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The two sides of sovereign balance sheets: how much do government financial assets tell us about $r - g$?*

José Alves† Clarisse Wagner‡

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Abstract

Governments hold large stocks of financial assets, yet debt rules and sustainability assessments look almost only at what they owe. This paper asks whether the asset side of public balance sheets tells us something about the interest–growth differential, $r - g$, that debt alone does not. We follow 17 advanced economies from 2000 to 2026 with sparse dynamic factor models, which let the states governing $r - g$ emerge from the data instead of being imposed through debt ratios. The answer is more uncertain than either side of the debate assumes. On whether the asset side adds information beyond debt, a pre-specified forecast test is inconclusive: it finds no evidence that it does for $r - g$ a year ahead, though it could not reliably have detected information large enough to move a one-year debt projection by the smallest annual adjustment the EU debt safeguard demands. A second comparison, on a consistently measured $r - g$, can neither confirm nor rule out information of that size. Nor is the asset side shown to matter differently when debt is high: the link between asset accumulation and long-term yields is not established to change with the debt burden. Countries respond to common forces in ways a weak test cannot tell apart, yet their coefficients differ for every variable. Applied to fiscal anchors, net and gross debt measures disagree in about one country-quarter in seven, showing where each would bind. The paper maps how much the asset side can, and cannot yet, tell us about debt sustainability.

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1 Introduction

What determines government capacity for promoting economic growth while ensuring long-term debt sustainability? The literature focuses mainly on how much debt a government owes. Yet, sustainability depends fundamentally on the interaction of economic growth and financing conditions – the interest-growth differential $(r - g)$.

Another issue from recent crises is that the mapping from fiscal interventions to $r - g$ is not linear. Our empirical approach therefore treats $r - g$ as jointly shaped by growth dynamics and sovereign risk pricing, with non-linear regime changes.

Governments increasingly act on their balance sheets – through loans, equity stakes and other financial asset operations – especially in periods of financial stress. Whether such interventions expand or erode fiscal capacity depends on improving $r - g$ with higher economic growth compared with borrowing costs.

This paper argues that crucial components are still missing from the empirical sustainability literature: not only the asset side of the public balance sheet, but also underlying emerging factors. The empirical question of the paper is: does the asset side add information about $r-g$ once gross debt is observed, and how does it co-move with $r - g$ across countries and between debt-stress regimes? Neither part concerns the effect of acquiring or selling assets, but both matter for theory and policy. If asset-side interventions systematically accompany favourable $r - g$, that would be consistent with a channel through which the State can support productive capacity and mitigate the fiscal burden of debt. If they instead coincide with deteriorating $r - g$, that would be consistent with interventions that raise sovereign risk premia or fail to promote economic growth, pushing the economy closer towards rollover-risk states, challenging fiscal limits. Balance-sheet interventions can be procyclical, occurring precisely when growth, interest rates or fiscal adjustments are jointly moved by underlying relevant state variables – financial conditions, risk appetite, credibility, institutions – which are only imperfectly observed. Latent state dependence of this kind suggests that a single linear model with constant parameters is unlikely to capture how balance-sheet policies map into $r - g$ across countries and over time.

We study quarterly data for 17 advanced economies from 2000Q1 to 2026Q1, estimated jointly as a single panel of 119 series – 104 quarters, 2000Q2–2026Q1, once the stationarity transformations of Section 5 are applied, of which the 79 quarters from 2000Q2 to 2019Q4 are the identification window of the pooled estimates – so that the latent factors are common and the loadings comparable across countries, and model the joint dynamics of $r - g$, sovereign interest rates, changes in public debt, primary balances, government financial assets and two key macroeconomic transmission channels of gross capital formation and the current account. The differential $r - g$ takes its growth term on the four-quarter GDP total on which every

ratio in the paper is built, as the accounting of the debt ratio requires (Section 4); the conventional measure, with growth taken on single-quarter GDP, is kept as a labelled alternative, and the nested forecast comparison of Appendix J, fixed before that choice, stays on it. Our empirical strategy applies a sparse dynamic factor approach that estimates underlying state-spaces, while results focus particularly on the co-movement of government financial assets with $r - g$. This strategy builds on a tradition in macroeconomics applying factor structures to recover underlying common components in large datasets (Forni and Reichlin, 1998; Forni et al., 2000). Three factors are extracted over the identification window 2000Q2–2019Q4, where four of the six selection criteria used agree on that number; the full-sample fit, over which the same criteria split, is reported as a companion at four factors. The country-level fits fixed six factors each, because with only seven series per country the same criteria have no interior solution; they are estimated and reported here as descriptive estimates, unidentified at that factor count, as Section 5 explains. The goal is not to replace economics with statistics, but to summarise in a few common factors the unobserved latent states – such as credibility, financial conditions, risk appetite – that simultaneously drive many of these series. No name is assigned to any estimated factor, because the rotation that would license such labelling is not identified.

The balance-sheet framing also bears on how fiscal positions are supervised. Conventional fiscal surveillance is predicated on gross debt and on the deficit that contributes to it, and the asset side of the balance sheet enters it only incidentally (Seiferling and Tareq, 2023). Three questions bear on a rule of that form, and this paper treats them as information and risk diagnostics rather than as conditions for its adequacy: whether a specified measure of government financial assets improves a specified forecast of $r - g$ beyond a baseline of gross debt and other fiscal and macroeconomic variables; whether comparable series have equal exposure to common variation across countries, or equal coefficients on the estimated factors; and whether the estimated common variation is symmetric, which is described and on which no verdict is claimed. We should be exact about the status of those three, because the paper's organisation depends on it and its conclusions do not extend beyond their scope. These questions are not derived here from any statute, surveillance procedure or government optimisation problem, nor do we claim that any particular framework presumes an answer to them. For instance, a ceiling on gross debt can be adopted despite heterogeneous transmission, asymmetric risk and full knowledge of the asset side, justified instead by reasons of observability, enforceability or political economy, and the European framework of Regulation (EU) 2024/1263 sets country-differentiated, risk-based adjustment paths rather than one homogeneous mechanism, so a common reference value is not evidence of a common estimated transmission.

Four things are worth keeping apart here, because the three questions bear on them unevenly and a reader is entitled to know which. A surveillance framework of the European kind contains a debt reference value, which is a threshold on the gross liability; an adjustment path, which under Regulation (EU) 2024/1263 is country-specific and risk-based; a debt sustainability analysis, which is a stochastic projection; and the assumptions underlying that projection, among them the distribution of shocks and the process for the interest–growth differential. The first of our questions bears on the reference value taken on its own. The second and the third bear on the kind of model that underpins the fourth element – specifically, a model featuring uniform coefficients across countries and a symmetric calibration. The second object, the adjustment path, is differentiated by design and none of our questions bears on it at all. Evidence on these questions therefore concerns a rule predicated strictly on gross liability and a common-coefficient, symmetrically calibrated model, rather than a comprehensive framework adopting all four elements. None of the three is answered here by assumption; rather, the estimates reported below turn each question into a statement that can be examined. An answer to any of these questions is evidence about these seventeen economies; it is not by itself a verdict on any rule currently in force. On the first question, what the pooled estimates measure is the co-movement between the asset side and the differential, and it is reported only descriptively: no confidence interval is computed for it within the specification the paper identifies from. The strength with which government financial assets and $r - g$ correlate with a shared latent factor is a continuous quantity, and the count of countries above a conventional threshold is too unstable a summary to be interpreted as a finding: the threshold is exceeded in two of the seventeen economies under the specification from which the paper identifies and in twelve under its full-sample counterpart. Meanwhile, the rotation-invariant version of the same measurement – the correlation between the two series' fitted common components – exceeds the threshold in absolute value in eight instances and is undefined in two countries. This is because the penalty leaves at least one of the two series with no common component at all — the financial-asset series in Germany, and both series in Greece. Neither that fact nor any count settles the first question: the absence of a common component between the asset side and the differential would not make the presumption of a gross-debt rule safe, and its presence would not overturn it. Estimated country by country, on separate fits that put six factors on each country's seven series, the scale-free correlation rule surpasses the threshold in sixteen of the seventeen and the raw loading rule in fifteen of them; but six factors on seven series is twice the row-deletion bound those estimations would have to respect to be identified at all, and they are estimated on each country's own full sample rather than on the identifying window, so the sixteen and the two are not counts of the same object. This paper reports those fits as descriptive

estimates only, not as evidence that the count of two is too low. Co-movement is not information, and whether the asset side adds information once debt is observed is examined separately by a nested forecast comparison specified prior to execution, which finds no evidence that it does for $r - g$ a year ahead, though it could not reliably have detected information large enough to move a one-year debt projection by the smallest annual adjustment the EU debt safeguard demands (Section 6.1). A second comparison, also pre-specified, takes the question to the differential as this paper measures it, with its own benchmarks and with fiscal and output data lagged one quarter, and leaves it unresolved: adding the government's financial-asset stock and its change to a forecast a year ahead that already uses the current differential and country effects makes its mean squared error 17.9 per cent higher (Clark–West $t = -0.28$, one-sided $p = 0.604$), and the size of the asset information in the projected contribution of the differential to the next year's change in the debt ratio lies between 0.00 and 0.79 points of GDP in root-mean-square terms, a 90 per cent interval that contains the 0.5 points of GDP of that smallest annual adjustment (Appendix K). On the second question, the restriction of common loadings is not rejected by the paper's composite rule under either null generator: $p = 0.23$ on the rebuilt null and $p = 0.28$ under the original generator. The form of that rule, the largest p-value over the cells, was fixed before any cell of the identifying specification was computed; the null generator under which it is evaluated was rebuilt only after the first cells were known, because the original generator itself carried loading dispersion. On a generator that satisfies (23) exactly, and sweeping the level of the common block rather than plugging in one value for a quantity the restriction does not identify, nineteen of the twenty-two cells reject at five per cent: all eleven cells of the rotation-invariant common-covariance statistic, all five correlation cells under Gaussian errors, and three of the six correlation cells under the dependence-respecting calibration. The three cells that do not reject are the correlation statistic under the dependence-respecting calibration at the lower levels, and because the rule is a maximum those three carry the verdict. The coefficient tests, by contrast, reject equality on all seven variables. What the composite reports is accordingly a property of the rule and not of the seventeen blocks, and the direction of the evidence and the verdict of the rule point opposite ways. The claim is accordingly not a formal rejection but the narrower one that pooling turns cross-country heterogeneity from a property of the design into a restriction the data can answer, and that the evidence on that restriction converges on cross-country differences which the composite rule does not declare established; the failure of the rule is not evidence that the seventeen blocks are common. On the third question, the estimated common factor space has a third moment in sample that is described here but not tested. The statistic we report is Mardia's multivariate skewness, which is invariant to any non-singular transformation of the factor block

and so does not depend on the rotation the estimator returns: on the identifying specification it is 6.83. We report it as a description and not as a test. The only reference distribution available for it assumes independent draws, and the three score columns have first-order autocorrelations of 0.88, 0.86 and 0.76, so no probability is attached to it and it is evidence neither for nor against symmetry; a symmetry test whose size is established under that dependence is the natural next step rather than something the present evidence supplies. No claim is made about the direction of the third moment, because the sign of an individual factor's skewness is not identified. A second property is examined alongside the third question, that the mapping from fiscal action to $r - g$ is linear; the two are kept apart because neither implies the other, and it is this linearity, not symmetry, that is rejected in the pooled panel at the one per cent level, on third-order block-bootstrap p-values running from 0.001 to 0.007 across the three specifications estimated. Those pooled probabilities are anti-conservative by an amount this design does not quantify, because the bootstrap draws its blocks separately within each country and so does not carry shocks common to several countries in the same quarter. That rejection concerns the equation for $r - g$ as a whole and is not located in its coefficient on financial assets: once the regime search and common quarterly shocks are accounted for, a difference between high- and low-debt states in that coefficient is not established, on the differential as measured here ($p = 0.171$) or on the conventional measure ($p = 0.550$). No criterion the paper sets for a verdict — the two forecast comparisons for the first question, the composite rule for the second — rejects the restriction it examines; the third question is described rather than tested; and no restriction is shown to hold. We do not develop an operative alternative rule here, beyond reporting in Section 6 what a net anchor would have implied on the balance sheets actually observed, which shows which country-quarters the two anchors would treat differently and does not rank them.

The paper makes the following contributions. It advances a state-dependent balance-sheet perspective of fiscal capacity, in which sustainability is evaluated through the joint behaviour of fiscal variables and the interest-growth differential $(r - g)$, rather than exclusively through debt stocks, debt ratios or primary balances (Blanchard, 2019; Bohn, 1998). Once sustainability is viewed as a balance-sheet issue rather than only a debt problem, it follows that government financial assets become financially relevant, yet their contribution may depend on prevailing underlying macro-financial regimes, motivating a state-dependent econometric framework. It measures the differential as the accounting of the debt ratio requires, with its growth term on the four-quarter GDP total, and documents what the conventional measure changes: through 2019Q4, taking growth on single-quarter GDP instead changes the sign of 132 of the 1,233 observed values of the differential (Appendix B). The

paper further develops a methodological approach that allows the mapping between sovereign policy instruments and $r - g$ to vary across debt-stress regimes, rather than imposing homogeneous or time-invariant transmission mechanisms across countries and periods. The paper then reports what the estimates deliver. The co-movement of financial assets and $r - g$ is reported as a description: on the specification the paper identifies from, the correlation between the two series' fitted common components exceeds 0.3 in absolute value in eight of the seventeen economies and is undefined in two – in Germany because the estimator leaves the asset series with no common component, and in Greece because it leaves both series with none. Furthermore, the count of countries in which the two have absolute correlations above 0.3 with the same factor moves too far across specifications to be read as a finding. Whether the asset side adds information once debt is observed is the separate question of the two nested comparisons described above. The restriction the model places on the asset side, that the association of the ten-year government bond yield, the rate the theoretical channel prices, with asset accumulation differs between high- and low-debt states, is tested with a specification and a decision rule fixed before it was estimated; once the regime search and common quarterly shocks are accounted for, that difference is not established ($p = 0.894$), nor is it on $r - g$ on either measure, and it is recorded as not established rather than refuted, because the power of the test against a difference of the size the model implies has not been measured (Section 6.4). Taken together, the primary tests the paper fixed in advance bound the role of the asset side rather than settle it: none establishes a difference between debt states or a forecasting gain, and the second forecast comparison leaves open contributions as large as the smallest annual adjustment the EU debt safeguard requires. Pooling also turns cross-country heterogeneity from a property of the design into a restriction the data can answer: equality of loadings is not rejected by the composite rule under either null generator, though all seven coefficient tests and nineteen of the twenty-two cells of the rebuilt exercise reject, so the evidence converges on cross-country differences that the rule does not declare established, and the rule's failure is not evidence that the blocks are common. The empirical evidence directly contributes to the literature on state dependence in fiscal transmission (Auerbach and Gorodnichenko, 2012; Nakamura and Steinsson, 2014) and the literature on non-linear sovereign risk mechanisms (Bocola and Dovis, 2019; Cole and Kehoe, 2000). Section 6 also reports a counterfactual that asks what a net anchor would have done: holding the observed balance sheets fixed, a net-financial-liability measure — gross debt less total general government financial assets — is given a threshold calibrated to bind on the same number of country-quarters as a sixty per cent gross-debt analogue, so that entries and exits are equal by construction and the only thing that can move is which country-quarters they are. The analogue is built on the all-instruments

debt definition this paper uses throughout and not on the Maastricht measure upon which the Treaty's reference value is based, so the sixty per cent is a calibration device and no claim about any country's actual excessive-deficit status is made. The exercise is accounting rather than causation and is reported as such: it establishes which constraint would have bound, not what any government facing it would have done, and it does not rank the two anchors. Because these empirical relationships sometimes emerge during exceptional policy episodes, identifying them requires event-sensitive identifications. Rather than assuming that macroeconomic variables evolve through smooth and continuously adjusting stochastic processes, the analysis argues that government interventions evolve through discrete, state-dependent and high-impact events, such as crisis interventions, bank recapitalisations, sovereign wealth reallocations and other exceptional events. This creates a tension with the estimator we use, and we address it explicitly rather than leaving it implicit. The dynamic factor model is itself a smooth, linear, Gaussian specification, and so cannot represent discreteness in its transition equation. Two implications follow. First, the estimated factor *scores* should not be interpreted as evidence of that discreteness: although the large excursions of the scores coincide with episodes of systemic stress, and leaving out the quarters of the global financial and euro-area crises reduces their multivariate skewness from 6.83 to 1.15, that moment describes autocorrelated scores of the whole common space and does not distinguish discrete interventions from common macro-financial shocks. Second, the state dependence that motivates the argument is tested separately, on a smooth-transition specification that nests linearity, and linearity is rejected in the pooled panel at the one per cent level under each of the three specifications we estimate, on third-order block-bootstrap p-values of 0.001, 0.007 and 0.001; on the full-sample companion two of the three yield 0.001 and the third 0.003, 0.001 being the floor that 999 draws can resolve. All of these pooled probabilities are anti-conservative by an amount this design does not quantify, for the reason given above. Country by country the rejection is more uneven: on the identifying window linearity is rejected in thirteen of the seventeen countries in first differences at the five per cent level and in fourteen at ten per cent, and in twelve and in thirteen countries at five per cent on the two specifications in levels. We report the first-difference specification as the primary model because it is the one of the three whose maintained linear benchmark comes closest to a balanced regression – its linear residuals pass an augmented Dickey–Fuller test at -3.5 in twelve of the seventeen countries, against three of seventeen in levels, where their median first-order autocorrelation is 0.80 – so a count in levels is not evidence of non-linearity on its own but a rejection against a null that is misspecified for an unrelated reason (Section 6). The factor model is therefore used as a measurement device for the common factor space, and the question of whether transmission actually differs across states

is examined by a test rather than inferred from the factor structure.

