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Population density, education spread and economic growth

By

José Pedro Pontes[1]

Abstract: We undertake a theoretical analysis of the spread of college education and economic growth, leading to three main findings.

First, by assuming that schooling is related with a productive activity that is *not* land-based, we draw the boundary of areas with formal education. We conclude that these areas expand when either population increases in each point or when technical progress primarily affects the modern productive sector.

Second, we characterise the evolution of schooling in a structurally stable economy and its impact on growth in a manner consistent with the empirical evidence. In particular, we find that the expansion of schooling in a structurally stable spatial economy promotes economic growth, albeit at a diminishing rate.

Finally, we provide a plausible explanation the apparent paradox of "education without economic development", which arises when the positive effect of a rapidly growing labour force on the education system is partially offset by a technological regression in the industrial sector.

Keywords: Education spread, economic growth, population density, education without development, space-time analysis

JEL Classification: R11, I20, O15

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1. Introduction

Significant positive elasticities of regional schooling levels with respect to demographic density are often reported in the literature. Differences in population density may either be substantial, as in the context of a “rural-urban gap” (Newbold and Brown, 2015) or more limited and arise within the same type of local area. In the latter case, differences in population density empirically affect educational attainment across both urban areas and rural regions.

Taking as a benchmark the empirically estimated range of productivity-density elasticities, the elasticity of educational attainment may lie slightly below the lower bound (Gibbons and Silva, 2008) or fall within the interval and close to its upper bound (Van Maarseveen, 2021).

This positive association follows clearly from the fact that education involves significant economies of scale, as most inputs have the nature of fixed assets. A trained professor can teach groups of varying sizes, and education typically relies on fixed infrastructure such as buildings, libraries and laboratories. A dense population allows these facilities to be shared by a larger number of students, thereby decreasing the cost per student.

Additionally, high population density fosters educational attainment by increasing the wage differential between individual with and without schooling. There is evidence that, in the case of higher education, the wage premium is significantly higher in urban than in rural areas; moreover, this gap has been increasing (Autor, 2019, Baum-Snow, Freedman and Pavan, 2018, Gould, 2007).

In this paper, in addition to explaining the spatial distribution of labour force and its educational level, we also highlight the main associated time trends. For this reason, this paper has a distinct space-time dimension. In what follows, we focus on universities, which correspond to post-compulsory education in most developed countries. However, it should be noted that population density also affects attainment in compulsory basic education (Zhang and Rozelle, 2022).

Pontes (2025), for Portugal, identifies four main trends in higher education in the recent past. First, the share of individuals with a completed higher education degree has increased, albeit at a decreasing rate. The annual growth rate of the share of tertiary-educated individuals rose from 6.4 per cent in 1981-2001 to 4.8 per cent in 2001-2021.

Second, the wage premium associated with higher education - measured as the difference in wages between graduates and individuals with only secondary education, has declined, sharply in some cases. The *OECD Employment Outlook* reports a decrease of about 23 per cent between 2006 and 2016.

Third, the spread of higher education has been a source of economic growth, although this effect appears to have weakened over time. The growth rate of real GDP per capita fell from 3.0 per cent in 1981-2001 to 0.4 per cent in 2001-2021.

Fourth, the expansion of higher education has led to a diffusion of college enrolment rates from main metropolitan areas to smaller cities and less densely populated regions. Growth rates of the share college-educated individuals were 3.0 and 7.7 for major metropolitan areas (Lisbon, Oporto and the region of Coimbra) and the less dense outer regions, respectively.

In this paper, we consider an asymmetrically populated spatial economy in which all individuals produce a composite consumption good and acquire education using a specific technology. The agent decides to be either a" farmer" or an "industrial worker". Following Tamura (2002), we emphasise the nature of this choice. Under the former option, the agent uses a technology with constant returns to scale in relation to labour and land, so that individual output decreases with population density. Under the latter, the individual adopts a land-free technology, with constant returns to scale in relation to labour only, so that productivity is independent of demographic density.

