types of equipment are used by each firm.

[2] We assume that when there is more than one good *j* produced by firm *i* it is possible using the accounting of the firm to determine the contribution of the production of each good to the value added of firm *i*.

[3] We simplify the analysis by ignoring taxes and subsidies.

For each firm *i* $k^i \equiv \sum_j k^i_j$ is the value of equipment goods owned by the firm. We call this value the stock of equipment of the firm and this leads us to a second Principle.

Principle 2 The value k^i is the value of the stock of equipment of the firm but it isn't the value of the stock of capital of that firm. To obtain the value of the stock of capital we have to add the value that is generated by the integration/combination of equipment goods in the production process.

Therefore, for each firm *i* we have to consider a value k^i_{n+1} that represents the increase of the value of the equipment that the firm obtains from integrating/combining this equipment in a more or less efficient way.

That is the stock of capital of firm *i* is the value k^{*i}

$k^{*i} \equiv k^i + k_{n+1}{}^i$

and for the whole economy summing all the k^{*i}

$K^* = K + K_{n+1}$

Remark 1.1. Surplus value of the equipment stock

We assume that there is a market for used equipment goods k^i so that it is possible to have a market price for every equipment good according to its age.

In what concerns the value of k^i_{n+1} it may be obtained from the value of firm *i* calculated by the usual procedures (namely present value of the future net benefits related to production) subtracting the market value of the equipment used and excluding all the financial assets and liabilities that are not directly linked with production.

The value k^i_{n+1} depends on the value of the rate of discount but as we assume that the rate of discount doesn't change in time this rate finally plays no role in the present analysis.

We call the value k^i_{n+1} the surplus-value (to simplify, the surplus) of equipment of the firm *i.*

Remark 1.2.

The consideration of the values k^i_{n+1} helps us to understand certain situations that are difficult to explain by the prevailing concepts. In particular situations where a firm continues to produce as usual even when the market price of its equipment goods goes down to zero (see Salter, chap IV, p. 48).

Remark 1.3.Enlarged capital/product coefficient. Income function

For each firm *i* the meaning of a given value va^i is the value-added that is generated in the firm which is not only a consequence of the value of the equipment but also of the surplus of that equipment.

Using for simplicity a linear function to describe those effects on the value-added (va^i) we have

1) $va^i = (1/c^i)\,(k^i + k^i_{n+1})$

where c^i is a positive constant that we call *enlarged capital /output coefficient*.

Note that *1)* is not a *production* function but an *income* function since it relates the income with the stock of capital. It has obviously a relation with the productive process but the properties of an income function are different of the properties of a production function as we will see later on at the macroeconomic level.

Remark 1.4. Stock of capital and technical progress

Technical progress impacts on the stock of capital and on the value of a firm.

An important effect of technical progress is to change the distribution of surplus functions between firms. Technical progress according to Schumpeter (1957, chap. IV) creates for firms that innovate a temporarily situation of monopoly. In general, this increases the surplus of equipment of those firms and reduce the surplus on non-innovative firms. However other, contrary effects may have to be taken in account as we shall see later on.

At macroeconomic level the situation is a little more complicated. Before tackling the macroeconomic case we have to look at investment.

Remark 1.5. Investment

Gross investment is the value of the acquisition of equipment goods in a given period of time (year).

However this concept needs some additional elaboration. For that purpose we introduce two additional assumptions:

a) All the equipment goods once acquired start participating immediately in the production process

b) Absence of wear and tear. The equipment goods maintain for the whole of their existence the same capacity to contribute to production that they had in the first moment of their economic life (this is not an essential assumption but it simplifies the analysis).

To understand better the impact of the gross investment on the stock of equipment we distinguish two alternative points of view on the value of the stock of equipment:

i) the stock of equipment as the value of the capacity to generate income assuming that assumption b) above applies.

ii) the stock of equipment as a financial asset.

The difference between the two points of view has a significant impact on investment.

Let’s us start with i).

When a firm makes an investment it changes the individual values of the equipment goods that the firm owns and the value of the surplus of equipment.

That is from year 0

$k^{*i0} \equiv k^{i0} + k_{n+1}^{i0}$

goes to year 1

$k^{*i1} \equiv k^{i1} + k_{n+1}^{i1}$

The firm obtains an increase (decline) off the stock of capital given by

$(k^{i1} + k_{n+1}{}^{i1}) – (k^{i0} + k_{n+1}{}^{i0}) = (k^{i1} - k^{i0}) + (k_{n+1}{}^{i1} - k_{n+1}{}^{i0})$.