We question the widespread assumption that macroeconomic dynamics are adequately characterised by a single stationary stochastic process. The pooled evidence answers a narrower question: it rejects linearity in the equation for the interest-growth differential, although it does not locate that non-linearity in the coefficient on government financial assets. This perspective is consistent with latent-state dynamics, in which the statistical relationship between policy instruments and macroeconomic outcomes can change across regimes rather than remaining constant over time, though the evidence here does not establish such a change for any one policy instrument. Representing such regimes requires an empirical framework capable of separating permanent structural characteristics from realised macroeconomic conditions. The factor model does not serve that purpose on its own; it is used as a measurement device and not as a source of regime labels: its loadings and scores are identified only up to a rotation, so the paper relies where it can on quantities invariant to rotation – the estimated common space, the series the estimator leaves with no common component, and the correlation between two series' fitted common components – although some reported figures do depend on rotation, among them the co-loading count, the Jarque–Bera count and the correlation statistic whose cells carry the composite verdict on common loadings. Whether transmission differs across regimes is tested separately, on a smooth-transition specification. Overall, our methodological contribution integrates state-dependent transmission mechanisms, applied empirically to the cross-country co-movement of financial assets and $r - g$ within a government balance sheet perspective.

The remainder of the paper proceeds as follows. Section 2 situates balance-sheet interventions within the literature on fiscal sustainability, sovereign risk and state-dependent transmissions. Section 3 develops the conceptual framework linking asset-side policies to growth and sovereign interest rates. Section 4 describes the data sample. Section 5 describes the empirical strategy. Section 6 reports empirical results and cross-country patterns. Section 7 concludes with implications for fiscal policy.

2 Related literature and contribution

2.1 Fiscal capacity, $r - g$, sovereign risk and state dependence

Modern debates on public debt sustainability increasingly emphasize that what matters is not merely the level of debt but the interaction between economic growth and financing conditions – summarized by the interest-growth differential $r - g$. In this view, debt dynamics can be stable even under persistent deficits, when borrowing

costs remain below growth, while sustainability can deteriorate quickly when the differential turns adverse, a point sharpened in the reassessment of fiscal costs in low-rate environments by (Blanchard, 2019). This perspective is consistent with the conventional but still central intertemporal discipline: solvency requires that fiscal policy eventually responds to debt accumulation, as documented in the fiscal reaction tradition initiated by (Bohn, 1998). Yet the $r - g$ object is not a primitive; it is shaped by equilibrium growth and by the pricing of sovereign liabilities, which can move sharply when rollover risk and pessimistic beliefs become prominent.

This link between fiscal outcomes and sovereign bond pricing is central in models where crises can be self-fulfilling. In the canonical framework of (Cole and Kehoe, 2000), economies can enter a crisis zone in which financial agents pessimistic beliefs raise interest rates and make default optimal, generating large spread movements that are disproportional to gradual changes in fundamentals. Quantitative evidence reinforces the importance of separating fundamental from non fundamental components of sovereign stress: Bocola and Dovis (2019) show that in practice the joint dynamics of spreads and maturity choices can help identify rollover risk. More broadly, nonlinear macro-financial amplifications – where financing constraints tighten endogenously and amplify downturns – provide a natural conceptual foundation for regime changes in sovereign bond yields and growth dynamics, as in Brunnermeier and Sannikov (2014). Empirically, risk premia respond to debt and fiscal fundamentals in ways that can worsen financing conditions even without large growth reversals, consistent with the evidence that debt dynamics affect premia (Liu, 2023).

A complementary strand emphasizes that fiscal space is endogenous and can shrink nonlinearly. In the "fiscal fatigue" approach, primary surplus capacity weakens as debt rises, generating endogenous debt limits and making rollover conditions especially fragile in high-debt states, as formalized and estimated in Ghosh et al. (2013). At the same time, the empirical literature on debt and growth warns against a single universal relationship. Euro-area evidence suggests nonlinearities and thresholds (Baum et al., 2013), while the robustness of threshold claims is contested (Égert, 2015). Subsequent work finds heterogeneous links between debt and growth across macro-financial and institutional environments (Gómez-Puig et al., 2022) and country-panel evidence remains mixed on sign and persistence (Asteriou et al., 2021; Lim, 2019). A related point is that short-run and long-run responses need not align: decompositions into permanent and transitory components can yield different conclusions about growth consequences of debt (Abubakar and Mamman, 2020). Finally, interpretations of $r - g$ depend on whether r is measured as a safe rate or a risky return, affecting the normative implications drawn from observed differentials (Barro, 2023).

2.2 State-dependent fiscal transmission in open economies: investment and the external balance

A second strand of literature emphasizes that fiscal transmission is state dependent, both across the business cycle and across countries, implying that linear constant-parameter approaches can obscure the mechanisms that connect policy instruments to growth. In an influential work, Auerbach and Gorodnichenko (2012) document that government spending multipliers can be substantially larger in recessions than in expansions, supporting the idea that the same fiscal intervention can have different growth effects depending on the macroeconomic state. Quasi-experimental identification strengthens the case that fiscal shocks can have large demand effects in some settings: Nakamura and Steinsson (2014) exploit regional variation in military procurement within a monetary union to estimate sizeable relative multipliers. However, historical evidence also cautions against universal "large multipliers in bad times", showing that estimates depend on the environment and the nature of spending (Ramey and Zubairy, 2018).

Cross-country evidence reinforces this state dependence and points to structural determinants that are particularly relevant for $r - g$. Openness, exchange-rate regime and debt levels shape multiplier sizes and can even reverse signs in high-debt environments, a result documented in Ilzetzki et al. (2013). These findings matter for fiscal capacity, because they suggest a feedback loop: high debt can worsen financing conditions and, in turn, alter growth effects of fiscal interventions, thereby shifting $r - g$ through both its growth and interest-rate components. In this sense, fiscal transmission cannot be separated from sovereign risk pricing, especially when economies are close to debt stress regimes, highlighted by Ghosh et al. (2013) or crisis-zone dynamics emphasized by Cole and Kehoe (2000).

In open economies, the external account is central to how fiscal actions translate into growth and financing conditions. The "twin deficits" hypothesis posits that fiscal deficits tend to widen current account deficits, but the strength and direction of this relationship depend on the persistence of fiscal shocks and on structural openness (Corsetti and Müller, 2006). In the new open economy macro tradition, nominal rigidities and expenditure-switching interact with intertemporal constraints in shaping external adjustment and policy spillovers (Obstfeld and Rogoff, 1995). Moreover, disruptions to trade costs and market access can generate persistent effects on trade flows and welfare, with implications for investment and growth trajectories (Dhingra et al., 2017). Recent European panel evidence similarly links of growth outcomes to openness and financial development in ways that suggest a joint role for external integration and domestic financial conditions (Mtar and Belazreg, 2023).

A further implication is that fiscal consolidation and fiscal expansions can have

persistent growth consequences, not merely temporary demand effects. Evidence on the long-run effects of consolidations suggests that fiscal policy can shift output paths, rather than simply smoothing cycles (Fatás and Summers, 2018). This is important for $r-g$, because persistent effects on growth can dominate the mechanical snowball term in debt dynamics, especially when borrowing costs are themselves sensitive to growth prospects and risk premia (Blanchard, 2019; Liu, 2023). Taken together, these strands imply that the growth component of $r-g$ is inherently state contingent and shaped by both domestic and external channels.

The state-dependent transmission literature motivates two modelling choices in our paper. First, we evaluate fiscal capacity through $r-g$ rather than through debt stocks alone, because fiscal interventions can shift both growth and financing conditions (Blanchard, 2019; Ilzetzki et al., 2013). Second, we treat gross capital formation and the current account balance as core transmission channels, rather than as controls, precisely because investment and external adjustment are the mechanisms through which policy affects growth in open economies (Corsetti and Müller, 2006; Obstfeld and Rogoff, 1995). This focus becomes particularly relevant once we incorporate asset-side interventions, which may influence growth not only through demand but also through credit provision, recapitalizations and competitiveness-enhancing investments.

2.3 The asset side of the sovereign balance sheet: liquidity provision, recapitalisation and contingent risk

While the literature above clarifies why $r-g$ is central and why fiscal effects are state dependent, it provides limited direct guidance on how the asset side of the public sector balance sheet enters the sustainability calculus. Yet, the theoretical case for balance-sheet interventions is strong. Under aggregate uncertainty, private financial markets may not supply sufficient liquidity, and government securities can command a liquidity premium, giving the State a role in liquidity provision and in stabilizing financial conditions (Holmström and Tirole, 1998). This mechanism rationalizes crisis-era policies that expand public financial assets through loans, credit lines and capital injections, with potentially large effects on investment and growth, when they prevent inefficient liquidations and preserve productive capacity.

Crisis research also clarifies that the design of recapitalization and asset interventions matters for incentives and equilibrium outcomes. Efficient recapitalization can relax private debt-overhang distortions and restore lending and investment, but only if the policy design mitigates opportunistic behaviour and aligns with private participation incentives (Philippon and Schnabl, 2013). Empirical event-studies evidence shows that large-scale interventions can create value by reducing bankruptcy probabilities, while redistributing risk across stakeholders and taxpayers, underscor-

ing that balance-sheet policy operates not merely through fiscal accounting but also through the pricing of risk and contingent liabilities (Veronesi and Zingales, 2010). These mechanisms imply that asset-side operations can lower $r - g$ by raising growth and stabilizing risk premia, but they can also raise $r - g$ if markets interpret them as stress-driven risk transfers to the sovereign.

Because changes in government financial assets may mechanically raise debt while also affecting investment and external competitiveness, their association with $r - g$ is theoretically ambiguous ex ante and likely to differ across institutional and macro-financial regimes (Brunnermeier and Sannikov, 2014; Ghosh et al., 2013).

Despite the importance of public balance sheets, the cross-country evidence falls short of the object studied here. That the asset side is priced at all is established, and by more than one study. Peppel-Srebrny (2024) finds that government assets, and not only liabilities, significantly determine sovereign borrowing costs; Henao-Arbelaez and Sobrinho (2017) relate government financial assets to sovereign spreads and to distress, including the part played by asset liquidity; Yousefi (2019) links public sector balance-sheet strength to yields and to recession outcomes; and Hadzi-Vaskov and Ricci (2016) ask whether gross or net debt matters more for spreads, allowing for nonlinearity and distress interactions. What remains scarce is evidence on the differential itself, rather than on its interest-rate component alone, and on how that relation varies across latent states. Against that work, what this paper adds is not the finding that the asset side matters, which those studies establish, but the object and the design: the interest–growth differential rather than the yield or the spread, and a common factor space in which the cross-country mapping becomes a restriction that can be tested. A separate strand of work typically treats public financial assets as residual fiscal items or focuses on liabilities and primary balances (Bohn, 1998; Ghosh et al., 2013). Our paper addresses this gap by bringing the change in government financial assets – an observed asset-position change, not a measured policy transaction – to the centre of the analysis and by evaluating its co-movement with $r - g$, jointly with debt dynamics, primary balances, investment and external balance, thereby connecting the balance-sheet literature to the $r - g$ fiscal-capacity framework (Blanchard, 2019; Holmström and Tirole, 1998).

Three studies are the closest to this one, and they differ from it in the data, in the asset and outcome measures, and in what is identified. Peppel-Srebrny (2024) uses advanced-economy data and relates government assets, liabilities and net worth to sovereign borrowing costs. Henao-Arbelaez and Sobrinho (2017) relate government financial assets to borrowing costs, to debt distress and to the liquidity of the assets, and find the protective association stronger in emerging markets. Hadzi-Vaskov and Ricci (2016) compare gross and net debt as determinants of emerging-market spreads, with differences under domestic distress. All three take a

market price of sovereign risk, a borrowing cost or a spread, as an outcome. This paper uses quarterly data for seventeen advanced economies; its asset measure is the change in general government financial assets as a ratio to GDP, an asset-position change rather than a transaction; its outcomes are the interest–growth differential, built on the effective cost of the debt, and the ten-year government bond yield; and it identifies co-movement through a common factor space estimated jointly for the seventeen economies, state dependence through a smooth-transition regression read as a conditional association, and information through two nested forecast comparisons specified before they were run. On that design, what the paper adds to them is a statement of how much of the asset side's role these data can resolve: once the regime search and common quarterly shocks are accounted for, a difference between high- and low-debt states in the yield's association with asset accumulation is not established, and the size of the asset side's information about the differential a year ahead is unresolved over a range that contains the smallest annual adjustment the EU debt safeguard requires.

The states that govern fiscal transmission and sovereign pricing – credibility, risk appetite, rollover conditions – are only imperfectly observed, and constant-coefficient models risk averaging across them. Dynamic factor methods recover low-dimensional latent forces from the co-movements of large systems (Bai and Ng, 2002; Forni and Reichlin, 1998), and enter here on the assumption that those states load on such forces.

3 Theoretical foundations

This section links balance-sheet policy – the primary balance, changes in public financial assets and gross debt – to the interest-growth differential $r - g$. The aim is not a fully structural model but a set of equations that make the sign ambiguity of asset-side interventions explicit, let institutions and debt stress condition it, and bridge to an empirical strategy that estimates latent common factors from the data. Derivations are in Appendix A.

For country i in period t, define growth net returns ($GNR_{i,t}$) as the interest-growth differential:

$$GNR_{i,t} = i_{i,t}^{eff} - g_{i,t}, \tag{1}$$

where $i_{i,t}^{eff}$ is the effective interest rate on government debt, the interest expense of the period over the debt stock four quarters earlier, and $g_{i,t}$ the growth rate of nominal GDP; $y_{i,t}^{10}$ denotes the market yield on new government borrowing, measured by the ten-year government bond yield, and the rest of the paper writes the differential $GNR_{i,t}$ as $r - g$. When $i_{i,t}^{eff} > g_{i,t}$, the debt burden tends to rise relative to the

tax base, unless offset by fiscal adjustments; when $i_{i,t}^{eff} < g_{i,t}$, debt dynamics are mechanically more favourable, though this does not eliminate risk, because $r - g$ can reverse and sovereign risk premia may be nonlinear. This logic underpins modern discussions of debt sustainability under low safe rates; the relevance of regime shifts and rollover risks (Blanchard, 2019; Bocola and Dovis, 2019; Cole and Kehoe, 2000); and remains consistent with the intertemporal solvency discipline emphasized by fiscal reaction evidence (Bohn, 1998; Ghosh et al., 2013).

A central premise is that focusing only on the liability side misses a crucial margin: governments increasingly intervene through the asset side of the public balance sheet – loans, equity stakes and other financial operations, and, off the balance sheet, guarantees, which enter the asset stock only when they are called – especially in crises. Whether those interventions expand or erode fiscal capacity turns on whether they lower $GNR_{i,t}$.

Let $GD_{i,t}$ denote gross public debt as a ratio to nominal GDP, $\Delta FA_{i,t}$ the change in government financial assets ratio to GDP, and $PB_{i,t}$ the primary balance ratio to GDP (positive for a surplus). Abstracting from valuation effects, a gross debt-dynamics representation accommodating financial-asset operations is:

$$\Delta GD_{i,t} = \frac{i_{i,t}^{eff} - g_{i,t}}{1 + g_{i,t}} GD_{i,t-1} - PB_{i,t} + \omega_{i,t}\Delta FA_{i,t} + SFA_{i,t}, \tag{2}$$

where $SFA_{i,t}$ collects stock-flow adjustments and measurement components, and $0 \leq \omega_{i,t} \leq 1$ captures the financing share of asset accumulation through debt issuance (with $\omega_{i,t} = 1$ corresponding to one-for-one debt financing of $\Delta FA_{i,t}$). We set $\omega_{i,t} = 1$ throughout: asset purchases are treated as debt-financed at the margin, which is the conservative case for the sustainability question, and ω is not separately estimated below. Three further things about $\Delta FA_{i,t}$ belong here rather than left for a reader to notice, because each imposes a constraint on how estimates should be interpreted. The first is algebraic. The debt accounting behind Equation (2), set out in Appendix A, records the asset term as a flow divided by output, while the series this paper measures, and that Appendix B documents, is the change in a ratio. The two are different objects even with no revaluation whatever, since, writing $fa_{i,t}$ for the nominal asset stock and $FA_{i,t} \equiv fa_{i,t}/Y_{i,t}$ for its ratio to nominal GDP $Y_{i,t}$, $(fa_{i,t} - fa_{i,t-1})/Y_{i,t} = \Delta FA_{i,t} + [\, g_{i,t}/(1+g_{i,t}) \,] FA_{i,t-1}$, so that they differ by a term, proportional to the lagged asset ratio, that vanishes only as growth approaches zero. We use the ratio change throughout and define the stock-flow adjustment $SFA_{i,t}$ to absorb the difference, so that (2) holds exactly; this makes $SFA_{i,t}$ a residual of the accounting rather than an independently measured quantity. The second is about measurement, and it is the sharper limitation. A change in a stock is not an acquisition: it is the sum of transactions, of holding gains and losses, and of other

changes in volume such as reclassifications, and the IMF's Government Finance Statistics Manual of 2014 separates precisely these three. The series available here does not separate them, so $\Delta FA_{i,t}$ is an asset-position change and not a policy action, and nothing estimated below identifies its transaction component. Where this paper speaks of interventions it means the model's $\Delta FA_{i,t}$, and not a measured purchase. The third is conceptual. A lower $r - g$ is neither necessary nor sufficient for greater fiscal capacity: a revenue-producing asset can raise future primary balances with $r - g$ unchanged, and an asset bought above its worth can destroy net worth while funding conditions improve. The model carries no asset payoff, no sale discount, no maturity, no earmarking and no matched obligation, so what it represents is the funding-cost margin alone. Identifying fiscal capacity with the sign of the $r - g$ response is therefore a simplification we adopt and state, and not one we defend.

