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Integration of Household Heterogeneity in Input-Output Modelling: A Comprehensive Literature Review*

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Abstract

Several studies highlight that standard Input-Output (IO) models do not accurately represent household consumption behaviour, namely due to the assumption of a representative household, which overlooks heterogeneity. This limitation affects, among other things, the estimation of induced effects, which are highly significant in economic impact analysis using IO methodology. This paper explores the importance of incorporating household heterogeneity into IO models and reviews methods for introducing such disaggregation. Demographic-Economic Extended IO Models are a common approach, with the Miyazawa and Batey–Madden models being particularly prominent. Alongside an analysis of these seminal models, the paper identifies recent developments and compares extended IO models with related frameworks that also incorporate household heterogeneity. It concludes that extended models not only enhance the realism of multipliers but also allow for the derivation of additional multipliers that reflect interactions between demographic and economic variables, making IO methodology a powerful analytical tool in suitable contexts.

Keywords: Demographic-Economic Extended Input-Output Models; Miyazawa Model; Batey-Madden Model

JEL codes: C67

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1. Introduction

A common application of Input–Output (IO) analysis is the evaluation of economic impacts through the estimation of demand-side effects triggered by exogenous spending changes.

In academic contexts, the application of the IO model for economic impact analysis includes the assessment of the impacts of major events, institutions, natural disasters, and the introduction of new products or technologies (Emonts-Holley et al., 2021). Despite the limitations of the standard IO model for long-term analysis, which are discussed in section 2 and in section 6 in comparison with other models, features such as its detailed sectoral disaggregation, data accessibility, and transparency of assumptions make this methodology highly relevant for quantifying the economic impact of a wide range of demand-side effects.

In the context of employing IO models for the purpose of economic impact analysis, it is common to consider not only the direct and indirect impacts of an activity but also its induced effects. Indirect impacts represent the effects generated upstream in the supply chain due to the demand linked to the economic activities associated with direct impacts, which are calculated using Type I multipliers derived from the so-called Open IO Model. Induced impacts arise from the stimulus created through final consumption expenditure on goods and services, driven by the wages earned by workers generated in the activities considered in direct and indirect impacts. These induced impacts are calculated using Type II multipliers, which are estimated by endogenizing household consumption, i.e. closing the IO model with respect to households.

In contrast to the quantification of indirect effects, the estimation of induced effects lacks a standardized methodology, as evidenced by the considerable variation in methodologies identified in the academic literature. Furthermore, the most widely applied methods often exhibit significant limitations that may compromise the reliability of the resulting estimates. Additionally, in comparative analyses such as those presented in Batey and Weeks (1989) and Emonts-Holley et al. (2021), it was found that the induced effect consistently exceeded the indirect effect in magnitude. Therefore, considering that induced effects can represent a substantial share of total impacts in economic impact assessments, appropriate attention must be paid to the methodology adopted for their estimation.

As observed in Batey and Hewings (2021), macroeconomic models, persist in overlooking the heterogeneity of economic agents and, given that households account for a significant share of total consumption, their disaggregation in macroeconomic modelling is particularly pertinent. Despite the relevance of household consumption in the economy, which is reflected

in IO modelling by the magnitude of induced effects, IO models with endogenous consumption, in their most common formulations, assume the consumption structure of a representative household, disregarding existing heterogeneity. These approaches consider an average propensity to consume across all households, limiting the model's ability to accurately reflect household consumption patterns. Consequently, the multipliers derived from such models, often used in economic impact analyses, may not accurately represent real-world dynamics or the true diversity of consumption behaviour. As identified in Chen et al. (2016), several studies point out that one of the reasons why household consumption behaviour is not accurately captured in the standard closed IO model is the neglect of differences in consumption patterns across household types.

The objective of this paper is to discuss the importance of incorporating household heterogeneity into IO modelling and to provide a comprehensive literature review of model formulations that have introduced such disaggregation. Additionally, given the importance of household consumption in the economy and the significance of induced effects in the IO analysis, this study also identifies IO model formulations aimed at improving the representation of households not only through disaggregation, but also through more refined modelling techniques.

A common approach to introducing greater detail in consumption within IO models has been through the use of extended IO systems (Emonts-Holley et al., 2021). These models, also referred to as demographic-economic extended IO models, not only enhance the representation of households, but also allow for the derivation of a much broader set of multipliers linking demographic and economic dimensions.

The focus of this study is on static extended IO models for a single region that allow for an improved representation of households. In addition, albeit less extensively, it also examines other models related to the IO framework that allow for a disaggregated representation of households (Social Accounting Matrices, Econometric IO models, and Computable General Equilibrium models) comparing them with the static IO models.

The paper begins with a brief presentation of the standard open model, typically used to compute Type I multipliers (section 2), followed by a review of the most common approaches used to endogenize household consumption and to calculate Type II multipliers considering one type of household (section 3). Following this, attention is given to seminal demographic-economic extended IO Models (section 4) and recent developments (section 5). Finally, a set

of other models related to the IO framework that allow for the incorporation of household heterogeneity is presented (section 6), followed by conclusions (section 7).

2. Open IO Model

Following Emonts-Holley et al. (2021), but with adjustments to the notation to ensure comparability with the models considered in this study, for an economy with n sectors, the conventional open IO model can be expressed as:

$$(I - A)x = f \qquad (1)$$

In Equation (1), A is the $n * n$ matrix of technical production coefficients of domestically produced intermediate inputs. The elements of matrix A are denoted by a_{ij} and represent the input from sector i needed to produce one unit of output in sector j. x and f are $n * 1$ vectors representing sectoral outputs and domestic final demand, respectively. I represents the identity matrix of appropriate dimensions.

Equation (1) can be reformulated in the following form, establishing a relationship between endogenous outputs to exogenous final demands:

$$x = (I - A)^{-1} f \qquad (2)$$

This version of the model implicitly relies on a set of assumptions generally acknowledged as limitations of the model, including constant technology in technical coefficients, linearity in the production system (which implies that the model assumes constant returns to scale), no supply constraints, and the absence of price effects (West, 1995; Sargento, 2009). Given the structural assumptions of the standard IO model, its limitations become more apparent in long-run contexts (Sargento, 2009). In situations where the IO model is used for short-term economic impact evaluation, where the magnitude of the shock is insufficient to generate significant price effects or compromise the economy's supply response, the model's assumptions become less constraining. A more detailed discussion of these limitations and their implications is provided in West (1995) and Sargento (2009). In addition, section 6 includes a brief discussion comparing this model with other approaches commonly used in the literature.

The component $(I - A)^{-1}$ in Equation (2) is known as the Leontief Inverse. Its elements (denoted by $\alpha_{i,j}^{I}$) represent the direct and indirect output effects in sector i required to satisfy one additional unit of exogenous final demand in sector j. Consequently, the sum of the elements in column j represents the total output effect across all sectors resulting from a unit

increase in exogenous final demand for the output of sector j. This sum corresponds to the Type I multipliers and, mathematically, is defined as:

$$M_j^I = \sum_{i=1}^{n} \alpha_{i,j}^I \qquad (3)$$

Assuming a linear relationship between sectoral output and another variable, the multipliers derived from Equation (3) can also be used to estimate the impact in terms of that alternative variable, by applying the following modification: replacing the elements $\alpha_{i,j}^I$ with their product by the ratio between the alternative variable and the output in sector i. Mathematically, the multipliers with respect to any variable can be represented as in Equation (4), where O denotes the alternative variable:

$$M_j^{I*} = \sum_{i=1}^{n} \alpha_{i,j}^I * \frac{o_i}{x_i} \qquad (4)$$

This modification can be used to estimate impacts in terms of economic variables such as employment or gross value added, as well as physical variables such as environmental emissions (Emonts-Holley et al., 2021).