We assume that the choice of production technology is linked to the type of education received. Typically, a "farmer" is educated within the family, whereas a "worker" receives formal schooling. Within this framework, densely populated areas host an industrial activity and formal education, while sparsely populated regions are characterised by agriculture (or housing services) and informal, family-based education. In addition to demographic increase, technical progress in the modern sector expands the region with industrial production and a school educated labour force.

Regarding the time dimension, individual decisions about production and education are independent across locations, so any ordering could in principle be assumed. However, since the expansion of education empirically leads to decentralisation - colleges spreading from major metropolitan areas to smaller cities and less dense regions - we assume that individuals make decisions in order of decreasing population density. Industrial activity and schooling thus emerge first in densely populated locations and subsequently spread to less dense areas. This assumption allows us to account for the main empirical trends associated with the expansion of universities.

In Section 2, we define the boundary between regions characterised by industry and schooling and those characterised by traditional technology and education.

Then, in Section 3 we derive the dynamic patterns of the schooling rate and per capita income. Section 4 concludes.

2. The geographical structure of the economy

We assume a spatial economy defined on the interval $[0,1]$. Let r be a location measured by its distance from the origin. Population density is given by a strictly decreasing function $L(r)$ that satisfies the condition $L(1)=1$ and is plotted in Figure 1.

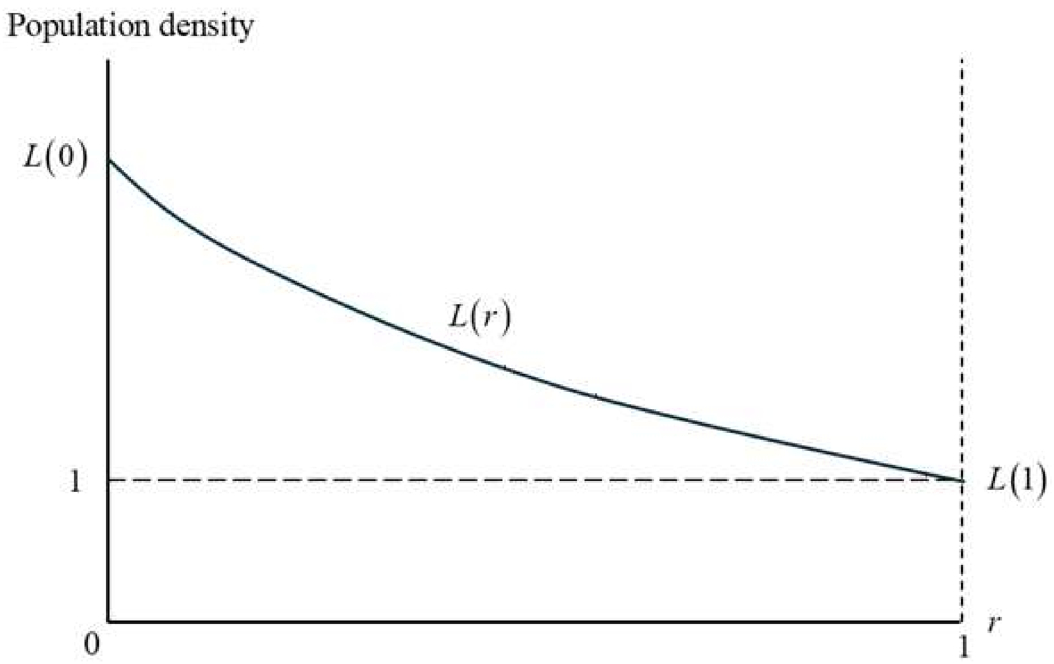

Figure 1: Population density curve

The economy produces a composite consumer good under constant returns to scale and uses education as an intermediate non-tradable service. For this reason, each good is produced and used in the same location.

At location r , $L(r)$ individuals decide on their occupation. They may be either "farmers", or (industrial) "workers".