The data of this paper are four-quarter totals, and on them (2) is applied as the annual identity (2a), with rates as fractions, the debt ratio and the primary balance taken over nominal GDP summed over the four quarters to t, $g^A_{i,t}$ the growth of that sum over four quarters, and $SFA^A_{i,t}$ the stock-flow residual, computed from the data as what the other terms leave unexplained rather than assumed zero, so that it contains the asset operations that (2) writes separately as well as valuation changes, other changes in volume and measurement differences:

$$\Delta_4 GD_{i,t} = \frac{i^{eff}_{i,t} - g^A_{i,t}}{1 + g^A_{i,t}} GD_{i,t-4} - PB_{i,t} + SFA^A_{i,t}. \tag{2a}$$

Equation (2) makes the paper's core ambiguity explicit: accumulating financial assets can mechanically raise gross debt through financing needs, yet can improve fiscal capacity if it raises growth or lowers borrowing costs, and can worsen fiscal capacity if markets read it as an assumption of contingent liabilities.

Rearranging (2) gives the growth-net return as:

$$GNR_{i,t} = i^{eff}_{i,t} - g_{i,t} = \frac{1 + g_{i,t}}{GD_{i,t-1}} \left(\Delta GD_{i,t} + PB_{i,t} - \omega_{i,t} \Delta FA_{i,t} - SFA_{i,t} \right). \tag{3}$$

The fiscal effects of balance-sheet operations depend on imperfectly observed states – credibility, institutions, administrative capacity, global risk appetite, rollover conditions – represented by a latent state vector:

$$s_{it} \equiv (s_{1,it}, \ldots, s_{m,it})', \tag{4}$$

which governs the pass-through strength of transmission from policy instruments to the real economy and to sovereign bond pricing.

Institutional quality plays a dual role in the $r - g$ mechanism. On the real

side it governs how far balance-sheet actions translate into investment and external competitiveness: the same capital injection or development-bank loan reaches viable borrowers and is withdrawn when conditions normalise where administrative capacity is strong, and is diverted or misallocated where it is weak.

On the financing side, weak institutions shift the distribution of future fiscal outcomes that bondholders price, amplifying fiscal-fatigue concerns as debt rises (Ghosh et al., 2013) and widening the scope for rollover-type shifts in confidence (Cole and Kehoe, 2000). Institutional quality therefore scales both the growth effects of policy and the risk-premium sensitivity of the market yield $y_{i,t}^{10}$ to fiscal and debt dynamics.

We represent institutional effectiveness and debt stress with smooth bounded transition functions. An institutional quality index $I_{i,t}$, such as a governance or government-effectiveness measure, is mapped into a bounded effectiveness scaler $S_I(I_{i,t}) \in (0, 1]$ by a logistic form:

$$S_I(I_{i,t}) = \left[1 + \exp\left(-\kappa_I(I_{i,t} - \bar{I})\right)\right]^{-1}, \qquad \kappa_I > 0. \tag{5}$$

Institutional quality above the threshold $\bar{I}$ pushes S_I toward one, reflecting stronger implementation and credibility; below it S_I falls, reflecting leakage, misallocation and higher perceived sovereign risk (Ghosh et al., 2013).

Debt conditions enter through a bounded debt-stress scaler δ_{GD}, built on a country-specific debt threshold GD_i^{th} beyond which rollover risk, fiscal fatigue, and premia sensitivity intensify, with a smooth, bounded hyperbolic tangent (tanh) transition (Teräsvirta, 1994):

$$\delta_{GD}(GD_{i,t}) = \frac{1 + \tanh\left(\kappa_{GD}(GD_{i,t} - GD_i^{th})\right)}{2}, \qquad \kappa_{GD} > 0, \tag{6}$$

so that $\delta_{GD} \in (0, 1)$ increases towards one as debt rises above the threshold. This encodes the idea that sovereign risk can be nonlinear in debt when economies move into "crisis zones" , where beliefs and rollover considerations matter (Bocola and Dovis, 2019; Cole and Kehoe, 2000).

The latent state $s_{i,t}$ and the observable scalers $\{S_I, \delta_{GD}\}$ jointly determine the multipliers of policy on growth and on borrowing costs. We model growth as driven by two channels, gross capital formation $GCF_{i,t}$ and the current account balance $CAB_{i,t}$, each a share of GDP:

$$g_{i,t} = \alpha_1(s_{i,t})GCF_{i,t} + \alpha_2(s_{i,t})CAB_{i,t} + \varepsilon_{i,t}^g. \tag{7}$$

The coefficients $\alpha_1(s_{i,t})$ and $\alpha_2(s_{i,t})$ are allowed to vary with the underlying latent state space, because the growth content of investment and external adjustment

depends on macro-financial regimes and structural features, including openness and competitiveness (Corsetti and Müller, 2006; Obstfeld and Rogoff, 1995).

Balance-sheet policy enters through two instruments, the primary balance $PB_{i,t}$ and the change in government financial assets $\Delta FA_{i,t}$.

Asset-side interventions affect the real economy through several mechanisms. Public liquidity provision can mitigate inefficient liquidations under stress (Holmström and Tirole, 1998); recapitalizations can relax debt overhang distortions and revive lending (Philippon and Schnabl, 2013); but interventions can also shift tail risks to the sovereign and change perceptions of contingent liabilities (Veronesi and Zingales, 2010). These mechanisms motivate a reduced form mapping from policy instruments to the two growth channels, evidencing the scalers $\{S_I, \delta_{GD}\}$:

$$GCF_{i,t} = S_I(I_{i,t})\left(1 - \delta_{GD}(GD_{i,t})\right)\left[\gamma_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \gamma_{2,i,t}(s_{i,t})PB_{i,t}\right] + \varepsilon^I_{i,t}, \quad (8)$$

$$CAB_{i,t} = S_I(I_{i,t})\left(1 - \delta_{GD}(GD_{i,t})\right)\left[\tau_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \tau_{2,i,t}(s_{i,t})PB_{i,t}\right] + \varepsilon^X_{i,t}. \quad (9)$$

Strong institutions raise $S_I(I_{i,t})$ and with it the effectiveness of ΔFA and PB, while $(1 - \delta_{GD})$ weakens the real-side multiplier under debt stress, through crowding out, debt overhang and credibility constraints (Ilzetzki et al., 2013; Krugman, 1988).

Substituting (8)–(9) into (7) yields a reduced form for growth:

$$g_{i,t} = \Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})\Delta FA_{i,t} + \Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t})PB_{i,t} + u^g_{i,t}, \quad (10)$$

taking multipliers of financial assets as:

$$\Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t}) \equiv \left[\alpha_1(s_{i,t})\gamma_1(s_{i,t}) + \alpha_2(s_{i,t})\tau_1(s_{i,t})\right] S_I(I_{i,t})\left(1 - \delta_{GD}(GD_{i,t})\right), \quad (11)$$

and of the primary balance as:

$$\Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t}) \equiv \left[\alpha_1(s_{i,t})\gamma_2(s_{i,t}) + \alpha_2(s_{i,t})\tau_2(s_{i,t})\right] S_I(I_{i,t})\left(1 - \delta_{GD}(GD_{i,t})\right). \quad (12)$$

In turn, we represent the sovereign borrowing cost, the market yield $y^{10}_{i,t}$, as the sum of a global safe component and a country-specific premium:

$$y_{i,t}^{10} = r_t^* + \pi_{i,t}. \tag{13}$$

The premium $\pi_{i,t}$ captures compensation for sovereign risk, rollover risk, and contingent liabilities. Motivated by nonlinear sovereign risk logic (Bocola and Dovis, 2019; Cole and Kehoe, 2000) and fiscal fatigue considerations (Ghosh et al., 2013), we model the premium as increasing in debt accumulation and in stress-interpreted balance sheet operations:

$$\begin{aligned}\pi_{i,t} = & \underbrace{\beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})\Delta GD_{i,t}}_{\text{debt/rollover channel}} \\ & + \underbrace{\beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1 - S_I(I_{i,t}))(\Delta FA_{i,t} - PB_{i,t})}_{\text{contingent liabilities and deficit/signalling channel}} + \varepsilon_{i,t}^{\pi},\end{aligned} \tag{14}$$

with $\beta_{GD}(s_{i,t}) > 0$. The second term captures the idea that assets accumulation and persistent deficits can raise premia when markets interpret it as stress-driven risk transfer to the sovereign – especially when debt stress is high and institutions are weak (so that monitoring, governance, and eventual non-default become less credible). This term operationalizes, in a simple way, the "signalling/contingent liability" mechanism discussed in crisis intervention research (Veronesi and Zingales, 2010) and the broader logic that institutional weaknesses amplify premia sensitivity (Ghosh et al., 2013).

The model abstracts from the maturity of the debt, so that within it the effective cost of the debt equals the market yield, $i_{i,t}^{eff} = y_{i,t}^{10}$; in the data the two differ, because the effective cost is the average cost of a legacy portfolio and follows the yield only as maturing debt is refinanced (Section 4). Substituting (13)–(14) for $i_{i,t}^{eff}$ and (10) for $g_{i,t}$ in (1) gives an equation for growth net returns that is not a reduced form, since $\Delta GD_{i,t}$ remains on its right-hand side and the debt identity (2) determines it jointly with $GNR_{i,t}$ (Appendix A):

$$\begin{aligned}GNR_{i,t} = & r_t^* + \underbrace{\beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})\Delta GD_{i,t}}_{\text{debt channel}} \\ & + \underbrace{\beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1 - S_I(I_{i,t}))(\Delta FA_{i,t} - PB_{i,t})}_{\text{premium channel}} \\ & - \underbrace{\left(\Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})\Delta FA_{i,t} + \Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t})PB_{i,t}\right)}_{\text{growth channel}} + \eta_{i,t}.\end{aligned} \tag{15}$$

Equation (15) isolates the paper's central economic ambiguity: the net effect of $\Delta FA_{i,t}$ on $GNR_{i,t}$ is the balance between a growth channel, which raises $g_{i,t}$

and lowers $r - g$, and a premium/financing channel, through which debt needs and signalled contingent liabilities raise $y_{i,t}^{10}$ and so $r - g$.

This trade-off is state-dependent, since both channels depend on $s_{i,t}$, on $I_{i,t}$ and on $GD_{i,t}$. Under the monotonicity assumptions of Proposition 1 below, stronger institutions and lower debt stress strengthen the growth channel and weaken the premium channel, and weaker institutions and higher debt stress do the opposite. Which channel dominates at a given state depends as well on the size of $\beta_{FA,PB}$ relative to the growth multipliers, which the model leaves free and the paper does not estimate.

Proposition 1. Suppose $\Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})$ is increasing in $S_I(I_{i,t})$ and decreasing in $\delta_{GD}(GD_{i,t})$, while the premium sensitivity in (14) is increasing in $\delta_{GD}(GD_{i,t})$ and increasing in $(1 - S_I(I_{i,t}))$. Then, holding $\Delta GD_{i,t}$ and $PB_{i,t}$ fixed, the partial derivative of $GNR_{i,t}$ with respect to $\Delta FA_{i,t}$ in (15) tends to a negative value (improves fiscal capacity) as institutions become strong and debt stress vanishes, $S_I \to 1$ and $\delta_{GD} \to 0$, and to a positive value (deteriorates fiscal capacity) as institutions become weak and debt stress becomes maximal, $S_I \to 0$ and $\delta_{GD} \to 1$; by continuity it keeps each sign in some neighbourhood of the corresponding limit. The claim follows from (15): holding $\Delta GD_{i,t}$ and $PB_{i,t}$ fixed, $\partial GNR_{i,t}/\partial \Delta FA_{i,t} = \beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1-S_I(I_{i,t})) - \Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})$. Writing $K \equiv \alpha_1\gamma_1 + \alpha_2\tau_1$, so that $\Phi^{FA} = KS_I(1 - \delta_{GD})$ by (11), the two monotonicity assumptions amount to $K > 0$ and $\beta_{FA,PB} > 0$, and the derivative tends to $-K$ in the first limit and to $\beta_{FA,PB}$ in the second. This is a sign restriction on (15) under assumptions that are imposed rather than derived from primitives, not an equilibrium result. Away from the limits those assumptions do not fix the sign: the derivative is negative exactly when $S_I > \beta_{FA,PB}\delta_{GD}/(\beta_{FA,PB}\delta_{GD} + K(1 - \delta_{GD}))$, a boundary that depends on parameters the paper does not estimate. At $S_I = 0.99$ and $\delta_{GD} = 0.01$, which would ordinarily be called strong institutions and no debt stress, the derivative is still positive if $K = 1$ and $\beta_{FA,PB} = 10{,}000$. Nothing in the empirical sections identifies where a country sits relative to that boundary, so the proposition is not used below as a prediction about observed regimes, and where Section 6 tests a sign it tests the debt-stress half of the first prediction alone. The derivative is also partial, and (2) does not leave $\Delta GD_{i,t}$ free while $\Delta FA_{i,t}$ moves. Letting $\Delta GD_{i,t}$ adjust (Appendix A) adds a financing term, a term through growth that vanishes in both limits, and a feedback multiplier $(1 - A_{i,t})^{-1}$, with $A_{i,t} \equiv \beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})GD_{i,t-1}/(1 + g_{i,t})$. Since $A_{i,t} \to 0$ as $\delta_{GD} \to 0$, the first limit is unchanged; the positive sign in the second survives only if $A_{i,t} < 1$, a restriction the paper assumes and does not test.

The model yields four testable predictions. First, sign switching with institutions and debt stress: the estimated association of $\Delta FA_{i,t}$ with $r - g$ should be more negative in high-institution, low-stress regimes and more positive in low-institution,

high-stress ones. Second, the decomposition into y^{10} versus g: in good regimes ΔFA should operate mainly through higher g, in bad ones through higher y^{10}. Third, channel alignment: where ΔFA improves $r-g$ it should co-move with higher GCF and improved external performance, and where it worsens $r-g$ with debt accumulation and deteriorating sovereign pricing. Fourth, latent-state interpretation: the extracted factors should align with interpretable macro-financial states and predict when the sign switching occurs. Of these four, only the first is taken to the data, and then only in its debt-stress half; the second and third are not tested with these data. The first is tested on two outcomes, the differential $r-g$, that is $i^{eff}_{i,t} - g_{i,t}$, and the market yield $y^{10}_{i,t}$ (Section 6.4). The fourth is not tested, because no factor in this paper is identified well enough to be named (Section 6.5). The system estimated in Section 5 contains no institutional index, and the smooth-transition specification uses only lagged debt-stress measures as transition variables, so the institutional margin through which $S_I(I_{i,t})$ operates in (11)–(12) and (14) is not tested; institutional quality enters the discussion below as an interpretive category and not as a measured regressor.

These predictions directly motivate the empirical model in Section 5, where dynamic factor methods estimate common factors from the joint dynamics of $r-g$, y^{10}, debt changes, primary balances, financial asset operations, investment and the current account balance (Bai and Ng, 2002; Forni and Reichlin, 1998; Forni et al., 2000; Stock and Watson, 2002).

The theoretical object $s_{i,t}$ is intentionally general: it collects the low-dimensional latent forces governing (i) the transmission of ΔFA and PB to investment and the current account, and (ii) the sensitivity of sovereign premia to debt and contingent liabilities. Its link to the empirical model is an assumption: $s_{i,t}$ is taken to load on a few factors common across countries, which the dynamic factor model estimates only up to a rotation, without identifying $s_{i,t}$, any of its components or which channel of (15) dominates.

4 Data and stylized facts

We investigated the interaction of government financial assets with growth net returns, focusing on the transmission channels of gross capital formation, current account balances and government interest rates. For cross-country comparability we used the same sources for all countries and let each country contribute the full extent of its own coverage rather than truncating the panel to the window they all share, thereby estimating country fixed effects on a common sequence of global regimes (albeit with heterogeneous intensity effects). The panel is unbalanced: France reaches back to 2000Q1, the other sixteen countries begin in 2001Q1, all seventeen

run to 2026Q1, and inside those country windows the individual series open and close at different dates, as Table 2 and Figure 2 show. Because the countries are estimated jointly as one system (Section 5), those ragged edges are carried into the estimation and absorbed by the Kalman filter rather than removed by dropping quarters, following Bańbura and Modugno (2014). Taking these considerations, the estimation sample was assembled from a quarterly annualized panel of 17 advanced economies, spanning the whole of the period the database covers, 2000Q1 to 2026Q1. Three windows should be kept apart. Table 3 counts the country-quarters of each series at each stage, from the raw inputs to the identifying window. The first is the database window: the span of quarters a country occupies in the dataset, within which the stationarity tests of Appendix C are run, series by series. The second is the estimation window, shorter because differencing incurs a loss of leading observations: once the transformations of Section 5 are applied, France is estimated from 2000Q2 over 104 quarters; the thirteen countries whose transformations take at most a first difference are estimated from 2001Q2 over 100; and Portugal, Spain and the United States, whose transformations take a second difference, are estimated from 2001Q3 over 99, all seventeen running to 2026Q1. The country-level fits estimated on those samples cover all seventeen countries and are reported as descriptive estimates, unidentified for the reason given in Section 5. The third is narrower than either and is the one the pooled estimates of Section 6 identify on: the pooled system is fitted to the 79 quarters from 2000Q2 to 2019Q4, so that the pandemic quarters do not enter the common factor space, with the fits that run to 2026Q1 reported alongside it as companions. That window was chosen on 14 September 2026, after results on the full sample to 2026Q1 had been seen, because the factor-count criteria, the error specifications and the structural-break tests agree on it and not on the full sample (Section 5); it was kept on 17 September 2026, when the companion fit that sets the pandemic quarters to missing and runs to 2026Q1 was assessed as an identifying space and rejected on the same grounds. The widest opening ragged edge is that of the change in Slovakia's gross debt and its $r - g$, which begin only in 2007Q4 because the debt level itself begins in 2006Q4, and the widest closing edge that of Greece's $r - g$, primary balance and financial assets, which end in 2024Q4; the United Kingdom alone carries all seven series through to 2026Q1. Two interest rates enter the system and they are not the same object. The differential $r - g$ is the debt-accounting one: the effective rate on government debt – the four-quarter sum of general government interest expense over the gross debt stock four quarters earlier – less the growth of nominal GDP summed over the four quarters to t, the GDP total on which every ratio in this paper is built (Table 1). It is therefore the differential of the annual identity (2a), the empirical counterpart of the differential in Equation (2) with its period read as the four quarters over which the change in the debt ratio is

measured. As the average cost of a legacy portfolio, it responds to current conditions only as maturing debt is refinanced. The conventional measure, which uses the year-on-year growth of single-quarter GDP, is kept as a labelled alternative: Figure 1 plots the two side by side, and Appendix B sizes the difference between them. The seventh estimated series y^{10} is instead the ten-year government bond yield, the marginal rate at which the sovereign borrows today and a *risky* rate rather than the safe rate of Blanchard (2019); the sovereign risk premium of Barro (2023) is priced there first and reaches $r - g$ only at the speed of the maturity structure, so both series are kept in the system.