3. **Closed IO Models: common formulations**

Following an exogenous shock to final demand, the open IO model presented in the previous section quantifies the multiplier effect of that shock throughout the value chain (indirect effect). However, both the direct and indirect effects resulting from the exogenous shock generate employment, which is remunerated through wages that, in turn, translate into household consumption expenditure, triggering a new round of effects in the economy. The multiplier effect of consumption in the economy, consistent with the Keynesian model (a concept introduced in Kahn, 1931), is captured by endogenizing household consumption within the IO framework. The standard procedure for this endogenization involves treating households as one of the endogenous sectors of the economy - a process known as closing the model with respect to households (Miller & Blair, 2022).

Following Emonts-Holley et al. (2021), and after some adjustments to the notation, harmonising it with Miyazawa model (Miyazawa, 1976), as formulated in Miller and Blair

(2022), the closed IO model, used to compute Type II multipliers, can be represented by the following equation:

$$\begin{bmatrix} I - A & -C \\ -(V + r\pi) & I \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f* \\ g \end{bmatrix} \quad (5)$$

In Equation (5), y represents the total household income, C is an $n * 1$ vector representing household consumption coefficients (defined as the ratio of household consumption expenditure to household income), V is a $1 * n$ vector representing wage coefficients (defined as the ratio of wages paid to households to output levels, by industry); π is also a $1 * n$ vector denoting the coefficients of other components of value added; r is the share of other value added that is distributed, either directly or indirectly, to domestic households; f^* is a $n * 1$ vector of domestic final demand excluding household consumption; and g represents exogenous household income.

Equation (5) is solved with respect to the model's endogenous variables, through the standard matrix inversion procedure, resulting in the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} I - A & -C \\ -(V + r\pi) & I \end{bmatrix}^{-1} \begin{bmatrix} f* \\ g \end{bmatrix} \quad (6)$$

The element $\begin{bmatrix} I - A & -C \\ -(V + r\pi) & I \end{bmatrix}^{-1}$ in Equation (6) corresponds to the Type II Leontief Inverse. Following Emonts-Holley et al. (2021), the elements of this term can be denoted by $\alpha_{i,j}^{II}$, which capture the direct, indirect, and induced effects in sector i resulting from a unit increase in final demand that is not endogenized in the model (i.e., excluding household consumption but including government expenditure, investment, among others) in sector j. The Type II multipliers are mathematically defined as:

$$M_j^{II*} = \sum_{i=1}^{n} \alpha_{i,j}^{II} \quad (7)$$

As noted in Emonts-Holley et al. (2021), IO accounts do not provide information on income flows between domestic institutions, which implies that data for variables r, g and C are not available from IO accounts. In Emonts-Holley et al. (2021), different formulations of the model presented in (6) are compared, each addressing the lack of information on the missing variables using only the data available in standard IO accounts. Some of the formulations considered in

Emonts-Holley et al. (2021), due to their relative simplicity and the fact that they are based solely on data available in IO tables, are among the most widely used by practitioners. In particular, the most common specifications in the modelling of Type II multipliers consider only labour income.

It is also worth noting that several limitations have been identified in the model presented in Equation (6), and consequently in the formulations discussed in Emonts-Holley et al., 2021: it considers only one type of household, assumes a linear consumption function, treats government transfers as exogenous, and also assumes that wage income generated within the region is fully retained as household income within that same region (Emonts-Holley et al., 2021). The linearity assumption of the IO model is especially debatable when applied to consumption modelling, where income coefficients are assumed to represent average propensities, employment coefficients reflect average labour productivity rates, and household consumption is determined by average expenditure patterns, not capturing significant non-linear components of consumer behaviour (West, 1995)

In Batey and Weeks (1989), additional limitations of the standard closed IO model are identified: consumption propensities are assumed to apply exclusively to employed households (and, if consumption by unemployed households is considered at all, it is treated as part of final demand), and no explicit assumption is made regarding the source of newly employed workers (whether they are recruited from the local labour market or are migrants).

Also in Chen et al. (2016), another limitation of the common closed IO model with respect to households (also referred to as the semi-closed IO model, since only household consumption is endogenized) is identified: in the model, household consumption is assumed to be determined solely by current income. However, several well-established theories of consumption behaviour (such as the relative income hypothesis and the life cycle-permanent income hypothesis) argue that current consumption is influenced not only by current income but also by a variety of other factors. Chen et al. (2016) argues that the standard closed IO model is not consistent with these consumption theories and is therefore likely to overestimate the linkages between the household sector and the production sector.

One of the specifications highlighted in Emonts-Holley et al. (2021) is the procedure proposed in Miller and Blair (2022), which, in addition to tracking only labour income, also assumes that all income is endogenous, which implies that $r, g = 0$, and consequently, household income corresponds entirely to total wage payments. In this specification, C is

defined as the ratio of household consumption expenditure to wage payments. The main issue with the specification considered in Miller and Blair (2022) is that only around 60% of total income originates from wages, and certain income sources, such as pensions, are independent of the income generated through production (Emonts-Holley et al., 2015, 2021). It is also worth noting that, following Miller and Blair (2022), this procedure can be extended to distinguish different types of households by adding more than one labour input row and more than one household consumption column to matrix A. This procedure is consistent with the Miyazawa model, which is discussed in section 4.

Additionally, among the formulations identified in Emonts-Holley et al. (2021), particular attention may be given to the specification referred to as Batey2 (proposed in Batey, 1985). According to the comparative analysis conducted, the Batey2 specification provides the closest results to the constructed benchmark (estimated with values obtained from a Social Accounting Matrix to address parameter gaps). In this formulation, household income is assumed to be equal to consumption, with $\mathbf{C}$ being defined as the ratio of household consumption expenditure in each sector to total household consumption. This approach also assumes $\mathrm{r} = 0$, and g is defined as the difference between household consumption and the wages paid to households.

Despite the practicality of common closed IO model formulations, Emonts-Holley et al. (2021) concludes that all the standard methods used to calculate Type II IO multipliers are necessarily defective, as data on income flows are missing from the IO accounts.

A relevant variation of the standard closed IO model is presented in Ferreira et al. (2025). It is explicitly assumed that only a portion of household consumption directly depends on labour income. In this way, final household consumption is disaggregated into four household types based on their primary income source: ‘labour income’, ‘real estate rental and other capital income’, ‘pensions’, and ‘other social transfers’. Within the model, only the consumption of households primarily dependent on labour income is endogenized. The vector $\mathbf{C}$ is thus calculated as the ratio of consumption to income for the household group living from labour income.

4. Seminal Demographic-Economic Extended IO Models

As illustrated in section 3, the standard closed IO model considers only a single household type, despite the substantial weight of household consumption within final demand and the

distinct consumption patterns associated with different household groups. This model can be seen as a rudimentary form of demographic–economic model (Batey, 2018).

A more in-depth modelling of demographic–economic linkages, particularly capturing the existing heterogeneity, can be achieved by extending the basic versions of IO models. The interactions captured by these extended IO models incorporating demographic and economic linkages not only have the potential to provide more realistic multipliers but also allow for the derivation of additional multipliers that reflect the relationships between demographic and economic variables.