By "farmer" we mean any agent who produces the composite consumer good using land; this includes not only those engaged in agriculture but also, for instance, providers of housing services. The output $Q_a(r)$ of the individuals living at point r is given by a production function that exhibits constant returns in relation to labour and land.

$$Q_a(r) = A_a L(r)^{\alpha} S^{(1-\alpha)} \tag{1}$$

where S is the total amount of land available in each location, parameter α belongs to $(0,1)$ and A_a is productivity in land-based production. Hence, a "farmer's" individual output in point r may be written as,

$$q_a(r) \equiv \frac{Q_a(r)}{L(r)} = A_a \left(\frac{S}{L(r)} \right)^{(1-\alpha)}$$

The output of a farmer is critically determined by the amount of land they can use. Since $S=1$ everywhere, $q_a(r)$ may alternatively be written as,

$$q_a(r) = A_a L(r)^{(\alpha-1)} \tag{2}$$

Thus, a farmer's output falls strictly with population density.

In addition to adopting a specific production technology, a "farmer" is also characterised by receiving a traditional form of family-based education at a constant unit cost g. Therefore, a farmer's net income is,

$$y_a(r) = q_a(r) - g$$

By substituting $q_a(r)$ from (2), the farmer's income becomes,

$$y_a(r) = A_a L(r)^{(\alpha-1)} - g \tag{3}$$

From now on, we assume that the productivity parameter A_a is sufficiently high to ensure that $y_a(r) > 0$ in each location.

By contrast, the individual may choose to be an (industrial) "worker". If point r is inhabited by workers, its aggregate output $Q_i(r)$ is generated by a production function that exhibits constant returns in relation to labour *only*.

$$Q_i(r) = A_i L(r) \tag{4}$$

where A_i stands for aggregate productivity in the industrial sector. Consequently, an individual worker's output is simply,

$$q_i(r) \equiv \frac{Q_i(r)}{L(r)} = A_i \tag{5}$$

which is independent of local population density.

In addition to applying technology (5) in production, each worker must be educated through formal schooling. For this purpose, they incur a fee $p(r) = \frac{F}{L(r)}$, where F denotes the fixed cost of establishing a school. For simplicity, we assume that the school operation entails zero variable costs.

Consequently, an individual worker's net output in location r may be written as,

$$y_i(r) = A_i - \frac{F}{L(r)} \tag{6}$$

where we again assume that A_i is sufficiently high to ensure that $y_i(r) > 0$.

Agents at point r will choose to be workers rather than farmers if,

$$y_i(r) > y_a(r)$$

or equivalently,

$$A_i - \frac{F}{L(r)} > A_a L(r)^{(\alpha-1)} - g \tag{7}$$

Conversely, they will become farmers if,

$$A_i - \frac{F}{L(r)} < A_a L(r)^{(\alpha-1)} - g \tag{8}$$

They are indifferent if

$$A_i - \frac{F}{L(r)} = A_a L(r)^{(\alpha-1)} \tag{9}$$

We can prove the following proposition.

Proposition 1: Assume that the spatial economy meets the two following conditions,

1. $L(0)$ is sufficiently high and,

2. $A_i - A_a < F - g$, i.e., the difference between educational costs outweighs the productivity advantage of industrial technology over the land-based production.

Then there exists a unique location $\tilde{r}$ interior to $[0,1]$, such that the agents in $r < \tilde{r}$ will prefer to become workers, whereas those in $r > \tilde{r}$ choose to be farmers, the marginal producer in $\tilde{r}$ being indifferent.

Proof: From (3) and (6), we may compute the first derivatives of individual incomes with respect to location r . Since $L'(r) < 0$, we obtain respectively

$$y_a'(r) = (1-\alpha) A_a L(r)^{(\alpha-2)} |L'(r)| > 0 \tag{10}$$

and

$$y_i'(r) = FL(r)^{(-2)} L'(r) < 0 \tag{11}$$

Consequently, there is *at most* a location r^* such that,

$$y_a(\tilde{r}) = y_i(\tilde{r})$$

We now show that the location r^* indeed exists.

From (3) and (6), both $y_a(r)$ and $y_i(r)$ are continuous throughout the domain $[0,1]$. Furthermore, if $L(0)$ is sufficiently high, the following inequality is satisfied:

$$A_i - \frac{F}{L(0)} > A_a L(0)^{(\alpha-1)} - g$$

In addition, since $L(1)=1$, the inequality

$$A_i - \frac{F}{L(1)} < A_a L(1)^{(\alpha-1)} - g$$

is equivalent to,

$$A_i - A_a < F - g \tag{12}$$

QED

Proposition 1 is plotted in Figure 2.