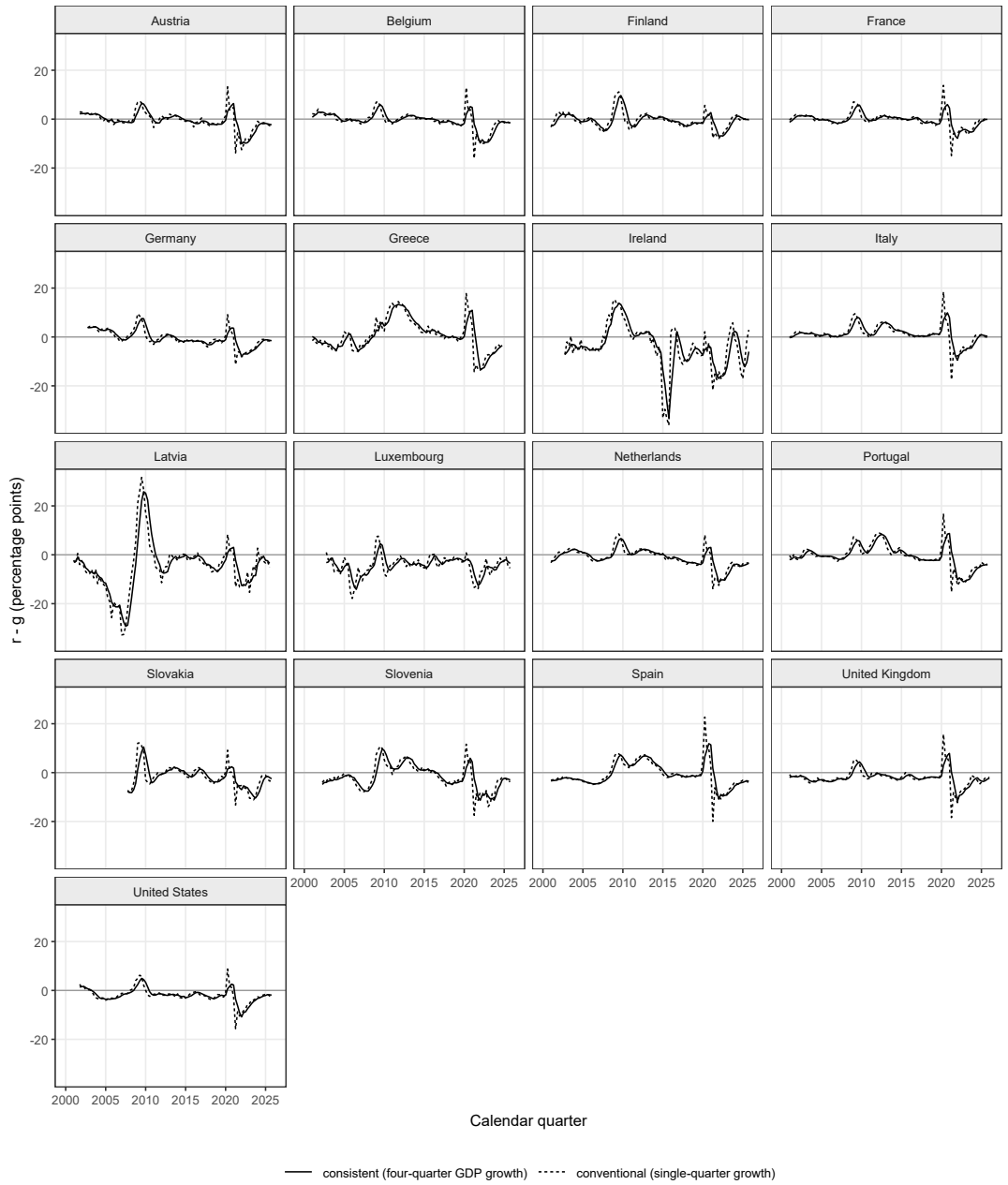
Table B1 (Appendix B) is informative about variables, their nature and sources. The series are drawn from the sources that Table B1 sets out: current-price GDP and gross capital formation from the IMF Quarterly National Economic Accounts (International Monetary Fund [IMF], 2026a); general government net lending/borrowing, interest payable and financial assets from the IMF Quarterly Government Finance Statistics (IMF, 2026b); general government gross debt, all instruments at nominal value, from the World Bank Quarterly Public Sector Debt database (World Bank, 2026); the ten-year government bond yield from the OECD financial market statistics (OECD, 2026b); and the current account balance from the OECD balance of payments (OECD, 2026a), while the net-anchor exercise of Section 6.6 also uses the Eurostat quarterly series of general government financial assets by instrument (Eurostat, 2026a) and, for its robustness construction, the archived IFS series (IMF, 2025). Figure 1 plots $r - g$ for each country by calendar quarter, on the consistent and on the conventional measure, and Figure I1 in Appendix I plots the consistent measure against a within-country observation index, with a smoothed trend. The differential was negative in most quarters: in 82 per cent of those plotted for the United States, 88 per cent for the United Kingdom and 62 per cent of the euro-area country-quarters. In the United States it was positive from the last quarter of 2008 to the end of 2009 and in only two quarters from 2010 to 2019, turned positive again in the last three quarters of 2020, and from 2022 averaged -4.7 points against -2.2 over 2014–2019. Nine of the United Kingdom's twelve positive quarters fall in 2008–2009 and 2020–2021. Grouped by calendar date, the euro-area observations were positive in about a third of country-quarters before the Great Financial Crisis, in most of them during that crisis and the sovereign debt crisis that followed, in just over a quarter from 2014 to 2019 and in fewer than one in ten from 2022. Dispersion of observations in Euro countries illustrates substantial country-specific heterogeneity. Two features of Figure I1 should be read with care, and its note states both: the horizontal axis is a within-country observation index and not a calendar date, so the euro panel does not align its fifteen members in time and its smoothed line is not a euro-area time series; and the span plotted is not the transformed sample

from which the estimates in the rest of the paper are drawn. Figures I2 and I3 in Appendix I illustrate an association of $r - g$ with increases in government debt that is broadly increasing, monotonically so in the two euro panels and with reversals in the four single-country ones. The untreated euro line is nonetheless uninformative at its left-hand end, where it is drawn over a sparse tail of large debt reductions: the thirteen observations below -15 percentage points of GDP, seven Greek from 2022Q1 to 2023Q3 and six Irish from 2014Q4 to 2016Q1. Winsorizing the upper and lower 5% of both variables removes that tail and leaves a relation that increases over most of the retained range, strictly so in the euro panel. Both figures use the series as reported, without the stationarity transformations of Section 5; the winsorization cut-offs and the pooling of the fifteen euro members into one scatter of 1,423 country-quarters are set out in their notes, as is the smoother — a penalised cubic regression spline in the euro panels against a LOESS in the single-country ones, which is why the euro lines look smoother than the single-country ones.

5 Empirical strategy

The paper studies whether government interventions through financial assets were systematically associated with growth net returns, measured by the interest-growth differential $(r - g)$. Interventions occur in environments shaped by latent macro-financial states – global financial conditions, credibility, institutional constraints, market sentiment, country-specific growth models – that are not directly observed but plausibly drive interest rates, growth, external balances, investment and public balance sheets jointly, so a reduced-form regression with fixed coefficients risks conflating interventions with the regimes in which they occur. The strategy is therefore a dynamic factor representation, whose key objects are the loadings linking $(r - g)$ and changes in government financial assets ΔFA to the same latent factor(s). When $(r - g)$ and ΔFA are both strongly correlated with the same factor, the two share a source of common variation, which the design does not identify with any one latent state or regime; the sign and magnitude of that association clarify whether asset accumulation tends to be associated with improving net returns (negative $r - g$) or deteriorating (positive $r - g$). In the pooled specification the criterion is applied to the correlation between a series and a factor rather than to the loading itself, because the estimator normalises each loading column across the N series, so that a fixed loading cut is not comparable across cross-sectional dimensions. The two are not interchangeable under a sparse estimator: a series can be strongly correlated with a factor on which its own loading has been shrunk to zero, as France's change in financial assets is, at -0.496, on a factor on which its loading is zero. This design is complemented by the remaining variables in the

Figure 1: The interest–growth differential by calendar quarter: consistent and conventional measures



Notes: $r - g$ in percentage points per year, one panel per country on a common vertical scale. Solid line: the consistent measure, the effective interest rate less the growth of nominal GDP summed over four quarters; dashed line: the conventional measure, which uses the year-on-year growth of single-quarter GDP (Table 1). Both are drawn on the country-quarters where both are defined, and the lines break at missing quarters.

Table 1: Measurement of the variables

Variable	Formula	Units	Timing	Sector	Consolidation	Valuation	Source
$r g^A$ (consistent $r-g$)	$i_t^{eff} - g_t^A$	pp	interest: flow over four quarters; debt: stock at end of $t-4$; GDP: four-quarter totals	S13 (interest, debt); total economy (GDP)	[to confirm]	debt at nominal value	as i^{eff} and g^A
$r g^{conv}$ (conventional $r-g$)	$i_t^{eff} - 100(GDP_t/GDP_{t-4} - 1)$	pp	as $r g^A$, but GDP growth of the single quarter t	as $r g^A$	[to confirm]	debt at nominal value	as i^{eff}; GDP (IMF, 2026a)
i^{eff}	$100\,\Sigma_4 Interest_t / Debt_{t-4}$	% p.a.	interest: flow over four quarters; debt: stock at end of $t-4$	S13	[to confirm]	interest: transactions; debt: nominal value	interest payable (IMF, 2026b); gross debt, all instruments (World Bank, 2026)
g^A	$100(Y_t^4/Y_{t-4}^4 - 1)$	%	flow over four quarters	total economy (S1)	n.a.	current prices, seasonally adjusted	GDP (IMF, 2026a)
Ten-year yield y^{10}	quarterly average of the long-term rate	% p.a.	average over the quarter	central government (ten-year bonds)	n.a.	market yield	long-term interest rate (OECD, 2026b); August 2026 and June 2025 retrievals by a vintage rule
Current account	$100\,\Sigma_4 CA_t / Y_t^4$	% of GDP	flow over four quarters	total economy vis-à-vis the rest of the world	n.a.	transactions, current prices, calendar and seasonally adjusted	current account (OECD, 2026a); GDP (IMF, 2026a)
Capital formation	$100\,\Sigma_4 GCF_t / Y_t^4$	% of GDP	flow over four quarters	total economy (S1)	n.a.	current prices, seasonally adjusted	gross capital formation and GDP (IMF, 2026a)
$\Delta_4 d$	$d_t - d_{t-4}$	pp of GDP	stocks at end of t and $t-4$, each over its four-quarter GDP	S13	[to confirm]	nominal value	gross debt (World Bank, 2026); GDP (IMF, 2026a)
pb (primary balance, surplus-positive)	$100(\Sigma_4 NetLending_t + \Sigma_4 Interest_t)/Y_t^4$	% of GDP	flow over four quarters	S13	[to confirm]	transactions	net lending and interest payable (IMF, 2026b); GDP (IMF, 2026a)
$\Delta_4 a$	$a_t - a_{t-4}$	pp of GDP	stocks at end of t and $t-4$, each over its four-quarter GDP	S13	[to confirm]	market value	financial assets (IMF, 2026b); GDP (IMF, 2026a)
d	$100\,Debt_t / Y_t^4$	% of GDP	stock at end of quarter	S13	[to confirm]	nominal value	gross debt (World Bank, 2026); GDP (IMF, 2026a)
a	$100\,FA_t / Y_t^4$	% of GDP	stock at end of quarter	S13	[to confirm]	market value	financial assets (IMF, 2026b); GDP (IMF, 2026a)

Notes: Y_t^4 is the sum of nominal GDP over $t-3,\ldots,t$; Σ_4 is the four-quarter sum ending at t. GDP, capital formation and the current account are national-currency, current-price series; debt and financial assets are end-of-quarter stocks of the general government (S13). Debt is at nominal value and financial assets at market value, so the two sides of the balance sheet are valued differently. [to confirm]: the consolidation basis of the QGFS and QPSD general-government series is not recorded in the project's files.

Table 2: Coverage of the panel by country and series: first and last quarter (filled cells)

Country	$r-g$	y^{10}	Ext. trade	Cap. form.	ΔDebt	Prim. bal.	ΔFA
	01Q4–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q4–25Q4	01Q3–25Q4
Austria	(97)	(101)	(101)	(101)	(99)	(97)	(98)
	01Q1–25Q4	01Q1–26Q1	03Q4–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Belgium	(100)	(101)	(90)	(101)	(99)	(100)	(98)
	01Q1–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Finland	(100)	(101)	(101)	(101)	(99)	(100)	(98)
	01Q1–25Q4	00Q1–26Q1	00Q3–26Q1	00Q3–26Q1	01Q3–26Q1	00Q3–25Q4	01Q3–25Q4
France	(100)	(105)	(103)	(103)	(99)	(102)	(98)
	02Q4–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	02Q4–25Q4	01Q3–25Q4
Germany	(93)	(101)	(101)	(101)	(99)	(93)	(98)
	01Q1–24Q4	01Q1–26Q1	02Q4–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–24Q4	01Q3–24Q4
Greece	(96)	(101)	(94)	(101)	(99)	(96)	(94)
	02Q4–25Q4	01Q1–26Q1	02Q4–26Q1	01Q1–26Q1	01Q3–26Q1	02Q4–25Q4	01Q3–25Q4
Ireland	(93)	(101)	(94)	(101)	(99)	(93)	(98)
	01Q1–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q1–25Q4	01Q1–25Q4
Italy	(100)	(101)	(101)	(101)	(101)	(100)	(100)
	01Q1–25Q3	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q3	01Q3–25Q3
Latvia	(99)	(101)	(101)	(101)	(99)	(99)	(97)
	02Q4–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	02Q4–25Q4	01Q3–25Q4
Luxembourg	(93)	(101)	(101)	(101)	(99)	(93)	(98)
	01Q1–25Q4	01Q1–26Q1	04Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Netherlands	(100)	(101)	(89)	(101)	(99)	(100)	(98)
	01Q1–25Q4	01Q1–25Q4	01Q1–25Q4	01Q1–25Q4	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Portugal	(100)	(100)	(100)	(100)	(99)	(100)	(98)
	07Q4–25Q4	01Q1–26Q1	04Q4–26Q1	01Q1–26Q1	07Q4–26Q1	01Q1–25Q4	01Q3–25Q4
Slovakia	(73)	(101)	(86)	(101)	(74)	(100)	(98)
	02Q2–25Q4	02Q2–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Slovenia	(95)	(96)	(101)	(101)	(99)	(100)	(98)
	01Q1–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–25Q4	01Q3–25Q4
Spain	(100)	(101)	(101)	(101)	(99)	(100)	(98)
	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q1–26Q1	01Q3–26Q1
United Kingdom	(101)	(101)	(101)	(101)	(99)	(101)	(99)
	01Q4–25Q4	01Q1–26Q1	01Q1–26Q1	01Q1–26Q1	01Q3–26Q1	01Q4–25Q4	02Q1–25Q4
United States	(97)	(101)	(101)	(101)	(99)	(97)	(96)