In the context of extending the IO framework to incorporate interactions between economic and demographic components, two main lines of research stand out: the Miyazawa model and the contributions of Batey and Madden, formalized in what is known as the Batey–Madden model (Miller & Blair, 2022).

According to Batey and Rose (1990), academic research on extended IO models can be broadly divided into two main areas: labour market analysis and income distribution. The Batey–Madden model is part of the research stream focused on labour market analysis, while the Miyazawa model represents the key contribution to the strand dedicated to income distribution. Batey and Rose (1990) provides a review of contributions to these two areas of research. Batey (1985) also identifies and compares different approaches for incorporating demographic–economic relationships into IO models.

Another important pioneering contribution is the formalisation, proposed by Batey and Weeks (1989), of a more comprehensive inclusion of household characteristics within IO modelling.

The Miyazawa model (Miyazawa, 1976) restructures the standard closed IO model by enabling the capture of household behaviour through the differentiation of multiple household categories. In addition, it accounts for interactions both among household groups and between these groups and industrial sectors. This leads the model to generate various multiplier matrices (Miller & Blair, 2022). For example, as exemplified in Steenge et al. (2020), the Miyazawa model allows for the analysis of how increased consumption by one household group affects others. Consequently, the model has been widely used to investigate income distribution effects (Steenge et al., 2020). Steenge et al. (2020) classifies the Miyazawa model as a type II model, as it allows for the estimation of induced effects from household consumption using type II multipliers, similarly to the standard closed IO model. The key distinction between the two lies

in the treatment of consumption: in the Miyazawa model, consumption is disaggregated by household type.

Applications of the Miyazawa model, in both its basic form and modified versions, whether in a single-region or multiregional context, are widely documented in the literature. These include the investigation into the interdependence between income formation and output generation (Hewings et al., 2001), the analysis of the impact of structural changes on income distribution (Moreira et al., 2008; Alberti et al., 2023), the study of the linkages between the productive structure of consumption and income formation (Gutierre et al., 2012), and the assessment of the economic impact of events such as earthquakes (Okuyama, 2004).

With regard to the Batey-Madden model, it is an extended IO model based on the activity-commodity framework (Batey & Hewings, 2021). Following Batey and Madden (1981) and Batey and Hewings (2021), in the activity-commodity framework, the demographic-economic system is characterised as a series of interrelated activities, each producing and consuming one or more commodities. In this framework, a matrix of coefficients, formed by dividing the entries in an activity vector by the corresponding activity level, is used to premultiply a column vector containing the activity levels, with the result set equal to a column vector of constraints. When the number of activities is equal to the number of commodities, the framework becomes merely a set of simultaneous equations, where the number of unknowns (activity levels) equals the number of equations, resulting in a single determinate solution (Batey & Hewings, 2021). Under this condition, the activity-commodity framework can be used to extend the conventional Leontief IO model by replacing the industrial sectors of the IO model with industrial activities (columns) and industrial commodities (rows), where gross output is defined as the activity level and final demand is specified as the vector of constraints. A detailed description of this framework is provided in the previously cited studies. As discussed in those studies, the Batey-Madden model, through the activity-commodity framework (or activity analysis, as it is also known), addressed a consistency problem in demographic-economic modelling related to the representation of unemployment levels, allowing the representation of transitions between different activity statuses of households (e.g., employed, unemployed). This framework automatically adjusts the weighting given to the employed and unemployed population in order to be consistent with the activity levels for unemployment (Batey & Madden, 1981).

The Batey-Madden model thus offers a mechanism for linking economic and demographic components within a single-region economy, which allows for the derivation of impact multipliers that are analogous to, but more comprehensive than, those obtained from a conventional Leontief model (Batey, 2018).

In the basic Batey-Madden model, population cohorts are not differentiated (e.g., by income or age), except in early versions that distinguish between two skill categories: non-manual and manual (Batey & Madden, 1983). Nevertheless, by incorporating a distinction between employed and unemployed individuals, the model already captures an important dimension of household heterogeneity. As noted in Batey and Hewings (2021), further disaggregation by cohort becomes possible when detailed consumption data are accessible. As presented in the following section, these authors also introduce a variation of the basic Batey-Madden model that incorporates more demographic detail.

The main applications of the Batey-Madden model, whether in its original or extended versions, are found in a regional context and includes its use in assessing the distributional effects of welfare payments (Batey & Madden, 1983) and the employment consequences of demographic changes, where the analysis includes the quantification of the economic impact of consumption by employed and unemployed individuals (Batey & Madden, 1999).

In the formalisation proposed in Batey and Weeks (1989) to enhance the representation of household characteristics in extended IO models, a four-stage process of elaboration is followed, where each stage builds upon the previous one. This sequence begins with the simplest form of the model, in which households are treated exogenously. The second stage involves endogenizing household consumption using the standard closure of the IO model with respect to households, which consists of adding a single row and column to the coefficient matrix, as discussed in section 3. The sequence culminates in a comprehensive Batey-Madden model that includes three types of household income and expenditure (Batey & Weeks, 1989; Batey & Hewings, 2021).

In this section, the Miyazawa model, the base version of the Batey-Madden model and the final two stages of the model proposed in Batey and Weeks (1989) are presented and discussed.

i. Miyazawa model

The presentation of the Miyazawa model follows the notation used in Miller and Blair (2022), which is in line with that used in Miyazawa (1976). The model assumes that households

can be classified into q income groups, so the augmented matrix of coefficients is represented by:

$$\bar{A} = \begin{bmatrix} A_{(n*n)} & C_{(n*q)} \\ V_{(q*n)} & 0_{(q*q)} \end{bmatrix} \quad (8)$$

Accordingly, the expanded IO system is expressed in the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} f* \\ g \end{bmatrix} \quad (9)$$

In Equation (9), V and C represent the same coefficients as before, but now account for the various consumer groups: $\mathrm{V}_{(q*q)} = [v_{gj}]$, where v_{gj} represents the ratio of wages paid to households in group g ($\mathrm{g} = (1, \ldots, \mathrm{q})$) to the output of sector j; $\mathrm{C}_{(n*q)} = [c_{ih}]$, where c_{ih} represents the ratio of consumption by household group h of products from sector i to that group's income. Variables y and g are also extended to account for the different household groups: $y_{(q*1)}$ is now a vector of total income for each of the income groups and $g_{(q*1)}$ is a vector of exogenous income for the income groups.