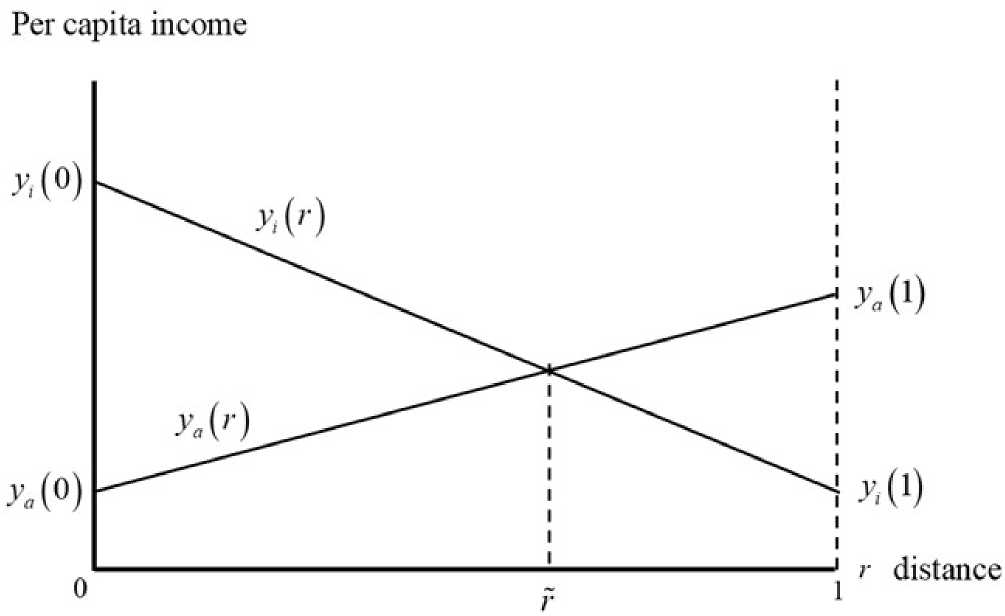

Figure 2: Region with schooling

In Figure 2, the per capita income curve is $y(r) = \max\{y_i(r), y_a(r)\}$, i.e., the upper envelope of $y_i(r)$ and $y_a(r)$, which is clearly given by

$$y(r) = \begin{cases} y_i(r) & \text{if} \quad r \le \tilde{r} \\ y_a(r) & \text{otherwise} \end{cases} \tag{13}$$

We note that $y(r)$ is a continuous function of r, even though it is not necessarily differentiable at point $\tilde{r}$, and that the conditions $A_i = A_a$ or $g = 0$ are sufficient to ensure that inequality (12) holds.

The economy typically undergoes a structural change characterised by an increase in the proportion of school-educated workers within the total population. In the following proposition we argue that this change may arise either from technological progress in the industrial sector relative to the land-based sector, or from an overall rise in population.

Following Galor and Weil (2000), we will assume that A_i may vary, whereas A_a is constant. The rise of A_i relative to A_a reflects technological progress, which tends to be biased to industrial activities rather than land-based production, and thus generally promotes formal education. A technological regression, with a decline in A_i, has the opposite effect on industry and schooling.

By contrast, an overall increase in population density reduces the per-student cost of schooling and raises its wage premium by limiting the amount of land available to a “farmer”.

Proposition 2: Label the initial state of the economy by 0. Suppose that at time 1 the following changes occur:

$$A_i^1 > A_i^0 \text{ or}$$
$$L_1(r) > L_0(r) \text{ for each } r \in [0,1]$$

Then the region with industrial activity and formal schooling expands relative to the area with traditional technology and in-house education, i.e., we have $\tilde{r}_1 > \tilde{r}_0$.