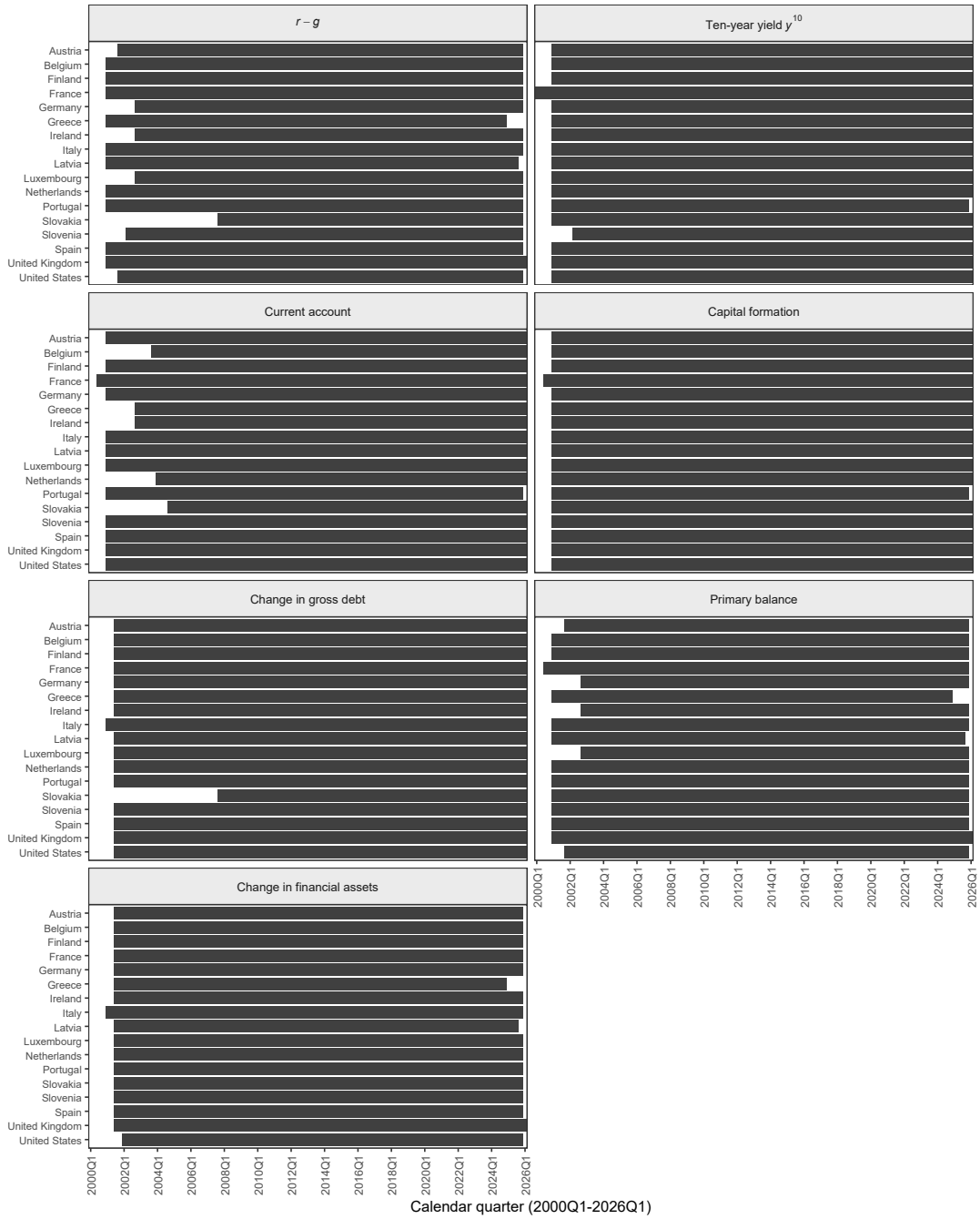
Notes: panel 2000Q1–2026Q1; a series' filled cells can be fewer than its span where the database leaves a cell empty. Quarters are written with two-digit years (01Q4 is 2001Q4).

Table 3: Sample flow: country-quarters by series and stage

	$r-g$	y^{10}	Ext. trade	Cap. form.	ΔDebt	Prim. bal.	ΔFA	All seven
Raw input observed	1,780	1,842	1,772	2,337	1,790	1,768	1,824	–
Ratio constructed	1,661	1,842	1,721	1,868	1,722	1,717	1,756	–
Panel, 2000Q1–2026Q1	1,637	1,715	1,666	1,718	1,660	1,671	1,662	1,594
After transformation	1,621	1,698	1,648	1,700	1,646	1,654	1,651	1,580
Identifying window	1,217	1,274	1,224	1,276	1,221	1,250	1,247	1,176

Notes: 17 countries. Raw: quarters at which the series' main raw input is observed (interest for $r-g$, either yield retrieval for y^{10}, current account, capital formation, gross debt, net lending, financial assets). Ratio: quarters 1999Q1–2026Q2 at which the construction rule gives a value. Panel: filled cells of the database. Transformation: each country's order of differencing from its stationarity tests. Identifying window: 2000Q2–2019Q4. All seven: country-quarters at which every series is defined.

Figure 2: Coverage of the panel by calendar quarter



Notes: for each of the seven series, one row per country and one column per calendar quarter from 2000Q1 to 2026Q1; a filled cell marks a value in the panel and a blank one a missing value. Table 2 gives, for each country and series, the first and the last quarter with a value and the number of filled cells.

system – interest rates, debt changes, capital formation, current account balances, and primary balances – , which locate that shared source within the common space the seven series span. Each factor is identified only up to a rotation of that space, so the states are not labelled as sovereign-pricing, investment, external-competitiveness or fiscal-consolidation regimes; the common space itself is the object read (Doz et al., 2011; Fan et al., 2021; Mosley et al., 2024; Stock and Watson, 2002).

For each country i and quarter t, we construct the seven-dimensional vector of sample variables:

$$y_{i,t} = [(r-g)_{i,t}; r_{i,t}; CAB_{i,t}; GCF_{i,t}; \Delta GD_{i,t}; \Delta FA_{i,t}; PB_{i,t}]' . \tag{16}$$

The system is designed to mirror the conceptual structure of growth net returns: $r - g$ combines a growth component (linked to investment and external trade competitiveness) and a financing-cost component (linked to sovereign bond pricing and debt dynamics). In the country-level specification, CAB is the current account balance as a share of GDP; GCF gross capital formation as a share of GDP; ΔGD and ΔFA are annual changes in those stocks (debt and financial assets, respectively), typically expressed in percentage points of GDP; and PB the primary balance as a share of GDP. The asset variable is therefore the variation in the asset position, which moves with revaluations and other changes in volume as well as with transactions; transactions are analysed separately in Appendix H.

Dynamic factor estimation is more reliable when inputs are stationary (Bai and Ng, 2002). Because quarterly macro-fiscal series can be persistent or trend-like, we applied unit-root testing at country level using standard augmented Dickey–Fuller procedures. The transformation follows each country's own test outcomes, series by series: a series enters at the order at which the unit-root null is first rejected at the ten per cent level. The rule is uniform across countries even though the transformations it produces are not, which is the sense in which the treatment is systematic rather than ad hoc. Appendix C reports the tests, Table C1 the order of integration of every series, and Table C2 sets the transformation actually applied against both that rule and the stricter five per cent classification, under which 15 series require second differencing rather than 5. Two consequences follow. Differencing costs observations, so the estimation samples differ in length across countries; and the mix of transformations differs across countries, so the factors extracted for one country are not measured on the same footing as those extracted for another, and the cross-country comparisons of individual factors made below are indicative rather than exact.

After transformation, each variable is standardized within each country as:

$$\tilde{y}_{i,t,j} = \frac{y_{i,t,j} - \bar{y}_{i,j}}{s_{i,j}}, \tag{17}$$

where $\bar{y}_{i,j}$ and $s_{i,j}$ are the sample mean and standard deviation for variable j in country i. Standardization prevents the factor extraction from being dominated by measurement units and aligns the analysis with the factor model framework (Bai and Ng, 2002; Stock and Watson, 2002). Missing observations lie at the edges of the sample rather than inside it. Across the seventeen countries' seven series the transformed samples carry 300 empty cells, with no gap in the interior of any series and no gap in the quarter labels of any country. They arise because the individual series do not all begin, and do not all end, with the country window that contains them, as Section 4 describes, and because differencing consumes a further observation or two. We do not impute them; interpolation would manufacture co-movement, whereas the Kalman filter of the state-space form updates the latent states from whichever series are observed at each date (Bańbura and Modugno, 2014). That route is not a formality here: deleting every country-quarter incomplete on the seven series would discard 121 of the 1,701 country-quarters contained in the transformed samples.

The model is estimated on a pooled panel. The seventeen countries' seven series are stacked into a single system of $N = 119$ series, and one common factor space is extracted from it; the specification this paper identifies from observes those series over $T = 79$ quarters, 2000Q2–2019Q4, for the reasons given in the paragraph that follows, and the full-sample companion over $T = 104$. Countries enter with their own sample coverage, so the union panel is ragged at its edges even though no country's own sample has an internal gap; those edges are absorbed by the Kalman filter rather than by dropping quarters, which preserves the whole of the estimation window instead of the 49 quarters a balanced intersection would leave on the identifying specification, or the 69 it would leave on the companion. The case for estimating country by country instead is that pooling would average away the cross-country heterogeneity this paper is about, and it rests on a misconception. Pooling imposes a common factor *space*; it does not impose common *loadings*. Each country retains its own $7 \times m$ block of loadings on the shared factors, so cross-country heterogeneity in fiscal-financial transmission — the premise of this paper — survives pooling intact. What changes is its evidentiary status. Estimated country by country, heterogeneity is imposed by construction and can therefore never be evidence for anything; estimated on a pooled panel it becomes a restriction that can be tested, and Appendix F reports that test. Three further considerations point the same way. First, the estimator: approximate factor models are justified asymptotically in both N and T, and $N = 119$ sits inside that regime where $N = 7$ does not. Second, factor

selection: with seven series the admissible range of the information criteria extends only to six and the minimum falls on that boundary in every country, whereas on the pooled panel the criteria have a genuine interior minimum. Third, comparability: seventeen separate models carry seventeen arbitrary rotations and seventeen score scalings, so a loading estimated for one country is not measured on the same footing as a loading estimated for another, and the cross-country comparisons this paper draws are not well defined under that design. A single rotation removes the problem by construction. The seventeen country-level fits were estimated and are reported, as descriptive estimates, in Section 6 and in Appendices D and E, and re-estimated for the comparison of Appendix G, with the full estimation output retained in the replication package. They are not identified, for the reason set out below, and no claim of this paper rests on them: every cross-country claim of this paper rests on the pooled specification.

One choice of specification governs every pooled number reported below, and it is stated here rather than left to an appendix. The panel is constructed over the full window, but the paper identifies on its pre-pandemic part: the common factor space is estimated on 2000Q2–2019Q4, $T = 79$ quarters of the same $N = 119$ series, at three factors, with an independent idiosyncratic error and a penalty of 3.73 chosen by the Bayesian Information Criterion. Five probes, none of them designed to settle the question, agree on that window and on that dimension: four of the six live selection criteria return three factors there, where on the full sample the same battery yields divergent results; ending the four-factor fit at 2019Q4 leaves the first three canonical correlations against the full-sample space at 0.9995, 0.9946 and 0.9703 while the fourth falls to 0.3038; the independent and the AR(1) error specifications agree on all three dimensions, at 0.9996, 0.9967 and 0.9698, despite selecting penalties forty-three times apart, where at four factors on the full sample they diverge on the fourth at 0.9215; the coefficient-equality tests of Section 6.2 reject on seven of seven variables on this window and on seven of seven on the full sample; and structural-break tests at known dates are correctly sized on this window, where on panels simulated from its fitted state space they reject in 5.7 per cent of tests at their nominal five per cent and, at the programme dates, in 4.2 per cent of the cells of the twelve countries without a programme, whereas on the full-sample space those cells reject in 17.7 per cent, significantly more often ($p = 0.001$), even though the same test keeps its nominal size, 5.05 per cent, on panels simulated from that space, so that the excess rejections reflect instability in the full-sample data rather than a defect of the test. The cost is plain and none of that offsets it. The identifying sample stops before the pandemic, the energy shock, the 2024 reform of the Stability and Growth Pact, the United Kingdom's switch to public sector net financial liabilities and the 2025 amendment to the German debt brake — that is, before the episode in which the

fiscal rules this paper discusses were suspended and rewritten. The full-sample fit, at four factors on $T = 104$, and a third fit in which the eight pandemic quarters are set to missing, are both retained and reported as companion specifications; no pooled number below is taken from either unless it is labelled as one, and the exclusion of the last six years is returned to as a limitation in Section 7.

Write $\tilde{y}_{i,t} \in \mathbb{R}^7$ for country i's standardised vector at quarter t, constructed as above. The pooled system stacks the seventeen countries into a single observation vector

$$y_t = \left(\tilde{y}'_{1,t},\ \tilde{y}'_{2,t},\ \ldots,\ \tilde{y}'_{17,t}\right)' \in \mathbb{R}^N, \qquad N = 17 \times 7 = 119, \tag{18}$$

observed for $t = 1, \ldots, T$ with $T = 79$ on the identifying window and $T = 104$ on the full-sample companion. The measurement equation for the pooled model is then

$$y_t = \Lambda F_t + \varepsilon_t, \qquad \Lambda \in \mathbb{R}^{N \times m}, \quad F_t \in \mathbb{R}^m, \tag{19}$$

with $m = 3$ factors common to all seventeen countries on the identifying specification, and $m = 4$ on the full-sample companion. The loading matrix partitions by country into seventeen blocks,

$$\Lambda = \left(\Lambda'_1,\ \Lambda'_2,\ \ldots,\ \Lambda'_{17}\right)', \qquad \Lambda_i \in \mathbb{R}^{7 \times m}, \tag{20}$$

so that Λ_i records how country i's seven series load on the common factors. This is the sense in which pooling imposes a common factor space without imposing common loadings: F_t is shared, Λ_i is not. The factor dynamics are again a stable VAR(1), now single and common,

$$F_t = AF_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, Q), \qquad \varepsilon_t \sim \mathcal{N}(0, R), \tag{21}$$

with $A \in \mathbb{R}^{m \times m}$, Q the $m \times m$ innovation covariance and R an $N \times N$ idiosyncratic covariance restricted to be diagonal, as in the country-level specification and for the same reason. Sparsity is imposed on the pooled loading matrix by the same l_1 penalty as before, applied to Λ as a whole rather than country by country:

$$\hat{\Lambda}(\alpha) = \arg\min_{\Lambda} \left\{ \frac{1}{2T} \sum_{t=1}^{T} \left\| y_t - \Lambda \hat{F}_t \right\|_2^2 + \alpha \left\| \Lambda \right\|_1 \right\}, \tag{22}$$

with α selected by the same Bayesian Information Criterion, over the logarithmically spaced grid at (31) rather than over the thirteen-point subset the country fits use, though here too the search stops short of that grid's upper end. Countries enter with their own sample coverage, so y_t is ragged at its edges: whole quarters fall

outside a country's window, and individual series open and close at their own dates inside it. On the identifying panel the two kinds of edge together leave 703 of its 9,401 cells unobserved, the 7.5 per cent recorded in Appendix F. Those entries entail no loss of information here, because the Kalman recursion treats a missing element of y_t as unobserved at that date and updates F_t from the elements that are observed, following Bańbura and Modugno (2014). One feature of the implementation qualifies (22) and is stated here rather than left to the code. The estimator exempts a leading block of series from the penalty, and in the pooled fit that block has size one: the first row of Λ, which is Austria's $(r - g)$ and no other series, is never shrunk, and it is accordingly the only one of the 119 series whose loadings on the identifying specification are all non-zero, against the 57 per cent of that fit's 357 pooled loadings that the penalty sets exactly to zero. The adopted fit keeps the exemption, so (22) is the objective actually minimised for the remaining 118 series. Refitted with the exemption removed, and again under five random orders of the country blocks, the identifying specification recovers the adopted factor space, every canonical correlation exceeding 0.99 against a threshold of 0.95 fixed beforehand (Appendix H).

Both restrictions carried by (21) — the diagonality of R and the single lag in the factor transition — are imposed for tractability rather than established from the data, and neither survives testing. Appendix F reports that on the identifying specification the idiosyncratic residuals are serially correlated in 103 of the 119 series at the five per cent level, that residual cross-correlations exceed 0.3 in absolute value in 5.8% of the 7,021 pairs against the 1.0% that sampling error alone would produce at the median pairwise overlap of 73 quarters, and that a likelihood-ratio test of VAR(1) against VAR(2) on the estimated factor paths gives 143.6 on 9 degrees of freedom. The standard errors implied by (19)–(21) are accordingly optimistic, which is why the tests reported below are bootstrapped rather than read off an asymptotic distribution. Only half of the first restriction can be lifted with the estimator used here: its AR(1) option relaxes the requirement that each idiosyncratic component be uncorrelated with its own past, but leaves R cross-sectionally diagonal, so the residual cross-correlations just reported go untested. Re-estimating the identifying panel under that option confirms the serial dependence — the fitted idiosyncratic AR(1) coefficients have a median absolute value of 0.288 — and establishes the one thing that lets the independent specification carry the results. The penalty is identified on this panel, an independent cold start at every point of a twenty-one-point grid returning an interior minimiser at $\alpha = 0.0869$; but what matters is not the penalty but what it leaves. The two error specifications select penalties about forty-three times apart, 0.0869 against 3.73, leave 35.0 per cent of the loadings non-zero against 42.6, and recover the same factor space, at canonical correlations of 0.9996, 0.9967 and 0.9698. The span is the only object this design identifies,

and on the identifying specification it is invariant to the error assumption; that is what allows the independent fit to stand as the headline quasi-maximum-likelihood estimator, with the AR(1) fit reported beside it as a co-equal column rather than as a correction to it (Doz et al., 2011). The one thing the re-estimation does not settle is the co-loading count, and it should not be read as a check on it. Appendix F rests nothing on that count and neither does this paper: the note to Table F2 shows it to be conditional, in degrees that are measured rather than asserted, on the loading threshold, on the panel, on the error specification and on the optimisation. One qualification runs the other way. The invariance of the span to the error assumption is a property of the specification this paper identifies from and not of the panel at large: on the full-sample companion at four factors the fourth canonical correlation is 0.9215, so there the error assumption does move what is estimated, and it moves the dimension the pandemic and the selection criteria already leave suspect. That is a further reason to identify where this paper identifies, and not a further degree of freedom. What a non-diagonal R would cost is still not established here.