But gross investment includes the value of new equipment goods that replace obsolete goods and goods that end their economic life, so that assuming that replacement investment is a fixed proportion of the stock of equipment and writing i^i as the gross investment of firm *i* we have

$i^i = (k^{i1} - k^{i0}) + (k_{n+1}{}^{i1} - k_{n+1}{}^{i0}) + r^i k^{i0}$.

For the economy as a whole we write similarly

$I = \Delta K + \Delta K_{n+1} + rK$

where $\Delta K_t \equiv K_{t+1} – K_t$ and the same for K_{n+1} .

An important point to emphasize is that the value of the stock of equipment as capacity to generate income doesn't necessarily coincide with the value of the stock of equipment at market prices, since market prices even in the absence of wear and tear decline with the age of equipment as the end of economic life approaches. But this difference doesn't cause any difficulty for the model.

Let' us now look at ii).

In this case if we write k^ for the stock of equipment as a financial asset so that we may write

$(k\hat{}^{i1} – k\hat{}^{i0}) = i^i – r^i k^{i0} - d^i$

where d is the value of the depreciation of the equipment that was in use in year 1 and did not end its economic life in that year.

Contrary to what sometimes is mentioned, the value of the fund of amortization that the firm maintains to face depreciation is not an investment. It is saving of the firm (that is non-distributed profits) that facilitates the acquisition of new equipment when the old one ends its economic life and is replaced.

Depreciation has no role to play in the value of the stock of equipment as capacity for generating income (point of view i), with assumption b)). This value is not affected by the existence or the absence of an amortization fund, but it is very much affected by the end of economic life and by obsolescence of the equipment goods.

Both ways of looking at the stock of equipment may coexist in a macroeconomic model. However for our purposes we exclude from the model the stock of equipment as a financial asset.

Based on these considerations we describe in the next section the macroeconomic model and the respective assumptions..

2. The macroeconomic model

Assumption 1. Income function and technological waves

Let Y^*_t be the maximum gross income (GDP) that can be generated in year t.

The value of Y^*_t is determined solely by technical progress and by the value of the stock of capital. Technical progress is described by a technological wave characterized by an increasing proportion of innovative capital (written as a_t with $a_t \leq 1$) in total capital (Amaral, 2022).

That is

2) $Y^*_t = f(a_t)K^*_t$

with $K^*_t \equiv K_t + K_{n+1t}$ and $a_t \equiv K^{*1}_t / K^*_t$ where K^{*1}_t is the stock of innovative capital and f é an increasing function of a_t.

Remark 2.1.

The income function as already stated is not a production function both at micro and macro levels. In the first place it doesn’t give us a description of the productive process but only an upper limit to the income that can be generated. As already seen this limit depends on the stock of capital and on technical progress.

Secondly because Y^* is a variable that represents an income and not a production value (there is a relation between production and income but the form of that relation is not specified in the model).

Seen from another angle, a more sociological one, the function of 2) gives us a quantification of the power of capital owners to generate income (and appropriate a large part of it). *The income function is an economic (social) function and not a technical function as the production function is supposed to be*.

The traditional one-sector growth theory tries to avoid the problem created by the concept of production function postulating that the only good that is produced is a final good so that production equals income.

But this doesn't work.

It is a fact that the income generated in a (closed) economy equals the value of final goods produced. But that doesn't mean that there aren't intermediate goods and it is surely a waste of time to study an economy that doesn't produce intermediate goods.

Remark 2.2.

Note that an income function doesn't have to include other factors aside capital. But even if we introduce other limiting factors the question of increasing or decreasing returns cannot be discussed in the same terms as the case of a production function. It would be a much more complex, partially nonsensical and far from interesting discussion.

Assumption 2

The period that is the object of the analysis is the ascending period of the technological wave. The period ends at year T where a_t attains its maximum before starting to decline.

The values of a_t till a_T are the values of a logistic function (Amaral, 2022).

Assumption 3

Gross investment which is written as I is a constant proportion s of the GDP generated in the economy. That is

3) $I_t = \Delta K_t + r_t K_t = sY_t$

Remark 2.3.