Proof: Define the function $h(r)$ as the difference between the individual incomes of a worker and a farmer at location r:

$$h(r) \equiv y_i(r) - y_a(r) \tag{14}$$

By substituting (3) and (6) in (14), we may write,

$$h(r) = A_i - A_a L(r)^{(\alpha-1)} + g - \frac{F}{L(r)} \tag{15}$$

We may compute the following partial derivatives of $h(r)$.

$$\frac{\partial h(r)}{\partial A_i} = 1$$

$$\frac{\partial h(r)}{\partial L(r)} = (1-\alpha)L(r)^{(\alpha-2)} + FL(r)^{(-2)} > 0$$

Clearly, if $A_i^1 > A_i^0$ or $L_1(r) > L_0(r)$ for each $r \in [0,1]$, the curve $h(r)$ shifts upwards leading to an expansion of the area with industry and school education, i.e., to $\tilde{r}_1 > \tilde{r}_0$ (see Figure 3).

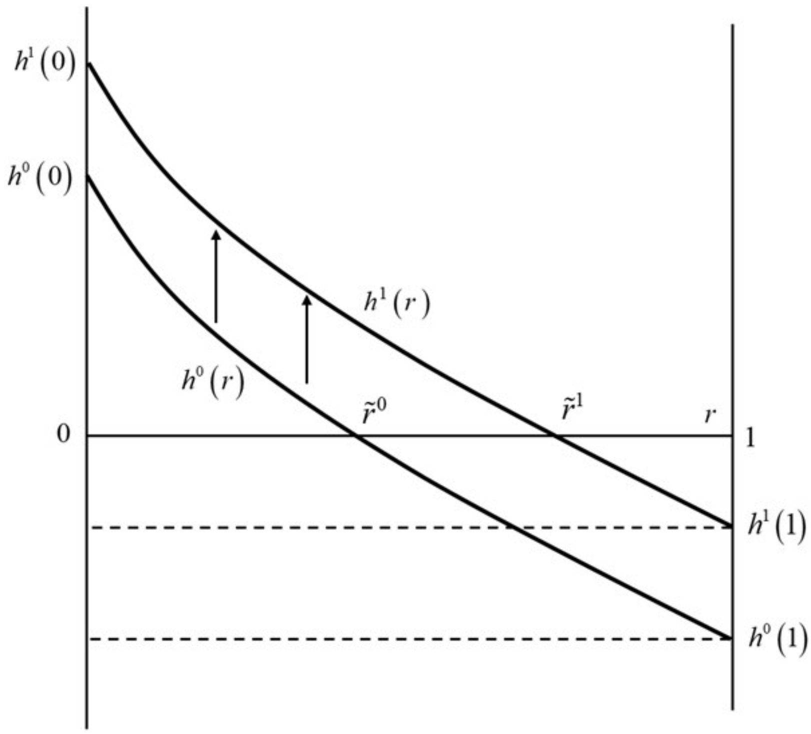

Figure 3: The spread of schooling and industry

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In Figure 4, we plot in detail the shift of schooling boundary from $\tilde{r}_0$ to $\tilde{r}_1$ resulting from an increase in A_i.