Assuming, $g = 0$, Equation (9) is thus reformulated as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} I - A & -C \\ -V & I \end{bmatrix}^{-1} \begin{bmatrix} f* \\ g \end{bmatrix} \quad (10)$$

Defining $\mathrm{B} = (\mathrm{I} - \mathrm{A})^{-1}$, $\mathrm{VBC} = \mathrm{L}$ and $\mathrm{K} = (\mathrm{I} - \mathrm{L})^{-1}$, and following Miller and Blair (2022), Equation (10) can be solved as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} B(I + CKVB)_{(n*n)} & BCK_{(n*q)} \\ KVB_{(q*n)} & K_{(q*q)} \end{bmatrix} \begin{bmatrix} f* \\ g \end{bmatrix} \quad (11)$$

Following Miller and Blair (2022), based on Equation (11) and using the equivalence $\mathrm{B(I + CKVB)} = B(I - CVB)^{-1}$[1], the Leontief inverse for the augmented system is given by:

$$(I - \bar{A})^{-1} = \bar{B} = \begin{bmatrix} \bar{B}_{11} & \bar{B}_{12} \\ \bar{B}_{21} & \bar{B}_{22} \end{bmatrix} = \begin{bmatrix} B(I - CVB)^{-1} & BCK \\ KVB & K \end{bmatrix} \quad (12)$$

The KVB matrix (from the Leontief inverse of the augmented system) captures the direct, indirect, and induced income effects for each household group generated by the initial final

[1] See Miller & Blair (2022, p. 271) for the derivation.

demand. Each element in K represents the total (direct, indirect, and induced) increase in the income of one household group resulting from the expenditure associated with each additional unit of income in another household group. The elements of BCK expresse the total production effect attributable to the increase in income of each household group, transmitted through consumption. Lastly, matrix $\mathrm{B(I-CVB)^{-1}}$ represents the effect of final demand on output. Therefore, and in line with the standard closed IO model, the Type II production multipliers derived from the Miyazawa model are calculated as the sum of the columns of matrix $\mathrm{B(I-CVB)^{-1}}$.

As previously mentioned, the Miyazawa model has been widely applied in the literature. Typically, studies employing this model consider only a few income groups (Alberti et al., 2023). To more thoroughly capture the heterogeneity addressed by the Miyazawa framework, the methodology presented in Alberti et al. (2023) incorporates a distinctive feature. In that study, a total of 4 947 households were considered, which were subsequently disaggregated into 356 904 individuals after each simulation. This high level of disaggregation was made possible through the use of a statistical matching technique, employed to combine microdata from the Family Budget Survey and the National Household Sample Survey. As highlighted by the authors, this degree of disaggregation allows for more accurate measurement of poverty and inequality, with the study producing estimates of changes in the poverty rate and inequality levels (measured by the Gini coefficient).

ii. Batey-Madden model

The description of the Batey-Madden model follows Batey (1985), Batey (2018) and Batey and Hewings (2021), with partial adjustments to the notation to align with the previously discussed models.

The basic version of the model considers three types of activity (industrial, employed workers and unemployed workers) and three types of commodities (industrial, labour and economically active population). The vector of activity levels consists of output, employment, and unemployment, and the input vector includes industrial final demand, commuting, and the economically active population. It is assumed that commuting between the study region and the surrounding area is in equilibrium and thus considered to be zero.

The model can be represented by the following equation:

$$\begin{bmatrix} I-A & -h_c^e & -h_c^u \\ -l & I & 0 \\ 0 & I & I \end{bmatrix} \begin{bmatrix} x \\ e \\ u \end{bmatrix} = \begin{bmatrix} f^* \\ 0 \\ \rho \end{bmatrix} \qquad (13)$$

In Equation (13), in the first matrix (denoted by A^*) , l is a 1 x n vector of employment-production functions, by sector, h_c^e is an n x 1 vector of household-consumption coefficients per employed worker, by sector, h_c^u is an n x 1 vector of household-consumption coefficients per unemployed worker, by sector. In the second matrix, e is a scalar measuring total employment and u is a scalar measuring total unemployment. Lastly, in the third matrix ρ is a scalar measuring total labour supply. The definitions of the remaining variables are consistent with those used in previous models.

Matrix A^* can be partitioned as follows, in order to separate the economic and demographic characteristics of the system:

$$A^* = \begin{bmatrix} I-A & -h_c^e & -h_c^u \\ -l & I & 0 \\ 0 & I & I \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* \\ A_{21}^* & A_{22}^* \end{bmatrix} \qquad (14)$$

In Equation (14), A_{11}^* corresponds to $(I - A)$, A_{12}^* to $[-h_c^e \quad -h_c^u]$, A_{21}^* to $\begin{bmatrix} -l \\ 0 \end{bmatrix}$ and, finally, A_{22}^* to $\begin{bmatrix} I & 0 \\ I & I \end{bmatrix}$.

The inversion of matrix A^* provides a set of impact multipliers, analogous to those resulting from the basic IO model, though more comprehensive in scope. These multipliers capture the effects on activity levels (output, employment, and unemployment) resulting from changes in input variables (final demand, commuting flows, or the economically active population). The inverse of matrix A can be expressed in the following form:

$$(A^*)^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \qquad (15)$$

The elements of matrix $(A^*)^{-1}$ can be computed using equations 6 to 9 presented in Batey (1985). Submatrix B_{11} represents the effects on gross output (direct, indirect, and induced effects) of unit changes in final demand, with the output multiplier calculated by summing the columns of submatrix B_{11}, as in the conventional closed IO model. Submatrix B_{21} indicates the direct, indirect, and induced effects upon employment of a unit change in final demand. In this version of the model, which includes only one type of worker, the first row of the submatrix

B_{11} represents the effects on employment, and the second row represents the effects on unemployment (both having the same values but with opposite signs). Submatrix B_{12} capture the effects on gross output of unit changes in demographic input variables (the first m/2 columns indicate the impact on gross output of a unit increase in locally resident unemployed individuals who take up employment outside the study area while continuing to reside within it[2]; the second m/2 columns show the impact on gross output of a unit increase in the size of the labour force). Lastly, the elements of matrix B_{22} represent the direct, indirect, and induced effects on local employment or unemployment resulting from an exogenous unit change in employment, as well as the probabilities of a new entrant to the labour force either obtaining employment or remaining unemployed. A detailed interpretation of the elements within these submatrices is discussed in Batey (2018).

iii. Extended input–output model with three household types (Batey & Weeks, 1989)

As previously discussed, the initial stage of the model presented in Batey and Weeks (1989) is the basic IO formulation with exogenous households, followed by the standard closure approach in the second stage.

The third stage of the proposed model contributes to addressing some of the criticisms directed at the standard closed IO model (discussed in section 3), particularly those related to the assumption of a homogeneous consumption function and the lack of an explicit assumption regarding the source of newly employed workers. The stage 3 model builds on the formulation proposed in Miernyk et al. (1967) allowing for a distinction between local and in-migrant workers and their respective consumption propensities. This model was applied to study the impact of the space program on Boulder, Colorado.

In Miernyk et al. (1967), growth in personal income is categorized into two types: extensive and intensive. Extensive growth reflects increases in output and employment without any increase in *per capita* income, whereas intensive growth results from gains in productivity. This distinction allows the induced effects to be separated into two components: those arising from extensive growth and those linked to intensive growth. In this model, it is assumed that

[2] It should be noted that in the version of the model presented, it is assumed that commuting between the study region and the surrounding area is in equilibrium and therefore considered to be zero, which implies that this effect is also zero.

in-migrants receive the same wage rates as local workers (extensive income), and that the productivity gains underlying intensive income occur among local workers.

Following Batey and Weeks (1989), which is based on the formulation of Miernyk's model proposed in Blackwell (1978), and using notation consistent with this study, stage 3 of the model can be represented as three blocks of simultaneous equations:

$$\begin{bmatrix} I-A & -C^m & -C^a \\ -V^{a-m} & I & 0 \\ -V^m & 0 & I \end{bmatrix} \begin{bmatrix} x \\ y^i \\ y^e \end{bmatrix} = \begin{bmatrix} f^* \\ g^i \\ g^e \end{bmatrix} \qquad (16)$$

In Equation (16) superscripts i and e denote intensive income and extensive income, respectively. V^m represents the wage coefficients for in-migrants, while V^{a-m} represents the wage coefficients for local workers associated with increased productivity[3].