We assumed *s* as a proportion of GDP and not of NDP (Net Domestic Product).This and the exclusion of the point of view of ii) above, explains why we consider unnecessary to introduce a variable representative of the depreciation of the equipment.

Assumption 4

We consider the rate of replacement *r* is a function of the path that the technological wave follows, that is,

4) $r_t \equiv r(a_t)$

Remark 2.4.

Assuming as we have been doing that the technological wave follows the path of a logistic curve till year T we may divide that path in three sequential periods:

a) a period called initial where the values of a_t are low and the rate of growth of a_t is also low

b) a period called medium where the rate of growth of a_t increases strongly

c) a period called final where the values a_t are high and the rate of growth of a_t , although non-negative declines rapidly. .

In the initial period the low levels of a_t and the relatively low rate of growth imply that the rate of replacement *r* in this period remains approximately constant.

In the medium period there is a rapid replacement of equipment due to technical obsolescence so that *r* growths as a_t growths.

In the final period most of the equipment has been replaced so that *r* is again approximately constant till year *T*.

Remark 2.5.

It is possible to obtain approximately a function $r(a_t)$ based on functions $r_1(a_t)$ and $r_2(a_t)$ relative respectively to innovative and non-innovative capital (see Appendix).

Assumption 5

This assumption is relative to the behaviour the ratio $K_{nt}/(K_t+K_{nt})$.

We assume that a determining component of this ratio is technical progress so that there is a function *g(.)* such that

5) $K_{n+1t}/(K_t+K_{n+1t}) = g(a_t)$

The justification is the following:

If there is technical progress that takes the form of a technological wave, when a new production process is installed based in a new kind of equipment replacing an obsolete one, the value of the stock of capital changes. Therefore there are reasons to assume that there is a function $g(a_t)$.

The behaviour of this function may also be discussed for each of the periods of the path of the wave.

In the initial period the replacement of the old equipment is not very pronounced so that a large number of firms continue to use the old equipment. However this old equipment loses value in the market because there are generalized expectations that the new innovative equipment will replace the old one. Therefore in this period the ratio $K_{n+1t}/(K_t+K_{n+1t})$ grows because of the loss of value of K_t and the function $g(a_t)$ is an increasing function of a_t.

In the medium period the replacement of equipment is rapid and the acquisition value of the new equipment is high due to strong demand. Although the banalization of the new equipment may have in due time an effect of reducing its price it is probable that the effect of demand will prevail so that $g(a_t)$ will be a decreasing function of a_t.

In the final period the large majority of interested firms has already replaced the old equipment so that the ratio $K_{n+1t}/(K_t+K_{n+1t})$ remains approximately constant and so is the function $g(a_t)$ till year T.

Remark 2.6.

As was the case for the function $r(a_t)$ it is possible with sufficient approximation to get a function $g(a_t)$ from the two functions $g_1(a_t)$ and $g_2(a_t)$ (See Appendix).

3. The results

The study of the behaviour of the economy focuses on the growth of the stock of equipment and of the utilization of the capacity to generate income.

3.1 Determination of the rate of growth of the stock of equipment

If Y^* is the maximum income that can be generated we have with some calculations and using the previous assumptions

6) $Y_t/Y^*_t = (\Delta K_t + r(a_t) K_t)/[sf(a_t)(K_t + K_{n+1t})]$

where $Y \leq Y^*$ so that

$\Delta K_t + r(a_t) K_t \leq sf(a_t)(K_t + K_{n+1t}) = \{sf(a_t)/[1-g(a_t)]\}K_f$

Dividing and multiplying the right-hand side of 6) by K_t we get

$Y_t/Y^*_t = [(\Delta K_t /K_t)+ r(a_t)][K_t /(K_t + K_{n+1t})][1/sf(a_t)]$

and since $K_t /(K_t + K_{n+1t})] = 1-g(a_t)$,

$Y_t/Y^*_t = [(\Delta K_t /K_t)+ r(a_t)][1-g(a_t)][1/sf(a_t)]$

so that

7) $\Delta K_t /K_t = sf(a_t)(Y_t/Y^*_t)/[1-g(a_t)] – r(a_t)$

In the left-hand side of this equality we have the rate of growth of the stock of equipment. The right-hand side includes the ratio of the generated income to maximum income.

Since $f(.)$ is always positive for every t and respective a_t the rate of growth of the stock of equipment is higher the higher is the utilization of the capacity to generate income as of course should be expected.