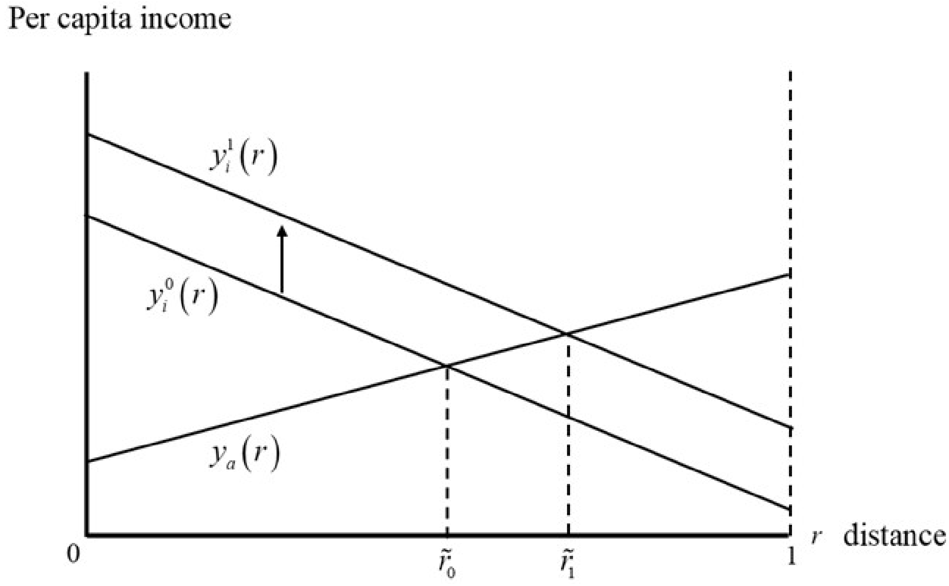

Figure 4: Schooling expansion due to a rise in A_i

In Figure 5, we plot the expansion of the region with formal education driven by an overall increase of population density. From (3) and (6), it is straightforward that the curve $y_a(r)$ shifts *downwards* with a population increase, whereas the curve $y_i(r)$ moves *upwards*.

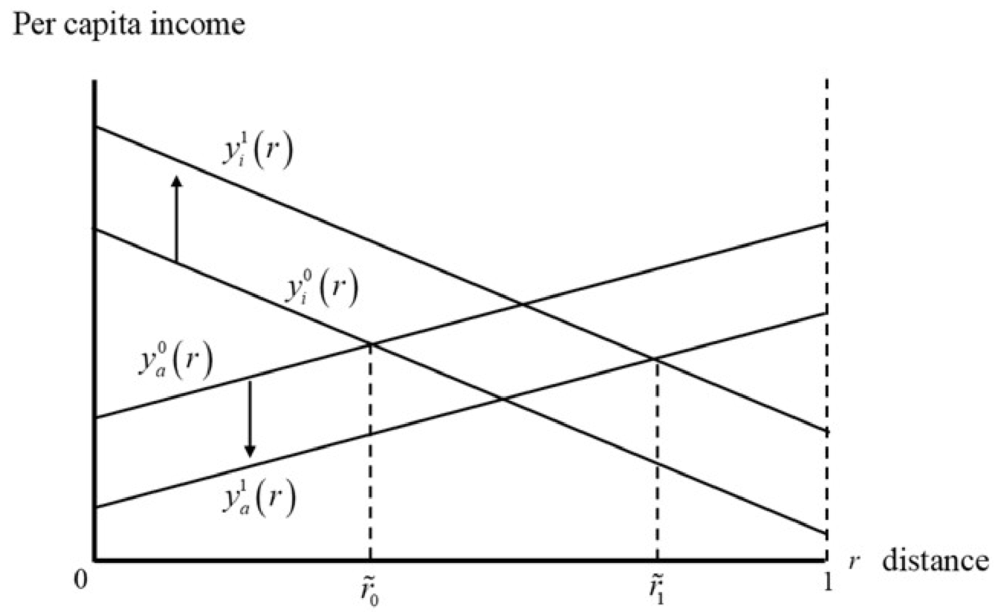

Figure 5: Expansion of schooling due to an overall rise in population density

From Figures 4 and 5, it is clear that a rise in A_i increases per capita everywhere whereas a uniform rise in population density does not not necessarily have the same effect.

A special case in which the expansion of formal education actually *reduces* per capita income, thus giving rise to "education spread without economic development" - is addressed in the following proposition.

Proposition 3: When population density increases in each location while maintaining approximately the same gradient, and the area with schooling does not expand significantly, then per capita income decreases everywhere.

Proof: As it is shown in Figure 5, if for each $r \in [0,1], L^1(r) > L^0(r)$, then we have $y_i^1(r) > y_i^0(r)$ and $y_a^1(r) < y_a^0(r)$. The area with schooling and industrial production expands to $\tilde{r}_1 > \tilde{r}_0$.