In Batey and Weeks (1989), it is argued that the model presented in Equation (16) is appropriate for contexts of rapid growth where there is a shortage of workers in the local labour market, but less applicable in situations where a substantial pool of unemployed workers exists. Building on this, the fourth stage of the model incorporates the explicit inclusion of unemployed households' income and consumption. After some adjustments to the notation and following Batey and Weeks (1989), this version of the model is represented by the following equation:

$$\begin{bmatrix} I-A & -C^m & -C^a & -sC^u \\ -V^{a-m} & I & 0 & 0 \\ -V^m & 0 & I & 0 \\ L\hat{\rho} & 0 & 0 & I \end{bmatrix} \begin{bmatrix} x \\ y^i \\ y^e \\ u \end{bmatrix} = \begin{bmatrix} f^* \\ g^i \\ g^e \\ \rho \end{bmatrix} \qquad (17)$$

In Equation (17) , L represents the employment-to-output ratio by sector of activity; $\hat{\rho}$ is a diagonal matrix of probabilities indicating the proportion of vacancies filled by previously unemployed local workers; s is a scalar indicating the unemployment benefit rate; C^u represents the consumption propensities of unemployed households; and ρ and u retain the same meaning as in the Batey–Madden model. In this model, the unemployment benefit payments foregone by local unemployed workers when they move into paid employment can be seen as a redistributive effect.

[3] The method for calculating these coefficients is detailed in Blackwell (1978).

The intensive and extensive effects from the third stage of the model give rise to what is referred to in the literature as type III income multipliers, whereas the induced effects considered in the fourth stage of the model (intensive, extensive, and redistributive) correspond to the so-called type IV income multipliers (West, 2002).

Batey and Weeks (1989) compares employment and income multipliers across the model stages for the Greater Cork region, Republic of Ireland. The analysis finds minimal differences between the standard closed IO model and the stage 3 formulation, suggesting that the additional effort involved in implementing stage 3 household disaggregation may not be justified. As anticipated, a comparison with the stage 4 model confirms that the standard closed IO model considerably overestimates impacts, with employment multipliers exceeding actual values by over 20 percent. The differences in income multipliers are more irregular, and relatively small in labour-intensive or low-wage sectors.

5. Recent developments in Demographic-Economic Extended IO Models

This section aims to present and discuss recent developments in Demographic-Economic Extended IO models. It analyses an extension of the basic version of the Batey-Madden model that incorporates population cohorts and a mechanism that permits a more in-depth representation of the role of households in the economy by reconciling IO analysis with modern consumption theory.

i. Batey-Madden model with more demographic detail

In Batey and Hewings (2021), the basic version of the Batey-Madden model (as presented in Batey, 1985) is extended to incorporate population cohorts through a hybrid framework that combines the Batey-Madden structure with Schinnar's Eco-Demographic model (Schinnar, 1976). After harmonizing the notation with that used in previous models, this extended model can be represented by the following equation:

$$\begin{bmatrix} I-A & -h_c^e & -h_c^u \\ -REA_0p & I & 0 \\ 0 & I & I \end{bmatrix} \begin{bmatrix} x \\ e \\ u \end{bmatrix} = \begin{bmatrix} f^* \\ 0 \\ \rho \end{bmatrix} \qquad (18)$$

In this version of the model, the terms h_c^e and h_c^u from the Batey-Madden model retain a similar meaning, representing household consumption propensities, but are now expressed as

an $\mathrm{m} * \mathrm{n}$ matrix, where m corresponds to the number of population cohorts. Likewise, the variables e, u, and ρ are now represented as $\mathrm{m} * 1$ vectors rather than scalars.

The REA_0 term in the Schinnar model relates gross output by sector to population by cohort. Following Batey and Hewings (2021), in the Schinnar model, R is a matrix relating the active and inactive portion of the total population, E is a matrix of actual employment patterns reflecting differences in labor force participation rates by cohort, and A_0 is a diagonal matrix of employment coefficients by sector.

Additionally, a diagonal matrix of probabilities, denoted by p, is introduced, indicating the proportion of vacancies in each sector that are filled by previously unemployed local workers, since not all job openings are assumed to be filled from the regional unemployment pool.

This new model is presented by the authors as a source of demographic-economic multipliers with considerable potential, being capable of capturing the economic impact of changes in specific population cohorts. Furthermore, by introducing alternative values for p, the model allows for an assessment of the effects of varying levels of dependence on in-migrant labour.

It is worth noting that in Batey and Hewings (2021), the model is only introduced in a relatively superficial manner, and the derivation of the resulting multiplier matrices is not presented.

ii. Reconciling input–output analysis with modern consumption theories

As noted in section 3, one of the criticisms of the standard closed IO model with respect to households is the assumption that current consumption is entirely determined by current income, which is inconsistent with widely accepted theories of consumption behaviour. According to theories such as the relative income hypothesis (Duesenberry, 1949) and the life cycle/permanent income hypothesis (Friedman, 1957; Ando & Modigliani, 1963; Hall, 1978), household consumption is also influenced by past consumption patterns, expectations about future income, and various other factors (Chen et al., 2016). This issue is addressed in Chen et al. (2016), which proposes a new model aimed at reconciling IO analysis with modern consumption theory. In this new model, household consumption is divided into endogenous and exogenous components: endogenous consumption is determined by current household income, while exogenous consumption is not. According to the authors, this model can simulate

changes in household consumption behaviour when exogenous stimulus policies lead to higher levels of disposable income, making it a valuable tool for evaluating short-term GDP responses. In Chen et al. (2016), the model is applied to analyse the impact of a large stimulus package on the Chinese economy.

In the model proposed in Chen et al. (2016), a single representative household is assumed, with household consumption being split into endogenous and exogenous parts, as discussed. In this formulation, only the endogenous portion of consumption is incorporated into the input coefficient matrix. In matrix form, and with notation adjusted to ensure comparability with the previous models, the model in Chen et al. (2016) is represented as follows:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} A & C^{end} \\ V & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} f* \\ g \end{bmatrix} \tag{19}$$

The formulation of this model is closely aligned with the basic closed model presented in section 3, specifically in the version that only endogenizes labour income. However, in the model proposed in Chen et al. (2016), the numerator of variable C^{end}, which represents household consumption coefficients, includes only endogenous consumption. Additionally, final demand $f*$ comprises the exogenous component of household consumption, as part of the consumption is not being endogenized.

From matrix $\bar{A} = \begin{bmatrix} A & C^{end} \\ V & 0 \end{bmatrix}$, the conventional multipliers of the IO model can be derived. In Chen et al. (2016), the derivation and interpretation of these multipliers are examined in detail. With the necessary notational adjustments to facilitate comparability across models, and following Chen et al. (2016), the extended Leontief inverse matrix can be expressed as:

$$(I - \bar{A})^{-1} = \bar{B} = \begin{bmatrix} \bar{B}_{11} & \bar{B}_{12} \\ \bar{B}_{21} & \bar{B}_{22} \end{bmatrix}$$

$$= \begin{bmatrix} B(I + C^{end}\dfrac{1}{1 - VBC^{end}})VB & BC^{end}\dfrac{1}{1 - VBC^{end}} \\ \dfrac{1}{1 - VBC^{end}}VB & \dfrac{1}{1 - VBC^{end}} \end{bmatrix} \tag{20}$$

In Equation (20), K retains its previous meaning ($B = (I - A)^{-1}$).