One statistic carries the co-movement evidence of this paper, and it is defined here so that Section 6 can report its distribution rather than only a verdict. Write $F_{k,t}$ for the kth of the m common factors and, for country i, define $\rho_i = \max_{k \le m} \min\Big(|\mathrm{corr}((r-g)_{i,t}, F_{k,t})|,\ |\mathrm{corr}(\Delta FA_{i,t}, F_{k,t})|\Big)$. On each common factor in turn ρ_i takes the weaker of that factor's two correlations, with country i's $(r-g)$ and with its ΔFA, and keeps the factor on which that weaker correlation is largest, so a large ρ_i requires both series to be correlated with the same common factor and not merely one of them. It lies in $[0,1]$, is unaffected by the sign and the scale of the factors and so is comparable across countries within a fit, and is defined for every country whether or not that country meets any threshold. The co-loading criterion applied below is $\rho_i > 0.3$, which makes the number of co-loading countries a summary of the distribution of ρ_i rather than a statistic in its own right; Section 6 accordingly reports the distribution of ρ_i across the seventeen countries and gives the count as one summary of it. No confidence interval has been computed for ρ_i on the specification the paper identifies from, so that distribution is reported without a measure of its sampling uncertainty.

Equation (20) renders the paper's central premise testable. Cross-country heterogeneity of fiscal-financial transmission is the hypothesis that the country blocks differ, so its negation is the restriction

$$H_0 : \ \Lambda_1 = \Lambda_2 = \cdots = \Lambda_{17}, \tag{23}$$

which is a restriction on a single estimated model and can therefore be tested. Under a country-by-country design no such restriction exists to be tested, because

each Λ_i comes from a different model with its own rotation. Appendix F reports the test of (23) and Section 6 its result.

The rule by which (23) is judged is enforced in the code that builds Appendix F rather than stated only in prose. Four dispersion statistics are computed there, two of which are independent of the scale of Λ: the dispersion across countries of each country's block of correlations between its seven series and the common factors, and the dispersion across countries of the common covariance that each block implies, $\Lambda_i \operatorname{var}(F_t)\, \Lambda_i'$. Only the second of the two is also invariant to the rotation of the factors. The correlation statistic is invariant to the signed permutations of the factor columns that the sparsity penalty leaves free, which is the indeterminacy the estimator actually carries; it is not invariant to a general rotation, because the correlation re-standardises each rotated column separately and that changes the metric when the factor covariance is not spherical. On the identifying fit the correlation dispersion is 20.39, and five random orthogonal rotations which leave the fitted common component numerically unchanged move it to values between about 18.7 and 20.5, the exact endpoints depending on the draw, while the common-covariance dispersion stays at 18.13 throughout. The composite rule below therefore pairs a statistic that is a property of the estimated common space with one that is a property of the representation the penalty selects, and the second is reported on that basis rather than as a property of the space.

The null is simulated on the scale on which (23) is written. Each simulated column is generated already standardised, as $z_{ij,t} = F_t'\lambda_j + \sqrt{1-q_j}\, e_{ij,t}$ with $q_j = \lambda_j' \operatorname{var}(F_t) \lambda_j$ and $\lambda_j = \kappa \Lambda_{\mathrm{RLS},j}$ imposed on every country, where Λ_{RLS} is the pooled restricted least-squares block and κ its size. The standardised blocks are therefore equal by construction whatever the idiosyncratic draw $e_{ij,t}$, which has unit variance and is free to carry each series' own dependence, and the dispersion of standardised loadings the null itself carries is 2.4×10^{-31}, zero to machine precision. The null is drawn under two calibrations. In the first, the idiosyncratic errors and the factor innovations are Gaussian white noise. The second is dependence-robust: the fitted idiosyncratic residuals are resampled in moving blocks of eight quarters on one block index for the whole panel, so that their serial and contemporaneous cross-series dependence is carried into the draw, and the fitted innovations of the factor transition are resampled in the same way and rescaled so that the simulated factor path carries the model's own stationary variances. The pattern of missing quarters is reimposed on every simulated panel under both. The size κ of the common block is not identified by (23) — equality holds exactly whatever that size is — so it is a nuisance parameter, and plugging in one value for it chooses the answer: across the sizes this panel admits, the cell on which the verdict turns moves from $p = 0.31$ to $p = 0.001$. It is therefore swept rather than plugged in. A 99 per cent confidence

set for κ is formed from a matching statistic that no rotation of the factors changes, the probability of each cell is taken as its supremum over that set, and the 0.01 the set may fail to cover is added, in the manner of Berger and Boos (1994). The restriction is recorded as rejected only if the resulting composite — the largest cell probability over the two statistics, the two calibrations and every size of the common block inside the confidence set, plus 0.01 — is at or below five per cent, so that the figure reported is the worst of those cells rather than the best of them. Each feature is deliberate: two statistics so that a rejection cannot rest on the more favourable of them, both calibrations and every admissible size of the block so that it cannot rest on a null that happens to make rejection easy, and the resampled calibration in particular because the Gaussian one assumes idiosyncratic errors that are white noise with a diagonal covariance, which they are not on this panel. The price is that the rule can fail while most of its cells reject, which is why Section 6 reports the outcome cell by cell and names the cell the verdict turns on.

The form of this rule — a maximum over cells, so that the restriction is rejected only if every cell rejects — predates both runs this paper reports, and it was left unchanged when, on an earlier run, it first failed to reject. The null generator it is evaluated under was not: it was rebuilt after the first run, because the generator first used did not satisfy (23). The sweep over the size of the common block, the confidence set for it and the 0.01 added for that set were fixed with the rebuild, after every cell of the first run was known. That generator imposed the common block on the series in their original units, kept each series' own idiosyncratic variance and then standardised every simulated column, so the standardised loading its refit targeted, $\bar{\lambda}_j \big/ \sqrt{\bar{\lambda}_j' \operatorname{var}(F_t)\, \bar{\lambda}_j + \sigma_{ij}^2}$, varied across countries with σ_{ij}^2; evaluated at the fitted parameters, the dispersion of standardised loadings that null carried was 0.047 and 0.176 under its two Gaussian calibrations, against 0.536 observed. The second of those calibrations rescaled the idiosyncratic variance to match the panel's signal-to-noise ratio. Under the rebuilt generator that ratio is set by the size of the common block alone, so the calibration is not kept as a separate null; its intent is one size of the block within the range swept. Section 6 reports the outcome, and Appendix F reports the first run beside the rebuilt one.

The specification set out in the remainder of this section is estimated country by country, for all seventeen countries, and is reported as description: the loadings, scores and diagnostics are tabulated in Appendices D and E, and summarised in Section 6.5; Appendix G compares the regime classification across specifications, taking the country-by-country one from seventeen fits re-estimated for that appendix. No substantive claim of this paper rests on the country-level fits. They are retained because they give each country its own factor paths, and so its own dated episodes, which a pooled specification whose factors are common to all seventeen cannot,

and because they show why co-loading counts read off them are not on a common scale with the pooled ones; what is drawn from them is descriptive, and they cannot establish any stabilisation, investment, or other functional role of government financial assets. The specification is set out because the pooled specification is this same system of measurement and transition equations estimated on one stacked panel rather than on seventeen, and it is written with a country subscript throughout. The footing of these country fits is set by identification, the first of the four limitations stated below: the factor count m_i is fixed rather than selected, at $m_i = 6$ on $N = 7$ series, twice the Anderson–Rubin row-deletion bound of $r \leq (N-1)/2 = 3$, so the resulting structure is one admissible rotation among many rather than an identified object.

For each country i, the measurement (observation) equation is:

$$\tilde{y}_{i,t} = \Lambda_i f_{i,t} + \varepsilon_{i,t}, \tag{24}$$

where $\tilde{y}_{i,t} \in \mathbb{R}^7$ is the standardized vector of observables, $f_{i,t} \in \mathbb{R}^{m_i}$ is a vector of latent factors, and $\Lambda_i \in \mathbb{R}^{7 \times m_i}$ is the loading matrix. The factor dynamics are specified as a stable VAR(1):

$$f_{i,t} = A_i f_{i,t-1} + u_{i,t}, \tag{25}$$

where A_i is an $m_i \times m_i$ transition matrix and $u_{i,t}$ are factor innovations. This specification is parsimonious but flexible enough to capture persistent latent regimes (Doz et al., 2011). As usually, mean-zero Gaussian innovations are assumed for quasi-maximum likelihood estimation:

$$u_{i,t} \sim \mathcal{N}(0, Q_i), \tag{26}$$

$$\varepsilon_{i,t} \sim \mathcal{N}(0, R_i). \tag{27}$$

In the country-level specification, R_i is restricted to be diagonal. The purpose of this restriction is practical and conceptual: it encourages the factor space to capture the dominant common covariance, rather than allowing residual cross-correlations to substitute for latent structures, which is especially important when T is limited (Bańbura and Modugno, 2014; Doz et al., 2011). We do not relax this restriction; a specification with a non-diagonal R is a natural robustness check and is not attempted here.

The principal empirical objects are the loadings in Λ_i, read jointly rather than one at a time. A rotation of the factors can change the size and the sign of any individual loading while leaving the fitted common component of each series unchanged, so no

single loading, and no single sign, carries an economic reading. The direction of the association between $(r-g)$ and ΔFA is read instead from the correlation between their fitted common components, which no rotation alters: a negative value indicates that financial assets tend to expand in the states in which $r-g$ declines (i.e., net returns improve).

For robustness, we also compute communalities. The communality of a series is the sum of its squared loadings. It equals the share of that series' variance attributable to the factor space only when the factors are mutually orthogonal with unit variance, which the estimated factors are not: on the country fits the ratio of $\lambda_j'\Sigma_F\lambda_j$ to $\lambda_j'\lambda_j$ runs from 0.93 to 3.62, and on the pooled fit the factor variances are 31.82, 51.77 and 35.00. The communality is therefore reported as a descriptive diagnostic and not as a variance share, and it is invariant to orthogonal rotations of the factors but not to the other non-singular transformations of them that leave the fitted common component unchanged, which are admissible here because the factors are not normalised to be orthonormal. The percentages quoted in Section 6 are not communalities: they are reconstruction R^2, one minus the residual sum of squares over the total, computed against the fitted common component, and they assume no such normalisation:

$$h_{i,j}^2 = \sum_{k=1}^{m_i} \hat{\lambda}_{i,j,k}^2. \tag{28}$$

Low communalities are informative: they signal that a variable behaves idiosyncratically relative to the system, which disciplines interpretation of its loadings. This is particularly relevant for interest rates in reserve-currency, safe-haven or, on the contrary, high indebted countries, where sovereign pricing may decouple from domestic macro-fiscal variables.

The purpose of the dynamic factor model is not to claim that changes in government financial assets cause changes in $(r-g)$. Rather, it provides a reduced-form description of common variation: latent factors are taken to summarise the unobserved states that jointly shape the behaviour of fiscal actions, sovereign pricing and growth outcomes. Under the assumption that a low-dimensional factor space captures the dominant common variation in $\tilde{y}_{i,t}$, co-loading indicates that changes in government financial assets and growth net returns are each correlated with the same estimated factor, even when the macro-financial states behind it are not directly observed; it is not itself a measure of their co-movement, and the co-loading count is not read as a finding (Section 6). Instead, the framework is designed to describe co-movement through the estimated common space, rather than unconditional average effects. Those states are not named: each factor is identified only up to a rotation of the factor block, so a label read off the variables that load on one factor would

describe the representation the estimator returned rather than the data.

The number of factors is fixed at $m_i = 6$ for every country. We compute the information criterion of Bai and Ng (2002) for each country and report it, with the share of variance explained, in Appendix E, but do not use it to select m_i, because at this cross-sectional dimension it cannot do so: with $N = 7$ series its admissible range extends only to $N - 1 = 6$, and its minimum is attained at that upper boundary in all seventeen countries. A criterion that declines monotonically across the whole of its admissible range has not identified a factor count; it has only signalled that the range is too narrow to contain one. On the pooled panel the criteria behave as intended: evaluated over $k = 1, \ldots, 15$ on the pre-pandemic window, four of the six live criteria, the IC_{p1} and IC_{p2} penalties among them, select three factors, and we adopt $m = 3$ for the pooled specification on that basis. The growth-ratio and eigenvalue-ratio statistics dissent, at one, the latter as it does on every space we estimate, which is the documented symptom of cross-correlated idiosyncratic errors rather than a statement about this window. Three qualifications belong with that statement: the criteria require a complete matrix, so they are evaluated on the 49 complete-case quarters rather than on the 79 the pooled model is fitted to; the margins are narrow, four factors being worse than three by 0.0035 on IC_{p1} and by 0.0134 on IC_{p2}; and the agreement is a property of this window rather than of the panel at large, since on the full-sample companion the same battery splits, IC_{p1}, IC_{p2} and the Marchenko–Pastur edge count taking four, Onatski's edge distribution two and the two ratio statistics one. That the criteria agree where the pandemic quarters are out of the sample and disagree where they are in is one of the grounds on which the identifying window was fixed. Something does turn on the count, and we explicitly acknowledge this rather than claiming a degree of stability that the fit lacks: on that companion, re-estimated over a grid of factor numbers running from two to fifteen, the pooled co-loading count runs from eight to thirteen of the seventeen countries, and no equivalent grid has been run on the identifying specification. That is one of the reasons Section 6 reports the distribution of ρ_i and treats the count as a summary of it. The criterion evaluated has the form:

$$IC_i(m) = \log\left(V_i(m)\right) + m \cdot g(N, T), \tag{29}$$

where $V_i(m)$ is the average residual variance when extracting m factors and $g(N, T)$ penalizes complexity; the aim is to balance fit and parsimony, avoiding over-extraction that would absorb noise and undermine economic interpretation (Bai and Ng, 2002).

Estimations are carried out in state-space form by quasi-maximum likelihood using an Expectation-Maximization (EM) algorithm. The logic is standard in macro DFMs and particularly convenient when missing observations are present (Bańbura

and Modugno, 2014). We proceed as follows.

We begin with initial values for Λ_i and $f_{i,t}$ based on a low-rank approximation, derived from an eigen-decomposition of the covariance matrix of $\tilde{y}_{i,t}$. These starting values are then refined iteratively:

E-step: Given current parameter estimates $\{\Lambda_i, A_i, Q_i, R_i\}$, the Kalman filter and smoother are applied to obtain the conditional expectations of the latent states and relevant second moments:

$$\mathbb{E}[f_{i,t} \mid \tilde{y}_{i,1:T}], \quad \mathbb{E}[f_{i,t}f'_{i,t} \mid \tilde{y}_{i,1:T}], \quad \mathbb{E}[f_{i,t}f'_{i,t-1} \mid \tilde{y}_{i,1:T}].$$

M-step: Using these smoothed moments, parameters are updated by closed-form least-squares-type expressions implied by the Gaussian likelihood, and convergence is declared when changes in the log-likelihood and parameter norms fall below a tight tolerance. After convergence the stability condition $\rho(\hat{A}_i) < 1$ is verified, where $\rho(\cdot)$ denotes the spectral radius, a direct check that the estimated latent dynamics are stationary. If the spectral radius is close to one, the factor is highly persistent (a slow-moving regime).

A standard DFM produces dense loadings, which in short macro samples can be difficult to interpret economically and may reflect over-fitting rather than stable structures, so we impose sparsity on the loading matrix Λ_i through l_1-penalization (Barigozzi et al., 2018; Mosley et al., 2024; Tibshirani, 2011).

Operationally, given smoothed factor estimates $\hat{f}_{i,t}$, the sparse loading matrix is obtained by solving:

$$\hat{\Lambda}_i(\alpha) = \arg\min_{\Lambda} \left\{ \frac{1}{2T} \sum_{t=1}^{T} \left\| \tilde{y}_{i,t} - \Lambda \hat{f}_{i,t} \right\|_2^2 + \alpha \|\Lambda\|_1 \right\}, \tag{30}$$

where $\|\Lambda\|_1$ is the sum of absolute values and $\alpha > 0$ governs the intensity of shrinkage. This step also bears on the rotational indeterminacy of factor models: many rotations fit the data similarly well, but sparsity effectively selects a rotation that yields a simpler structure, with loadings near zero and a subset of remaining non-zero loadings. The result is a set of factors that can be more plausibly interpreted as distinct channels rather than diffuse mixtures (Mosley et al., 2024). The rotation so selected is nevertheless a choice the penalty makes rather than one the data force, so this paper gives it no structural weight and names no factor.

The tuning parameter α is chosen minimizing the Bayesian Information Criterion over a logarithmically spaced grid, that spans weak to strong penalization:

$$\alpha_k = 10^{\text{seq}(-3,2,\text{length}=50)} \in [0.001, 100]. \tag{31}$$

For each α_k, the quasi log-likelihood and the effective number of parameters are computed (counting non-negligible loadings and dynamic parameters), and the α^* is selected minimizing BIC. Neither specification searches the whole of it, and the two depart from it differently. The pooled fit is given the full grid and evaluated along it with a warm start, each penalty initialised at the preceding one's solution, until the walk halts at the first penalty that empties a factor, short of the upper end. Because the paper treats an independent cold start as what identifies a penalty — it is how the country penalties are selected — that search was repeated on the identifying panel, every point of the grid fitted separately from a cold start. It returns a different penalty as an interior minimiser, 5.96 against the 3.73 selected along the warm path, but a factor space indistinguishable from the adopted one, at canonical correlations of 1.0000, 0.9993 and 0.9883; at its own penalty the cold fit leaves fewer of the loadings non-zero, 29.4 per cent against 42.6. The pooled estimates below keep the penalty the warm path selects: the factor space does not depend on that choice, but the share of loadings set to zero does. The country penalties are instead selected over thirteen of the fifty points, each of the thirteen fitted from a cold start, so that the penalty selected does not depend on the path the estimator takes through the grid; that subgrid runs up to 19.31. The minimum is taken over the fits that converge, which are not all of them: ten of the 221 country–penalty fits fail to converge, and at the largest of the thirteen penalties the estimator declines to return a fit in seven countries, one of the six factors having had all of its penalised loadings shrunk to zero. Appendix E plots the criterion for each country and marks the penalty selected. This ensures the degree of sparsity is disciplined by the data rather than chosen. One implementation detail must be recorded, because it bears directly on how the results below should be read. The estimator allows a leading block of series to be exempted from the l_1 penalty altogether, and in the country-level specification that block has size one and contains $r - g$; the corresponding statement for the pooled fit is made above. The loadings of $r - g$ are consequently never shrunk to zero, whereas those of every other series, including ΔFA, are: across the seventeen countries no estimated $r - g$ loading is set to zero, against between 14 and 37 per cent for the remaining six variables. Since the co-loading diagnostic below asks whether $r - g$ and ΔFA attach to a common factor, one side of that comparison is protected from shrinkage while the other is not. The country fits reported here have not been re-estimated with the exemption removed, so that asymmetry qualifies every co-loading reading drawn from them; those readings are description, and no substantive claim of this paper rests on them.