However, suppose that a decrease in the productivity of industry with $A_i^1 < A_i^0$ exactly offsets the effect of rising population at $\tilde{r}_0$, so that the boundary of the region with schooling remains unchanged. It follows straightforwardly that the resulting per capita income curve $\hat{y}_i^1(r)$ lies below $y_i^0(r)$, i.e., we have $\hat{y}_i^1(r) < y_i^0(r)$ for each $r \in [0,1]$. This holds at the point $\tilde{r}_0$ from the inequality

$$\hat{y}_i^1(\tilde{r}_0) = y_a^1(\tilde{r}_0) < y_a^0(\tilde{r}_0) = y_i^0(\tilde{r}_0) \tag{16}$$

Since $y_i(r) = A_i - \frac{F}{L(r)}$ and $L_1'(r) \approx L_0'(r)$, it is intuitive that $\hat{y}_i^1(\tilde{r}_0) < y_i^0(\tilde{r}_0)$ is a sufficient condition for the inequality $\hat{y}_i^1(r) < y_i^0(r)$ to hold for every $0 \le r \le \tilde{r}_0$. This result is illustrated in Figure 6.

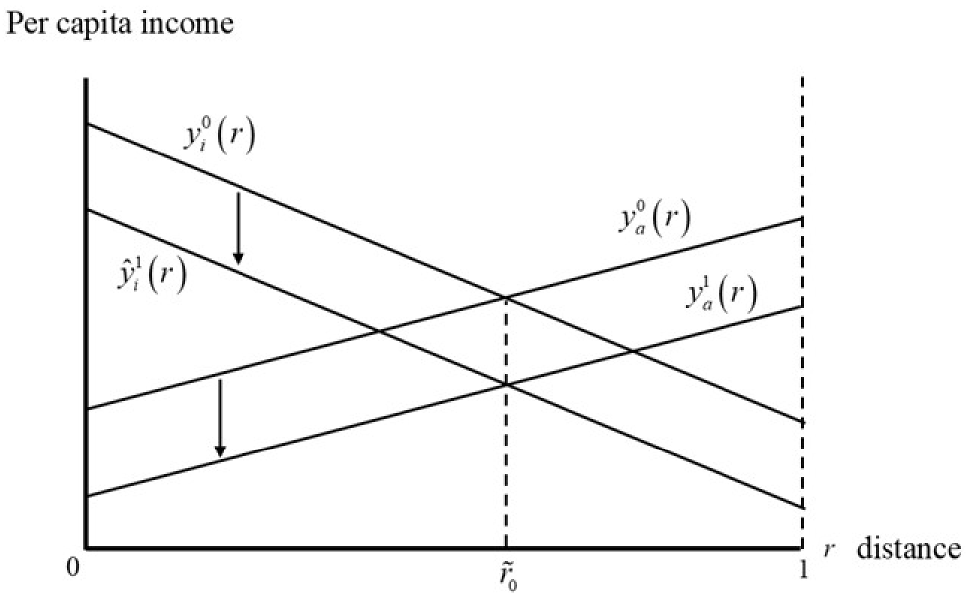

Figure 6: Formal education without productivity increase

As $y_i(r)$ is a continuous function of A_i, $\hat{y}_i^1(r) < y_i^0(r)$ will also be satisfied if the boundary shifts to $\tilde{r}_1 > \tilde{r}_0$ and $\tilde{r}_1$ lies in a neighbourhood of $\tilde{r}_0$.

QED

3. The Evolution of Schooling and Productive Activity

We now address the evolution of technology and education methods. We assume that, at the outset, each producer is a "farmer" and is educated within the family. The agents then make decisions on whether to continue using traditional methods of production and learning, or to switch to modern ones. They decide sequentially, starting with the individuals living at the origin and then subsequently including producers living further away.

It is important to note that, within our framework, while the decisions to become a "worker" and receive formal education are perfectly correlated across agents at the same location, they are completely independent across different locations. For this reason, we could theoretically assume any order of decisions among people living in various locations.

Nevertheless, evidence shows that (higher) education shows a decentralising time pattern, beginning in major metropolitan areas and then spreading out to smaller cities and rural areas. In the context of the monotonically decreasing population density curve $L(r)$ depicted in Figure 1, assuming that producers decide sequentially in order of increasing distance from the origin is a necessary condition for the theoretical analysis to align with the evidence.

Consequently, we will use a time subscript t and will assume that $t = r$. In addition, the threshold for modern methods of production and education will be designated indifferently by $\tilde{t} = \tilde{r}$ -

With this framework, we can prove the following propositions.