From matrix $\bar{B}$, the following multipliers can be derived: the column sums of $\bar{B}_{11}$ indicate the gross output of all industries required to satisfy one unit of exogenous final demand for the output of industry j (which gives rise to the Type II multipliers previously introduced); the

column sum of $\bar{B}_{12}$ represents the gross output of all industries induced by one unit of exogenous household income; the jth element of vector $\bar{B}_{21}$ indicates the total household labour income (earned across all production industries) induced by one unit of exogenous final demand for the output of industry j; and $\bar{B}_{22}$ represents the total household income induced by one unit of exogenous household income.

In Chen et al. (2016), the distinction between endogenous and exogenous consumption is estimated using an econometric model, represented by the following system of equations:

$$\begin{cases} c_{it} = \alpha_{it} y_t + \beta_i c_{i(t-1)} + \varepsilon_{it} \\ \alpha_{it} = \alpha_{it-1} + \mu_{it} \end{cases} \quad (21)$$

In Equation (21), c_{it} represents the aggregated household consumption of product i in year t; α_{it} is the endogenous consumption coefficient of product i; y_t, as before, denotes total household income; and β_i captures the effect of the previous consumption peak on current household consumption. The coefficients of Equation (21) are estimated using a maximum likelihood estimation method with the Kalman filter. Further details regarding the formulation and estimation procedure are provided in Chen et al. (2016). Endogenous consumption is thus obtained as follows, while exogenous consumption is derived by difference.

$$c_{it}^{end} = \alpha_{it} y_t \quad (22)$$

In Chen et al. (2016), as expected, the estimated results of the fiscal stimulus analysed through the proposed model are lower than those obtained using the standard semi-closed input–output model, in which all household consumption is endogenous. According to the new model, the estimated economy-wide value-added multiplier is 0.852, whereas in the standard semi-closed input–output model it is 1.003. In Chen et al. (2016), the multiplier obtained through the new model is also compared with the result obtained in Diao et al. (2012), where a multiplier of 0.82 was estimated using a Computable General Equilibrium model, suggesting that the results compare well despite the methodological differences. Lastly, it is also concluded in Chen et al. (2016) that the results obtained are in line with the actual changes in Chinese GDP following the introduction of the stimulus package.

6. Alternative models based on IO framework

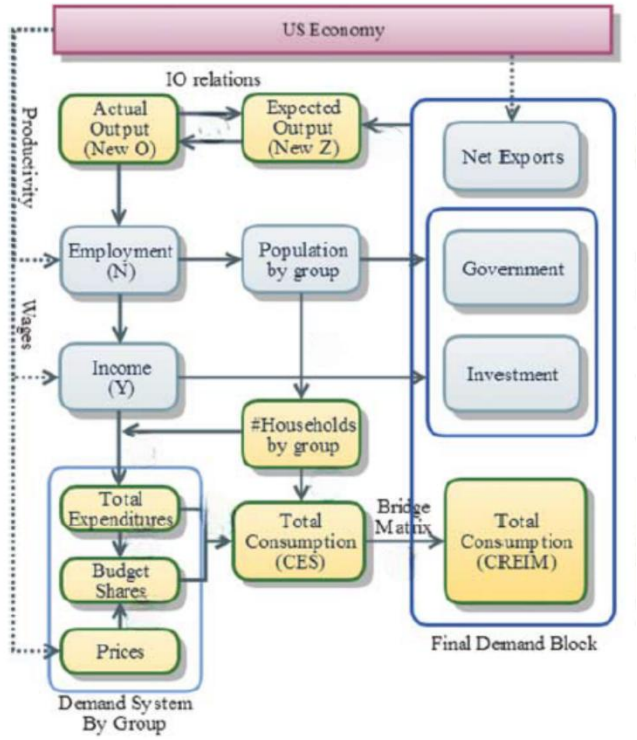
Even though Demographic–Economic Extended IO Models offer parsimony and perform well under certain conditions (West, 2002), they retain characteristics of standard IO models that represent some of their main limitations . For example, they preserve the linear structure, which in impact studies may lead to an overestimation of multiplier effects (West, 2002). It is worth noting that the linearity assumption in IO models is particularly problematic in capturing the complexity of consumption patterns (West, 1995).

To better capture demographic–economic interactions and address some of the limitations of IO models, other modelling approaches have become widely used, especially extended econometric IO models and computable general equilibrium (CGE) models (Batey, 2018). These modelling frameworks retain some fundamental features of the IO model, albeit in a more advanced and flexible form (Batey, 2018). It is important to note, as discussed in West (1995), that in practice the distinction between econometric IO models and CGE models is not always clear, since econometric IO models sometimes incorporate features typically associated with CGE models.

Econometric IO models belong to the class of models referred to in the literature as integrated models (West, 2002), which recognize the strengths and limitations of each individual approach and combine them to achieve a potentially more accurate representation of reality. The main benefit of integrating these two methodologies is the ability to develop a model that offers a dynamic, non-linear picture of economic change within a detailed interindustry framework (West, 2002). The way in which IO and econometric models are integrated depends on the specific modelling objective and can result in a wide range of specifications (West, 1995). In general, these models can be classified into two types: IO econometric (IO/EC) models, which are primarily based on the IO framework and incorporate econometrically estimated equations to endogenize the relationships between factor inputs and final demands (the closure); and econometric/IO (EC/IO) models, which use an econometric model as a starting point and substitute its output component with an IO block framework (West, 2002). However, the distinction between the two types of modelling is often not clear (West, 2002). In West (2002), several models constructed for the USA are identified as EC/IO type (e.g., Conway, 1990), while models developed for Australia are classified as IO/EC type (e.g., West, 1994). A more in-depth analysis of these models and their characteristics is presented in West (1995) and West (2002).

The consideration of household heterogeneity within the Econometric IO framework is addressed, for example, in Kim et al. (2015), which proposes an extension to the regional econometric IO model (REIM) [Conway, 1990], integrating a demand system with age and income parameters into the REIM. The extended REIM model offers a valuable approach to analyse how structural changes in expenditure patterns arise from shifts in demographic composition, serving as a useful way of assessing the economic impact of different population scenarios, particularly relating to various migration streams (Batey, 2018), as exemplified in Kim et al. (2015). A schematic representation of the extended REIM is represented in Figure 1.