To verify that factor extraction is warranted and that the dynamic structure captures main persistences of the data, we report a compact set of diagnostics. They are, first, residual serial correlation in $\hat{\varepsilon}_{i,t}$, to check that persistence is absorbed by the

factor dynamics rather than left in the idiosyncratic component; second, the stability metric $\rho(\hat{A}_i)$; and third, a comparison of fit and parsimony across PCA, dense DFM and sparse DFM on log-likelihood and BIC (Doz et al., 2011; Mosley et al., 2024).

Four limitations should be stated before the estimates are read. The first three apply to the country-level fits; they are the reason the pooled specification rather than those fits carries the cross-country claims, and the reason those fits are reported as descriptive estimates rather than as identified ones. They were estimated for all seventeen countries, and their loadings, scores, dated episodes and per-country diagnostics are tabulated in Appendices D and E and in Table H2 on that footing; the country-by-country classification of Appendix G rests on a re-estimate of the same specification, made for that appendix and read on the same footing. First, identification. With $N = 7$ observed series, classical factor-analytic bounds place the number of identifiable factors well below the six extracted here: the Anderson–Rubin row-deletion condition gives $r \leq (N-1)/2 = 3$, and a six-factor model of seven series carries more free parameters than the sample covariance matrix has distinct moments. Imposing sparsity improves interpretability but does not relax these bounds. Any loadings so obtained are therefore one admissible representation of the common variation rather than a uniquely identified structure, and neither their rotation nor the sign pattern of any individual factor should be given structural weight; that is why they are reported as description rather than as estimates of an identified structure. Second, dimension. Extracting six factors from seven series is closer to a rotation of the data than to a reduction of it, and any variance share computed from such a fit should be read in that light. Individual factors should not be treated as separately identified objects. Third, estimation regime. Approximate factor models of this type are justified asymptotically in both N and T; a country-by-country specification with $N = 7$ sits outside that regime, so the country-level estimates are best understood as descriptive summaries of within-country co-movement. This is precisely why the pooled panel, which places the estimation in the large cross-section regime these methods are designed for, is used for every cross-country statement below. Fourth, the relation to Section 3. The state-dependent model set out there motivates the empirical design and supplies the vocabulary of regimes in which the estimates are discussed, but its structural parameters are not estimated here, and the factor model is not a test of it. The two are complementary rather than nested. The factor model estimates a common factor space but does not by itself establish that transmission differs across states. To test that directly we estimate, alongside it, a smooth-transition regression in which the interest-growth differential is regressed on the change in financial assets, the change in gross debt, the primary balance and capital formation, with the coefficients weighted between two regimes by a logistic transition in a lagged measure of debt stress. The transition variable

requires a word, because it is not the object Section 3 specifies. The theory indexes stress by the level of the debt ratio; the database contains no debt level, only the four-quarter change in the debt-to-GDP ratio defined in Appendix B, and the transition variable is therefore the one-quarter lag of that change. It measures debt accumulation rather than debt burden, so a country carrying a high but stable debt ratio is placed in the low-stress regime, and the regimes should be read in those terms. Linearity is the restriction that the two regimes coincide, which is the null of the Luukkonen–Saikkonen–Teräsvirta test, applied here in its third-order form; the modelling cycle within which the test sits follows Teräsvirta (1994). Because the regressors are persistent we report block-bootstrap p-values, from 999 resamples in blocks of eight quarters, whose smallest attainable value is therefore $1/1000 = 0.001$. Those blocks are drawn separately within each country, so the resample preserves serial dependence but not any shock common to several countries in the same quarter. On a panel selected for its common component that is the wrong way round, and it leaves the pooled probabilities below anti-conservative by an amount this design does not quantify. The resampled calibration of the homogeneity test draws one block index for the whole panel and so carries such shocks; the linearity bootstrap does not, and every pooled linearity p-value in this paper should be read with that reservation. This regression is estimated on the identifying window of Section 5: each country's raw series are truncated at 2019Q4 before any lag, difference or cumulative proxy is formed, which leaves an estimation window of 2001Q2–2019Q4 and between 48 and 75 quarters per country, 73 at the median; the full-sample fit is retained as a labelled companion. The first-order form of the same test is computed alongside it and, on the identifying window, rejects the null less often rather than more: eleven of the seventeen countries at five per cent against the thirteen reported in Section 6, and nine countries reject under both forms, Portugal and Slovenia under the first-order form alone and Austria, Belgium, Greece and Luxembourg under the form reported there alone. This specification nests the linear model, so state dependence becomes a hypothesis rather than a modelling assumption, and it is the empirical counterpart of the regime weighting in Section 3. Results are in Section 6.

Three variants of that regression are estimated. They differ in whether the regression is written in first differences or in levels and, in the levels case, in whether the transition variable is the lagged change in gross debt or a proxy for the level of the debt ratio. A test of linearity carries information about the transition function only where its maintained linear benchmark is itself well specified, and in first differences it comes closest to being so: on the identifying window the residuals of the linear regression clear an augmented Dickey–Fuller test at -3.5 in twelve of the seventeen countries and carry a median first-order autocorrelation of 0.56, against three of seventeen and a median of 0.80 in levels. The first-difference specification is therefore

the one reported as primary, and the one on which the restriction implied by Section 3 is estimated; the levels specifications are reported alongside it, and a larger count of rejections there should not be read as more evidence of non-linearity.

The design is reduced-form throughout: it identifies regime-dependent co-movement, not causal fiscal multipliers, so the conclusions describe co-movement and do not by themselves establish that changes in financial assets cause changes in $(r - g)$. In addition, the quarterly sample is short, measurement conventions for public financial assets may differ across countries, and structural breaks can shift the mapping from latent regimes to observables. These considerations discipline interpretation and motivate the robustness checks reported above.

5.1 Selection of factor analysis approach

To determine the most appropriate factor analysis approach, we compared Principal Component Analysis (PCA), Dynamic Factor Models (DFM) and Sparse Dynamic Factor Models (Sparse DFM), taking the Bayesian Information Criterion (BIC) and log-likelihood diagnostics.

PCA reduces data dimensionality extracting orthogonal principal components from the eigen structure of the covariance matrix. It remains consistent for the factor space under an approximate factor structure (Bai and Ng, 2002), so the case for a dynamic specification is not that PCA is invalid but that it carries no transition equation, and so leaves factor persistence in the residual rather than absorbing it – which is what the comparison below measures. A DFM absorbs it with a vector autoregression on the factors (Doz et al., 2011), letting slow-moving regime shifts enter as latent states (Jordà et al., 2017); an l_1 penalty on the loadings further limits the bias that dense loadings induce in short samples (Uematsu and Yamagata, 2023) and aids interpretability (Despois and Doz, 2023; Erichson et al., 2020). All three are linear, so none captures nonlinearity; the state dependence at issue here is tested separately, on the smooth-transition specification described above.

Diagnostics statistics favoured Sparse DFM over DFM and PCA (Appendix E). PCA is rejected across all countries, with inferior log-likelihood values, often differing by several orders of magnitude relative to both DFM and Sparse DFM (Austria: -5.19×10^4 vs. -730; France: -1.04×10^6 vs. -634; Belgium: -2.01×10^5 vs. -681; USA: -1.83×10^6 vs. -732). Their BIC values are larger by more than an order of magnitude everywhere, running from 1.04×10^5 in Austria to 4.80×10^{11} in Ireland, against values between 1,719 and 2,270 for the two dynamic specifications. These results reflect PCA's inability to capture the dynamic and serial dependence structure of the data. These figures, and those of the three paragraphs that follow, are computed on the seventeen country-by-country fits set out earlier in this section, one system of seven series per country, and they are reported for one purpose only:

to record how the estimator was chosen. No result in Section 6 rests on them, and the seventeen tables that carry them are reported in Appendix E, on the descriptive footing set out above. The identification objection raised above bears on what those fits may be used to claim about countries, not on which of the three estimators fits these series better: the number of factors is held at six across all three specifications in the comparison, so it is differenced out of it.

The Bayesian Information Criterion (BIC) is lower for the Sparse DFM than for the standard DFM in sixteen of the seventeen countries (e.g., Ireland: 1889.7 vs. 1968.6; Spain: 1996.0 vs. 2064.4; UK: 2116.8 vs. 2188.1), by as little as 31.54 in Austria (2122.1 vs. 2153.6); in Greece it is higher, by 8.08 (2232.8 vs. 2224.7). The dense specification uses 106 parameters and 42 non-zero loadings in every country; the sparse specification uses fewer in all seventeen, from 93 parameters and 29 non-zero loadings in Ireland to 102 and 38 in Greece. The additional parameters of the dense model buy little fit: its log-likelihood exceeds the sparse model's by less than 5 in thirteen countries, and by 5.1, 9.3, 11.7 and 17.1 in Spain, the United States, Italy and Greece, the last the one country in which the BIC favours the dense specification.

Likelihood ratio tests of the DFM against the Sparse DFM are not reported, because they are not valid tests here. The sparse fit solves the penalised problem (30), which shrinks the loadings it retains as well as setting others to zero, so it is not the dense model's maximum-likelihood fit under a set of zero restrictions; and the zero pattern it imposes is selected from the same data. Twice the difference in log-likelihoods therefore has no chi-square reference distribution, and we report no p-values from it. The comparison that does support the sparse specification is the BIC comparison reported above, which weighs the small loss of fit against the parameters saved.

Residual and stability diagnostics do not favor more parameters of standard DFM. Average Ljung–Box p-values lie between 0.000 and 0.068 under both specifications (e.g., Austria: 0.023 vs. 0.023; Belgium: 0.004 vs. 0.014; Spain: 0.001 vs. 0.001). At the five per cent level these values *reject* the null of no residual autocorrelation in sixteen of the seventeen countries — Slovakia, at 0.065, is the single exception — so serial dependence is not fully absorbed by the factor dynamics in either model. A VAR(1) on the factors with independent idiosyncratic errors is too restrictive for these series, and the differencing applied to some of them will itself induce residual correlation. The two specifications fail this diagnostic to a comparable degree, so it does not discriminate between them, but neither passes it. Likewise, Breusch–Godfrey average p-values are consistently moderate to high – generally between 0.242 (Luxembourg) and 0.669 (Austria), similar for DFM and Sparse DFM – suggesting no systematic differences in remaining serial correlation. Regarding stability, the

spectral radius remains strictly below unity in all cases, ranging, for the sparse specification, from 0.476 (Spain) to 0.855 (Finland), with the Sparse DFM often exhibiting equal or similar values to DFM, confirming stable dynamics.

The sparse dynamic factor model is therefore retained as the baseline specification.

6 Empirical Analysis

The estimates below are organised around the three questions that Section 1 sets out as information and risk diagnostics rather than as conditions for the adequacy of any rule: whether a specified measure of government financial assets improves a specified forecast of $r - g$ beyond a baseline of gross debt and other fiscal and macroeconomic variables, whether comparable series have equal exposure to common variation across countries or equal coefficients on the estimated factors, and whether the estimated common variation is symmetric, which is described without a verdict. Section 6.1 takes the first, Section 6.2 the second and Section 6.3 the third; Section 6.4 then tests the restriction the model of Section 3 places on itself, and Section 6.5 reports the seventeen country-level fits. The third is reported as a statement about the third moments of the estimated common space and not as a rejection: the only reference distribution available for those moments is one for independent draws, which these scores are not, so Section 6.3 attaches no probability to them and does not read them as evidence against symmetry. The first is addressed both as a claim about co-movement and, through a nested forecast comparison fixed before it was run, as a claim about information (Section 6.1). On the second, equal exposure is not rejected by the composite rule, at $p = 0.23$, though nineteen of the twenty-two cells of the rebuilt exercise and all seven of the coefficient tests reject; the rule is a maximum over cells, so the three that do not reject carry the verdict. That is a failure to reject at the five per cent level the rule fixes and not evidence that the seventeen loading blocks are common; with nineteen of twenty-two cells against it, the composite does not support the reading in the other direction either, and what the paper says about heterogeneity rests on the coefficient tests. The form of the rule, the largest p-value over the cells, was fixed before any cell of the identifying specification was computed; the null generator it is evaluated under was rebuilt only after the first cells were known, because the original generator itself carried loading dispersion; and the restriction is not rejected under either generator ($p = 0.28$ under the original).

Interpretation of estimated factor structures centres on whether $(r - g)$ and ΔFA co-move through the estimated common space, measured by quantities that do not depend on the sign or the order of the factors. This structure makes the

empirical narrative transparent: no factor is named, and the configuration labels of Table H1 name sign patterns of individual loadings and scores under the magnitude conventions set out below; they are descriptive, and no economic regime or function is inferred from them, because an individual loading and its sign depend on the basis the estimator returned rather than on the data.

Economic interpretation of results was restricted to loadings above an interpretation threshold ($|loadings| > 0.3$). This threshold, and the score threshold introduced below, are magnitude conventions rather than statistical tests. No interval is reported for these loadings, which come from the seventeen country-level fits that Appendix D reports as descriptive summaries rather than as identified structures, for the identification reason set out in Section 5; and none has been computed on the specification the paper identifies from. The configurations below are accordingly a taxonomy of estimated co-movement under a stated convention, not a set of separately established coefficients. Applied macroeconomics typically interprets fluctuations at annual frequency, as is the case of the business-cycle literature, identifying expansions and contractions as multi-quarter phenomena. A minimum persistence window of four consecutive quarters would be the natural convention in that literature, ensuring that regimes reflect medium-term cyclical dynamics rather than high-frequency transitory noise. We do not apply it here, because government financial assets – comprising equity injections, recapitalizations, nationalizations, loans – exhibit a non-continuous, event-driven structure. Unlike primary budget flows, which evolve continuously, financial assets positions are characterized by sporadic, high-magnitude shifts, that do not occur on a continuous basis, hence we account for $|\hat{F}| > 0.9$ at any quarter. This approach keeps excursions of the scores lasting fewer than four quarters, which a persistence window would discard, although it does not identify any of them as a discrete intervention. Magnitudes below these thresholds are not interpreted. Table H2 and its continuation, in Appendix H, list every quarter meeting these thresholds and are generated directly from the estimated factor scores of the seventeen country-level systems, not of the pooled specification reported in Section 6.1, so the quarters reported there are exhaustive rather than illustrative. That tabulation is reported as a descriptive summary of the country-level fits, which place six factors on seven series, and no claim in this section rests on it; the identification limit is set out in Section 5. Appendices D and E and Table H2 are computed on the database this paper adopts, and so are the classifications of Appendix G, although none of those classifications rests on the fits behind Table H2: the country-by-country column of that appendix rests on a separate re-estimate of the seventeen country-level systems, and its two pooled columns on a four-factor fit of all 119 series over 2007Q4–2024Q4, which is not the pooled specification reported in Section 6.1. Individual loadings of the country-level fits should not be read as coefficients at all: an individual loading

of a system that places six factors on seven series is a property of the rotation the estimator returned and not of the data, which is the same reason no factor is named anywhere in this paper. What survives the rotation is set out in Section 6.5: which series the estimator leaves with no common component at all, the correlation between the fitted common components of two series, and, under an orthogonal rotation only, each series' squared-loading norm. The country material is offered on that basis alone.

As a disclaimer, empirical results must be interpreted with caution, since low communalities constrain estimates explanatory power, reflecting exogenous dynamics not captured by the data structure — and, as Section 5 notes, the country-level communalities reported in Section 6.5 are squared-loading norms, which are shares of variance only when the estimated factors are orthonormal, and these are not. Other latent drivers, such as institutions, credibility, financial markets or geopolitical conditions, may underlie the estimated common space; whether they do is not tested here, and no factor is read as any of them.

6.1 The information in the liability side

The first question is whether a specified measure of government financial assets improves a specified forecast of $r - g$ beyond a baseline of gross debt and other fiscal and macroeconomic variables, which is to ask whether the liability side alone carries the information about $r - g$ that such a forecast can use. What this section measures first is weaker than that question, and the gap matters. The absence of a systematic common component between government financial assets and the interest–growth differential would not make the presumption of a gross-debt rule safe, and its presence would not overturn it. Assets can co-move with $r - g$ and still add nothing once debt is observed: if both load on debt, their correlation is positive while the mean of $r - g$ conditional on debt and assets equals the mean conditional on debt alone. Nor is co-movement necessary for the liability side to be insufficient: were the differential to depend on the square of a centred asset position independent of debt, assets would predict it with no linear co-movement at all, and information specific to one country need not load on a common factor. What we report is therefore the strength of that co-movement, which is what this design measures, rather than a verdict on the informational claim, which the next paragraph takes to the data directly. For each country we take the smaller of the two absolute correlations that $r - g$ and ΔFA have with a pooled factor, evaluated on the factor that makes that smaller value as large as possible, and call it ρ_i; the statistic is scale-free, invariant to the sign and scale of the factors, and defined for every country. On the specification the paper identifies from, its distribution across the seventeen has a median of 0.208, a mean of 0.235 and a range from 0.086 in Austria to 0.669 in the Netherlands. No interval

for ρ_i has been computed on that specification. Where seventeen point estimates fall relative to a convention is in any case a summary of this distribution rather than a quantity these data resolve, and the sampling uncertainty of the distribution just reported is not quantified anywhere in this paper.