The upper limit of the rate of growth of the stock of equipment is attained when

$Y_t/Y^*_t = 1$ and is given by

$\Delta K_t /K_t \leq sf(a_t) /[1-g(a_t)] - r(a_t)$.

It is the task of short-term economic policy to manage demand in order to reach a satisfactory value for the ratio Y_t/Y^*_t.

Other conclusions may be obtained based on the joint behaviour of the functions $r(.)$ and $g(.)$. Three situations may be identified corresponding to the three periods mentioned above.

Situation 1: initial period

As we have seen r is approximately constant and $g(a_t)$ is increasing.

Based on equation 7) we can conclude that for a fixed value for the utilization of capacity (that is for Y_t/Y^*_t constant) the rate of growth of the stock of equipment growths with a_t .

Situation 2: medium period

In this period $r(a_t)$ is an increasing function and $g(a_t)$ is a decreasing one. The behaviour of the rate of growth of the stock of equipment is uncertain. It depends on the behaviour of $f(a_t)$.

Situation 3: final period

The rate $r(a_t)$ and the functions $g(a_t)$ are approximately constant, so that the rate of growth of equipment increases with $f(a_t)$ and therefore with a_t .

3.2 rate of growth of K. A new indicator of efficiency*

As $K^*_t = K_t/[1-g(a_t)]$

we have

8) $K^*_{t+1}/K^*_t =(K_{t+1}/K_t) \{[1-g(a_t)]/[1-g(a_{t+1})]\}$

And we may conclude that the rate of growth of the stock of capital is higher than the rate of growth of the stock of equipment in the initial period, inferior in the medium period and approximately equal in the final period.

On the other hand from equation 8),

$K^*_{t+1}/K^*_t > K_{t+1}/K_t$ if and only if $\{[1-g(a_t)]/ [1-g(a_{t+1})] > 1$

Provided that

$\Delta K_t /K_t \leq sf(a_t) /[1-g(a_t)] - r(a_t)$

The accumulation of the stock of capital K^* is crucial to generate more income. On the other hand the value of K may be considered as indicative of a sacrifice of consumption.

The number R

$R= [1-g(a_t)]/ [1-g(a_{t+1})]$

may be used as an indicator of efficiency of the economy. From 8) the higher the value of R the higher will be the capacity of producing income relatively to the sacrifice of consumption (we assume that to obtain the surplus of the stock of equipment we don't have to sacrifice consumption).

For the period $[0, T]$ we may use the indicator R in the form

$R = \Pi_0^{T-1} [1-g(a_t)]/ [1-g(a_{t+1})]= [1-g(a_0)]/[1-g(a_T)].$

Conclusion

The intention of this paper was to study the relations between the concept of technical progress of a certain type (technological wave with technical progress embodied in innovative capital) and the concept of surplus-value of the stock of equipment.

For that purpose we used a very simple model that however is susceptible of being utilized in empirical studies.

It is our conviction that that there is an important question that so far has not been satisfactorily answered by economic growth theory: *"is technical progress always associated with an increase of surplus-value of the stock of equipment or on the contrary there is technical progress that goes with a decline in the surplus of value of the equipment?"*

The present paper is a small step in the direction of the answering of the question. Additionally and notwithstanding the simplicity of the model it is possible to define an indicator of efficiency that relates the two concepts and that is susceptible of numerical quantification.

Appendix. Determination of a function $r(a_t)$ and of a function $g(a_t)$ using the same approximation

For the function $r(a_t)$:

Being 1 the index for innovative capital and 2 for non-innovative we have

$r(a_t)K_t = r_1(a_t)K^1_t + r_2(a_t)K^2_t$

Dividing both sides by K_t and using de approximation

$K^1_t/K_t = a_t$

$K^2_t/K_t = 1 - a_t$

we have

$r(a_t) \approx r_1(a_t)a_t + r_2(a_t)(1-a_t)$ □.

For the function $g(a_t)$:

we have $K^*_t = K_t / [1-g(a_t)] = K^1_t / [1-g_1(a_t)] + K^2_t / [1-g_2(a_t)]$

and using the same approximation

$K^1_t/K_t = a_t$

$K^2_t/K_t = 1 - a_t$

we get $(1-g(a_t) \approx 1/\{a_t/[1-g_1(a_t)] + (1-a_t)/[1-g_2(a_t)]\}$□.

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