Figure 1. Schematic representation of the extended REIM (Kim et al., 2015)



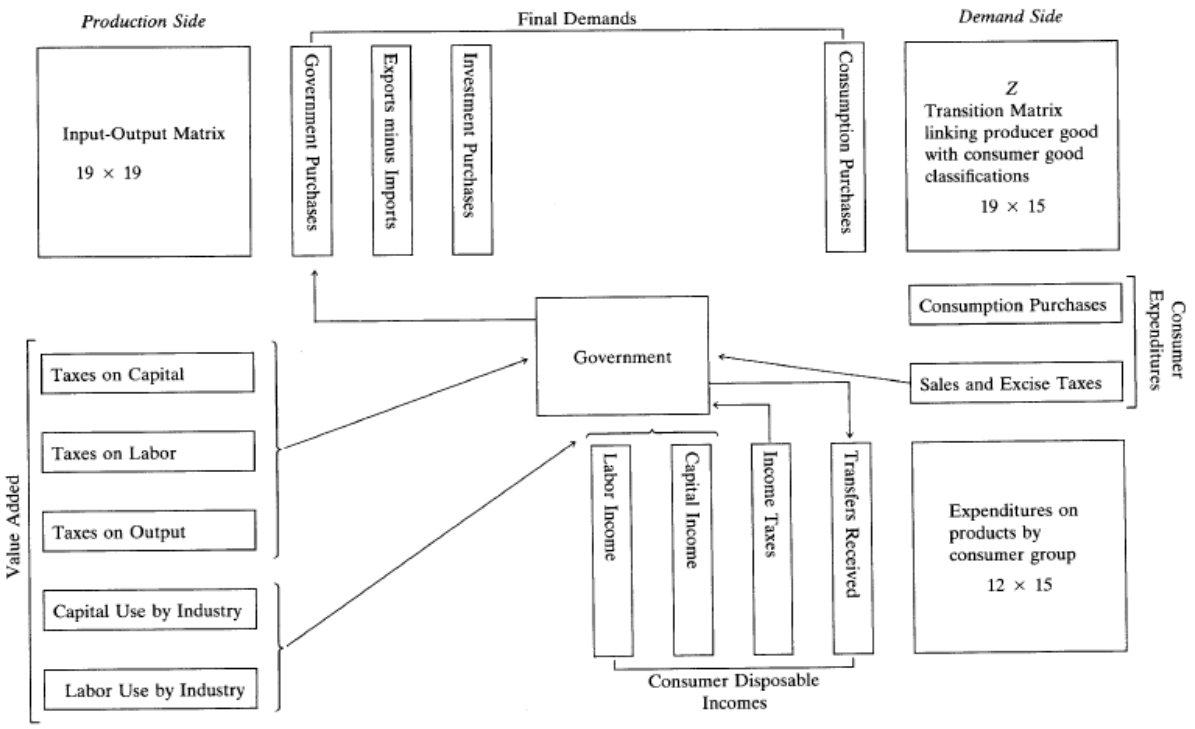
Source: Kim et al. (2015)

CGE models aim to capture a broader set of equilibria among economic variables than IO models, relaxing typical IO assumptions such as linearity and unlimited capacity, with a key feature being the endogenization of prices. The CGE model, formulated as an optimization model, determines the optimal set of endogenous variables in response to an exogenous shock and, unlike the demand-driven IO model, incorporates explicit supply constraints typically grounded in a neoclassical framework (West, 1995).

In CGE models, there is also the possibility of disaggregating households into several representative household types, according to different criteria such as sources of income,

location, or expenditure patterns (Burfisher, 2021). The model proposed in Ballard (1985) is an example of a CGE model with a high level of household disaggregation, considering twelve consumer groups classified by gross money income. This model has been used in several analyses of U.S. taxes. The basic structure of this single-period model is shown in Figure 2.

Figure 2. A CGE model with household disaggregation (Ballard, 1985))



Source: Ballard (1985)

The characteristics of these models are analysed and compared with each other and with IO models, for example in West (1995). From this analysis, it can be observed that Econometric IO models by introducing a dynamic structure address the persistent criticisms of IO models, such as their static nature and the time lag between the reference period of the coefficients and the period of analysis, and their non-linear formulation enhances the model's ability to reflect complex economic behaviour. On the other hand, a common criticism of Econometric IO models is their lack of adequate supply-side specification. Regarding CGE models, the main advantage is that both the supply side and the demand side are explicitly determined with full price responsiveness. Additionally, CGE models facilitate the imposition of capacity constraints, particularly in the short term, and they allow for better alignment with microeconomic theory. However, their implementation requires the specification of a large number of parameters and coefficients, which are often not available, introducing a significant element of uncertainty into the model with a considerable effect on empirical results. Another

feature of these models is their greater complexity in terms of understanding and application, especially when compared to IO models (West, 1995).

West (1995) also presents an empirical comparison of value-added and employment multipliers calculated using an IO/EC model, a CGE model, and the conventional closed IO model (which allows for the estimation of type II multipliers). Regarding value-added multipliers, the long-term IO/EC model generally yields the highest multipliers, averaging 109.4% of the type II IO multiplier values, while the short-term CGE model produces the lowest, averaging 72.6% of the IO multiplier values, primarily due to capital supply constraints. Additionally, in West (1995), the application of the models to a case study shows that the short term IO/EC model's estimate, in terms of value added, is slightly lower, at 96% of the IO model's result, while the short term CGE model produces the lowest estimate, at 62% of the IO model's value. Another important finding is that the share of value added allocated to wages and salaries decreases from 58% in the IO model to 53% and 56% in the short term and long term IO/EC models respectively and drops further to 29% and 39% in the short term and long term CGE models. This reflects these models' capabilities to redistribute resources and increase production without necessarily corresponding proportional increases in labour costs and employment. Regarding employment impacts, the results are significantly smaller relative to the IO model compared to those for value added, reflecting the greater influence of marginal labour productivity changes. For instance, the short term IO/EC model captures only 72% of the employment impact estimated by the IO model, while the short term CGE model accounts for just 48% percent.

Miller and Blair (2022) identifies several applications of CGE and Econometric IO models. CGE models have been applied in areas such as trade (e.g., Chen et al., 2022), environmental policy (e.g., Turner et al., 2012), fiscal planning (e.g., Madden, 2017), tourism and transportation (e.g., Wittwer, 2017), and economic planning and forecasting (e.g., Bardazzi & Ghezzi, 2018). In the case of Econometric IO models, these have been used to address policy issues such as regional labour productivity (e.g., Rey & Jackson, 1999), to study the impact of unexpected events (e.g., Donaghy et al., 2007), and to analyse regional growth (e.g., Capello, 2007).

Another type of model that enables a more detailed representation of households is the Social Accounting Matrices (SAM). SAM extends the IO tables by incorporating income and expenditure accounts and can be seen as an extended IO framework that captures not only

macroeconomic aggregates and their sectoral breakdown, but also the distribution and redistribution of income (Ross, 2017). As noted in Ross (2017), IO tables display payments to factors of production (such as wages and other value added components), but they do not capture the subsequent flows to institutions.

The additional information provided by the SAM, compared to IO tables, can be used to extend and improve multiplier modelling by incorporating the behaviour of non-production sectors and, in particular, by establishing a more explicit link between economic activity and household income, as the SAM multiplier captures all income flows to households, which is expected to improve the accuracy of the Type II multiplier (Ross, 2017).

Ross (2017) also notes that the SAM is a flexible framework in terms of the degree of disaggregation, which generally depends on the motivation behind its construction and the availability of data. In Figure 3, an example of a SAM in which households are disaggregated into three income-level groups is presented.

Figure 3. Example of a reduced form fully articulated SAM with household disaggregation (Miller & Blair, 2022)

	Industry			Households			Exogenous			
	Nat. Res.	Manuf.	Services	LIH	MIH	HIH	Capital	Govt.	RoW	Total
Industry										
Natural Resources	50	93	10	5	8	7	0	20	10	203
Manufacturing	70	900	400	350	650	600	40	200	350	3,560
Services	10	500	550	50	50	100	200	150	250	1,860
Households										
Low Income	6	150	60					200	(2)	414
Medium Income	12	510	250					89	(1)	860
High Income	25	550	260					35	(2)	868
Exogenous										
Capital	15	200	75	(20)	60	100		(10)	90	710
Government	10	400	125	20	50	25	140	25	(15)	780
Rest of World	5	257	130	9	42	36	330	71	0	880
Total	203	3,560	1,860	414	860	868	710	780	880	10,135

Source: Miller & Blair (2022, p. 521)

When SAM are used to quantify demand shocks, fixed prices are assumed, along with excess capacity and unemployment, which absorb the shock (Ross, 2017). Moreover, calculating multipliers requires specifying which variables are treated as exogenous. As noted in Round (1988), government, capital, and external sectors are commonly designated as exogenous in SAM-based impact modelling.