Proposition 3: The wage premium of formal education strictly decreases over time.

Proof: The wage premium in time t is,

$$\Delta w(t) = A_i - A_a L(t)^{(\alpha-1)} \tag{17}$$

Its derivative is,

$$\Delta w'(t) = (1-\alpha) A_a L(t)^{(\alpha-2)} L'(t) < 0 \tag{18}$$

QED

Proposition 4: The share of people with formal education increases steadily, albeit at a decreasing rate.

Proof: For $0 \le t \le \tilde{t}$, the schooling rate $s(t)$ is,

$$s(t) = \frac{\int_0^t L(r)\,dr}{\int_0^1 L(r)\,dr} \tag{19}$$

The first and second derivatives of $s(t)$ are,

$$s'(t) = \frac{L'(t)}{\int_0^1 L(r)\,dr} > 0 \tag{20}$$

and

$$s''(t) = \frac{L'(t)}{\int_0^1 L(r)\,dr} < 0 \tag{21}$$

For $\tilde{t} < t \le 1$, $s(t)$ is

$$s(t) = \frac{\int_0^{\tilde{t}} L(r)\,dr}{\int_0^1 L(r)\,dr} \tag{22}$$

which is a constant function of t. **QED**

Additionally, we can demonstrate the following Proposition.

Proposition 5: In this economy, per capita income increases in time, albeit at a decreasing rate.

Proof: Per capita income $y(t)$ is

$$y(t) = \frac{Y(t)}{\int_0^1 L(r)\,dr} \tag{23}$$

where $Y(t)$ denotes aggregate income.

Since the total population in the denominator of (23) is constant, the sign of $y'(t)$ and $y''(t)$ will be the same as that of $Y'(t)$ and $Y''(t)$, respectively.

For $0 \leq t \leq \tilde{t}$, $Y(t)$t is given by

$$Y(t) = \int_0^t L(r) y_i(r) dr + \int_t^1 L(r) y_a(r) dr \tag{24}$$

where $y_a(r)$ and $y_i(r)$ are the per capita incomes of the modern and traditional modes of production and education, respectively, as defined in (3) and (6).

The first derivative of $Y(t)$ is

$$Y'(t) = L(t)\left[y_i(t) - y_a(t)\right] > 0 \tag{25}$$

According to (10) and (11), the second derivative is

$$Y''(t) = L(t)\left[y_i'(t) - y_a'(t)\right] + L'(t)\left[y_i(t) - y_a(t)\right] < 0 \qquad (26),$$

For $\tilde{t} < t \leq 1$, $Y(t)$ becomes

$$Y(t) = \int_0^{\tilde{t}} L(r) y_i(r) dr + \int_{\tilde{t}}^1 L(r) y_a(r) dr \tag{27}$$

which is independent of t . For this reason, we have

$$Y'(t) = Y''(t) = 0 \tag{28}$$

QED

4. Concluding remarks

In this paper we performed a theoretical analysis of the spread of higher education and economic growth, leading to three main findings.

First, we associate the operation of the school system with industrial activity and education within the family with land-based production. We were able to delineate the boundary between regions that adopt modern and traditional methods of production and learning. We found that regions with formal schooling and production based on reproducible factors expand when technical progress primarily concerns the modern productive sector or when population rises in each location.

Second, we theoretically characterise the evolution of schooling in a structurally stable economy and its impact on growth in a manner consistent with empirical evidence. The school system starts in major urban centres and subsequently spreads to smaller towns and rural areas. The wage premium associated with education steadily declines throughout this process. The schooling rate consistently rises, even though growth proceeds at a decreasing rate. For the same reason, per capita income also increases steadily, albeit at a decreasing rate.

Finally, straightforward comparative statics from our model allow us to explain why the apparent paradox of "education without economic development" may arise. Such a situation derives from the combination of several factors. Strong population growth across all locations, possibly driven by mass immigration, reinforces economies of scale in formal education and increases the overall rate of schooling. However, this effect is offset by a concurrent decline in productivity in the modern sector associated with population growth, thereby leading to a reduction in per capita income at each location.

Declaration: The author declares that this paper does not incur any conflict of interest.

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