Multipliers derived from the SAM are generally larger than those from the IO model, and when final demand is also endogenized, they become even larger, as the SAM captures transactions not previously included in the IO interindustry accounts (Miller & Blair, 2022).

However, this relationship between SAM and IO multipliers is not observed in the analysis conducted in Ross (2017), which may indicate that SAM multipliers are highly sensitive to modelling choices. In Ross (2017), similarly to the analysis presented in Emonts-Holley et al. (2015), SAM-derived multipliers are compared with various closed IO model variants, and it is concluded that the mean SAM multiplier falls within the range of mean IO Type II values.

SAM-based models are widely applied in the literature to policy problems. In Miller and Blair (2022), some key applications are highlighted, including analyses of regional development policy (e.g., Ferreira et al., 2022), extensions to social and environmental indicators and the study of environmental policies (e.g., Mardones & Brevis, 2021) and the study of spatial patterns of income and wealth generation (e.g., Madsen & Jensen-butler, 2005).

Although SAMs can be used to analyse economic and social impacts in a comprehensive manner, these models present some of the limitations of IO models, particularly regarding the modelling of agents' behavioural responses to shocks and the incorporation of the supply side (Ross, 2017). CGE models, often using the SAM as a database, are adopted to address some of these limitations. However, it is important to note that, as discussed in this section, CGE models themselves also have their own limitations.

7. Conclusions

The estimation of induced effects in the context of economic impact evaluation using the IO methodology lacks a standardized approach. Induced effects are highly significant in impact analysis due to the central role of household consumption in the economic system and can even exceed indirect effects in magnitude. Therefore, appropriate attention must be paid to the methodology adopted for their estimation. Despite the relevance of induced effects, in the most common formulations, IO models with endogenous consumption assume the consumption structure of a representative household, thereby disregarding existing heterogeneity. Several studies have pointed out that one reason household consumption behaviour is not accurately captured in the standard closed IO model is the neglect of differences in consumption patterns across household types.

A common approach to introducing greater detail in household consumption within IO models has been the use of extended IO systems, referred to in the literature as Demographic-Economic Extended IO Models. The interactions captured by these extended models, which

incorporate both demographic and economic linkages, not only enhance the realism of multipliers but also allow for the derivation of additional multipliers that reflect the relationships between demographic and economic variables. These extended IO models, as mentioned in Batey (2018), have also the advantage of analytical clarity, including the ability to add extra layers as the model is progressively elaborated, which is a valuable feature in model design.

In the context of extending the IO framework to incorporate interactions between economic and demographic components, two main lines of research stand out: the Miyazawa model and the contributions of Batey and Madden, formalized in what is known as the Batey-Madden model. The Miyazawa model restructures the standard closed IO model by enabling the representation of household behaviour through the differentiation of multiple household categories. The Batey-Madden model provides a mechanism for linking economic and demographic components within a single-region economy. Through the activity analysis framework (also known as the activity-commodity framework), this model consistently allows for the modelling of transitions between different activity statuses of households (for example, employed and unemployed).

Additionally, the representation of household consumption in IO models can also be improved through more refined modelling techniques, as exemplified by the work of Chen et al. (2016), which develops a new model intended to reconciling IO analysis with modern consumption theory.

Although Demographic-Economic Extended IO Models offer parsimony and perform well under certain conditions, they retain characteristics of standard IO models that represent some of their main limitations. To better capture demographic and economic interactions and address these limitations, alternative modelling approaches have become widely used, particularly extended econometric IO models and CGE models. These frameworks retain some fundamental features of the IO model, albeit in a more advanced and flexible form.

Despite the theoretical foundations of these more complex models, as concluded in West (1995) and emphasized in Batey (2018), complexity is not necessarily a good indicator of performance, and large models tend to become a law unto themselves. For example, in the case of CGE models, their implementation requires the specification of a large number of parameters and coefficients, which are often unavailable. This introduces a significant element of uncertainty into the model, with considerable effects on the empirical results. According to the

comparison presented in West (1995), the differences in multipliers derived from extended econometric IO models and CGE models, compared to those from the standard IO model, are particularly significant in terms of employment. This is an indication that the assumption of linear structure in the IO model constitutes a major limitation in the quantification of employment effects.

Another type of model that enables a more detailed representation of households is the SAM. SAM extends the IO tables by incorporating income and expenditure accounts. The additional information provided by the SAM, compared to IO tables, can be used to extend and improve multiplier modelling by incorporating the behaviour of non-production sectors and, in particular, by establishing a more explicit link between economic activity and household income. On the other hand, these models retain some of the limitations of IO models, particularly with respect to modelling agents' behavioural responses to shocks and the absence of a supply-side component. Additionally, calculating multipliers requires specifying which variables are treated as exogenous. This decision can have a significant impact on the magnitude of the multipliers and may be somewhat arbitrary. Multipliers derived from the SAM are generally larger than those from the IO model, and when final demand is also endogenized, they become even larger, as the SAM captures transactions not previously included in the IO interindustry accounts. Although the greater detail provided by the SAM is relevant for certain types of analysis, in other cases there is less interest in representing such an extensive set of flows. Moreover, the increased complexity and data requirements of the SAM compared to IO models, combined with the fact that it still retains some of the same limitations and the degree of arbitrariness in the selection of endogenous variables, suggest that this model may not offer significant advantages over the Demographic-Economic Extended IO Models in certain contexts, for example in those that do not require detailed analyses of income distribution and redistribution.

As noted in West (1995), the trade-off lies between selecting a model that is simple and transparent (IO model) and one that is often harder to understand and apply, although it may be more theoretically appealing (e.g. econometric IO models and CGE models). The IO model has well-recognized limitations such as constant technology in technical coefficients, linearity in the production system, no supply constraints, and the absence of price effects, which make the model less suitable for long-term applications. However, when these limitations are acknowledged and the model is applied in contexts where they are less pronounced, aspects such as its transparency, relative ease of implementation compared to other models, and

sectoral disaggregation make the IO model a very powerful tool. Overall, it can be concluded that selecting a model for economic impact analysis invariably entails a trade-off. The aim should be to maximise the difference between the benefits and limitations of the models, considering both the purpose of the analysis and the resources available.

Regarding future developments on the topic of heterogeneity within the IO modelling context, Batey and Hewings (2021) concludes that a more expansive linkage between the activity analysis dimension offered by the Batey-Madden models and the associated income distribution impacts provided by the Miyazawa framework is needed. According to these authors, the Miyazawa disaggregation should be complemented by greater flexibility in the definition of households. One important dimension is their participation in the labour force (employed, unemployed), along with designators of their occupational or skill characteristics. A step toward this development may have been taken in the same publication, where a basic version of the Batey-Madden model is extended to incorporate population cohorts through a hybrid framework that combines the Batey-Madden structure with Schinnar's Eco-Demographic model.

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