The comparison reported next was fixed before it was run, and it was run on the conventional differential, $r - g$ measured with growth taken on single-quarter GDP, on which it is reported here. The informational claim is examined by a nested forecast comparison whose design, sample, test and reading were fixed before it was run and are set out in Appendix J. Adding government financial assets to a forecast of the interest–growth differential a year ahead that already uses gross debt, its change, the primary balance and the current interest and growth rates lowers its mean squared error by 0.1 per cent over forecasts made each quarter from 2010Q1 to 2018Q4 on final-vintage data, a gain not distinguishable from what estimation noise produces when assets carry no information (Clark–West $t = 0.30$, one-sided $p = 0.389$ against Student-t with four degrees of freedom). This comparison therefore gives no evidence that the liability side omits information about the next year's $r - g$. The test detects, with probability 0.80 at its five per cent level, only asset information that moves the projected contribution of $r - g$ to the next year's change in the debt ratio by 4.38 points of GDP or more in root-mean-square terms in the least favourable of three simulated forms — the asset level, the asset flow, and the square of the asset flow with no linear component — which is more than the 0.5-point minimum annual reduction required by the debt sustainability safeguard of Regulation (EU) 2024/1263, so contributions of the size that safeguard treats as a minimum are neither shown nor excluded. The comparison is pseudo-out-of-sample, on final-vintage data that earlier stages of this project had already examined in other specifications, including exploratory forecasts with the asset side added, run while the design was being chosen (Appendix J), and it concerns prediction: it says nothing about the effect of acquiring or selling assets or about which fiscal rule is better. Its design, and the sentence reported here for each possible outcome, were fixed before it was run (Appendix J); the nineteen secondary analyses specified with it are reported in Table J1 whatever they show, and none of them changed the classification of the result.

A second comparison, NF2, takes the same question to the consistent differential of Section 4, with its own benchmarks and with fiscal and output data lagged one quarter; its design and tables are in Appendix K. In a pre-specified pseudo-out-of-sample comparison, with forecasts made each quarter from 2010Q1 to 2018Q4 for the seventeen countries on final-vintage data that earlier stages of this project had examined in other specifications, and with fiscal and output data lagged one quarter, adding the government's financial-asset stock and its change to a forecast of the

interest–growth differential a year ahead that already uses the current differential and country effects makes its mean squared error 17.9 per cent higher (Clark–West $t = -0.28$, one-sided $p = 0.604$ against Student-t with four degrees of freedom), and the size of the asset information in the projected contribution of the differential to the next year's change in the debt ratio, net of estimation noise and to first order, is unresolved between 0.00 and 0.79 points of GDP in root-mean-square terms (90 per cent interval; point estimate 0.00), a range that contains the minimum average annual reduction of 0.5 points of GDP that the debt sustainability safeguard of Regulation (EU) 2024/1263 requires while debt is between sixty and ninety per cent of GDP. Relative to the no-change forecast, the mean squared prediction errors are 0.656 for the pooled autoregression with country effects, 0.899 for the debt-and-macro model, and 0.773 and 0.916 when the asset block is added to each (Table K2).

On the identifying specification no country is resolved as neutral — an asset side managed independently of the differential — because the design identifies co-movement, not the effect or the purpose of an asset position, and even on co-movement none falls below a cut by anything stronger than a point estimate, since no interval for ρ_i has been computed there. What this specification does report is a stronger statement: in two of the seventeen the estimator leaves the change in financial assets with no common component at all, which separates the asset side from the differential without a threshold having to be chosen. That separation is a statement about co-movement only: for the reason given above, it does not make the presumption of a gross-debt rule safe in those two countries, since information specific to one country, or carried through a nonlinear channel, need not load on a common factor. On the country-by-country design nothing is separated, because there sixteen of the seventeen values of ρ_i lie above the cut and the seventeenth, France's, at 0.295, just below it.

Read as a summary of the distribution of ρ_i, the count of countries above the 0.3 cut is too unstable to be the headline, and the identifying specification makes that plain. On it, two of the seventeen countries have $\rho_i > 0.3$ — Finland at 0.520, with opposite signs on the two series, and the Netherlands at 0.669, with the same sign — against twelve on the full-sample companion and seven when the pandemic quarters are treated as missing, at three factors and at four. Nor is the sample the only thing it turns on. Holding the factor space fixed and changing only the idiosyncratic error, the same count is two under the independent specification and three under the AR(1) one; and at the penalty the AR(1) criterion selects it runs 3, 5, 2, 4 and 5 across five expectation-maximisation starting points. A quantity that moves that far under changes which leave the fitted common component essentially alone should not be read as a finding, so we report the distribution beneath it and a rotation-invariant companion in its place. That companion is the correlation between

the fitted common components of the two series, which is unaffected by any rotation of the factor block and therefore measures the co-movement rather than a labelled factor's share of it. On the identifying specification it exceeds 0.3 in absolute value in eight countries, Finland at -0.597, Latvia at 0.732, the Netherlands at 1.000, Portugal at 0.498, Slovenia at 0.586, Spain at 1.000, the United Kingdom at 0.999 and the United States at 0.863; across the fifteen countries in which it is defined its median absolute value is 0.498 and its signed median is 0.252; and it is undefined in two, Germany and Greece, because at the selected penalty the estimator gives at least one of the two series no common component at all in those countries: the change in financial assets in Germany, and both series in Greece. That is a stronger statement than a weak loading, and it is the one this specification supports: on the space the paper identifies from, the estimator leaves the financial-asset series with no common component whatever in two of the seventeen countries. Several of the fifteen values that are defined say less about the country than their number suggests. In the Netherlands and Spain the estimator gives each of the two series a loading on the first factor and on no other, so the measure there is one by construction. In Italy it places $r - g$ on the first factor and the change in financial assets on the third and gives neither series a loading on the other's, so the measure there equals the sample correlation between those two pooled factors, 0.104; in Belgium, France and Slovakia, too, the two series load on no factor in common, and the measure is set by the correlations between the pooled factors and the weights the two series place on them. In the other eleven countries the two series load on at least one factor in common, and only there can the measure reflect co-movement particular to the country. No bootstrap interval for the rotation-invariant measure is computed on this specification, so the figures above are point estimates of a distribution and are not offered as a tested difference. Estimated country by country, with six factors fitted to each country's seven series, sixteen of the seventeen co-load on the scale-free correlation rule, France falling below it at 0.295, and fifteen on the raw $|\lambda| > 0.3$ rule, France and Latvia falling below it at 0.209 and 0.264; but those fits run on each country's own full sample rather than on the identifying window, and six factors on seven series leaves them close to saturated, the squared-loading norm of each country's seven series sitting close to one, against a mean reconstruction R^2 of 0.320 pooled. The two distributions are not on a common scale, the two counts are not comparable, and the sixteen is not evidence that the pooled figure is too low.

The identifying specification of Section 5 extracts three factors from the 119 series. Table F1 in Appendix F reports what loads on each, and what follows is a profile and not a set of names: the rotation that would license an economic label for an individual pooled factor is not identified, so none is given. The first factor is the broadest, active in 53 series across all seventeen countries; ranked by mean

absolute correlation across the seventeen country series of each variable, the variables closest to it are $r - g$ and gross capital formation, at 0.70 for the leading one, and of its active series fifteen are $r - g$, thirteen are primary balances and eleven are gross capital formation. The second is active in 42 series across sixteen countries and is led by gross capital formation together with the primary balance, at 0.34, with eleven ten-year yields, eleven gross capital formation series and eight primary balances among those it reaches. The third is the narrowest, active in 34 series across fifteen countries, and is led by the same two variables, at 0.31, with nine gross capital formation series, six primary balances and five each of changes in gross debt and in financial assets. Figure F1 plots the three score paths. Taken jointly the three account for 29.2 per cent of total variance. Taken one at a time their marginal shares are 16.1, 18.6 and 16.8 per cent, which sum to 51.4 per cent; the sum exceeds the joint share by a wider margin than in the four-factor companion, 47.9 against 36.9 per cent, which says that at the selected penalty the three estimated factors are far from mutually orthogonal in this sample and that the marginal shares double-count much of the explained variance between them. The joint figure, 29.2 per cent, is the quantity to compare with the series-by-series variance decomposition of Table F3.

What that common component explains, and what it does not, bears on the restriction under test, and on the identifying specification it does not divide the system the way the four-factor companion did. Averaged across countries, the three factors account for 58 per cent of the variance of the ten-year yield and 64 per cent of the variance of $r - g$; then, next in order, 30 per cent of gross capital formation; then the three fiscal flows, at 26 per cent of the primary balance, 22 per cent of the change in gross debt and 14 per cent of the change in financial assets; and last the current account balance, at 11 per cent (Table F3). The two rate variables therefore lead the common component by a wide margin, but the variable that comes next is a real one. Capital formation sits above all three fiscal flows, and the reading — that the common macro-financial component is a financial and fiscal phenomenon while the real-side variables are largely idiosyncratic to each country — is false on this specification. Only the current account balance behaves as that reading claimed, and it is the lowest share in the system. The dispersion beneath these averages is wide enough to matter and is reported with them: the same three factors explain none of $r - g$ at all in Greece and 86 per cent of it in France, and the country ranges are of that order on every one of the seven variables, so a mean share is a summary of a distribution and not a property any individual country has. What the argument of this paragraph needs, however, is not the ordering but the overlap, and the overlap is untouched by the reordering: the factors on which the asset side loads, in the countries where it has a common component at all, are ones on which $r - g$ or the yield also loads in the pooled system, though not always in the same country.

That is a sharper statement than the country-level fits can support, because there the corresponding quantities are not measured on a common scale, and, like every co-movement statement in this section, it does not show that the asset side adds information about $r - g$ once debt is observed.

One point of measurement deserves emphasis before the counts below are read. The 0.3 loading threshold stated at the head of this section cannot be carried over to the pooled matrix, and the reason is dimensional rather than empirical. Because loading columns are normalised, an equal-share loading is $1/\sqrt{N}$, which is 0.378 at $N = 7$ but 0.092 at $N = 119$, so a rule calibrated on the first is close to unreachable in the second: applied literally to the pooled system, the $|\lambda| > 0.3$ rule finds financial assets and $r - g$ on a common factor in none of the seventeen countries. Rescaling the threshold by the square root of the ratio of the two dimensions, to $0.3\sqrt{7/119} = 0.073$, finds it in two. The correlation rule, which is what the pooled results above use, finds $\rho_i > 0.3$ in two. Three conventions applied to one fit return zero, two and two, and the spread between them is the point rather than any one of the counts. We do not report the Euclidean norms of the fitted loading columns here. The pooled results above use the correlation between a series and a factor, which is invariant to how the factors are scaled and does not depend on the normalisation at all.

The configurations below are defined by patterns in the loadings and scores of financial assets and $r - g$, and their labels name those patterns rather than report functions. They are not evidence of a fiscal reaction of the kind estimated for the primary balance by Bohn (1998), nor of financial assets acting as stabilisers, buffers or strategic holdings: the design identifies co-movement, not the effect or the purpose of an asset position (Section 5). Whether correlated or orthogonal to $r - g$, financial assets cluster along the configurations (Table H1): One caution applies to everything that follows. The classification below rests on two fixed cut-offs, $|\lambda| > 0.3$ on loadings and $|\hat{F}| > 0.9$ on scores, and it is more sensitive to those conventions than to the data. Neither cut-off is scale-free. The loading threshold is dimension-specific: because loading columns are normalised, an equal-share loading is $1/\sqrt{N}$, which is 0.378 for the seven-variable systems estimated here but 0.092 in a pooled system of 119 series, so the same numerical rule is permissive in one and unreachable in the other – applied literally to a pooled specification it places all seventeen countries in the no-association category. The score threshold is not scale-free either, because a cut stated in raw score units is a different cut for every factor whose scores have a different dispersion. How far either effect moves the classification is not quantified here: the country-level fits on which it could be measured fall under the identification reason set out in Section 5, and on the pooled specification it cannot be measured on the same terms, because a pooled system of 119 series has no country-specific factor for either cut to be applied to, only factors common to all seventeen countries. The comparison of

the resulting labels across criteria is tabulated country by country in Appendix G as a descriptive summary carrying the same limit, with the first criterion applied to a separate re-estimate of the seventeen country-level systems and the other two to a four-factor pooled fit of all 119 series that is not the pooled specification: only three of the seventeen, Spain, the United Kingdom and the United States, hold the same label under all three criteria, and only two of the five configurations that appendix distinguishes, crisis liquidity provision and co-movement with capital formation or the current account balance, retain any members under all three. The configurations should therefore be read as a descriptive organisation of the estimates under a stated convention, not as a partition the data impose, and how much the convention carries is not established in this paper.

• **Economic growth:** when financial assets co-move with declining $r - g$ ($\lambda_{f,i}^{r-g} < -0.3$) and scores are positively activated ($\hat{F} > 0.9$); or the symmetric sign configuration.

• **Subdued growth:** when financial assets co-load with rising $r - g$ ($\lambda_{f,i}^{r-g} > 0.3$) with scores activation ($\hat{F} > 0.9$); or the symmetric sign configuration.

• **Orthogonal regimes:** when financial assets load on orthogonal factors relative to $r - g$. Throughout this section a variable is called *orthogonal* when it loads on a factor that the other series do not reach above the interpretation threshold. The term describes the factor's separation from the rest of the system, not the size of the variable's own loading, which is typically large precisely because the factor carries little else.

Two countries invite a reading of decoupling between sovereign financial assets and $r - g$, and the estimates do not settle it. On the specification the paper identifies from, Austria's ρ_i is 0.086, the lowest of the seventeen, and France's is 0.124, below the median of 0.208; both fall below the 0.3 cut. Neither is separated from the differential in the way the two countries reported above are, whose asset series the estimator leaves with no common component at all: on both, each of the two series retains a fitted common component and correlates with it. The country-level fits cannot settle the question, for the identification reason set out in Section 5; what remains is two low values of a statistic read against a conventional cut, which is why the classification is reported as a convention rather than as a partition the data impose. What separates the two countries is institutional rather than statistical. Austrian state-held assets are embedded within a broader strategic economic coordination and public corporate governance, functioning as instruments of economic management, particularly in energy and infrastructure; France holds one of the largest public financial asset portfolios among advanced economies, serving objectives of industrial sovereignty, technological autonomy, long-term capitalization and strategic sectoral coordination characteristic of mixed State and market economies. Those are statements about

what the asset side is for. They do not carry the further implication that the asset side evolves independently of $r - g$. Nor does anything here bear on Bohn (1998) in either direction: what that paper establishes is the response of the primary balance to the debt ratio, which is a different object from the coupling of the asset side to $r - g$.

6.1.1 Co-movement of financial assets and $r - g$

In two countries, sovereign financial assets and interest-growth differentials have absolute correlations above 0.3 with the same pooled factor ($\rho_i > 0.3$) on the specification the paper identifies from, one of them with opposite signs on the two series, but that count is too unstable to be the headline. The two are Finland, the one with opposite signs, and the Netherlands, and the figure is the one already reported in Section 6.1 rather than a second count of its own; across the samples, error specifications and starting points examined there it runs from two to twelve.

Whether periods of economic growth in which $r < g$ were used to increase financial assets, the configuration analysed by Blanchard (2019), to build permanent fiscal capacities as stressed by Bénassy-Quéré and Borgy (2026), is not established here, and no cross-country episode list is stated. Any such list would rest on the episode lists of fits that place six factors on seven series, whose dated extremes are properties of the basis the estimator returned rather than of the data (Section 6.5); that material is tabulated only as description, in Appendices D and H.

Asset expansions while $r - g$ is rising, through purchases of distressed securities, recapitalisations of financial institutions or credit to enterprises, and drawdowns of assets accumulated earlier in place of additional borrowing, are the forms a stabilising role of financial assets would take. Whether financial assets played that role in this panel is not established here: the design identifies co-movement, not the effect or the purpose of an asset position (Section 5), and the country-level fits on which any dating of such episodes would rest are identified only up to a rotation of their factor block (Section 6.5).

Positive co-movements of financial assets and debt are to be expected as a structural signature of the canonical sovereign balance sheet expansion/contraction cycles, observed during systemic crises. The estimates here do not say how widespread they are: no count of the economies whose financial assets and change in gross debt reach a common factor together is reported on the specification the paper identifies from, and the country-level fits cannot supply one, for the identification reason set out in Section 5.

Negative coupling of financial assets and $r - g$ appears in one of the seventeen countries on the specification the paper identifies from, Finland. Negative coupling is the narrower configuration in which $r - g$ and ΔFA attach to the same factor

with *opposite* signs, so that asset accumulation accompanies a compression of the differential rather than merely sharing a factor with it; it is therefore a subset of the countries whose two series have absolute correlations above 0.3 with the same factor, and on this specification, of the two that do, Finland carries opposite signs on the two series, at 0.520, and the Netherlands the same sign. Two further countries, France and Luxembourg, carry opposite signs without sharing a factor by the criterion used here, the smaller of their two absolute correlations being 0.124 and 0.197 against a cut of 0.3; a configuration resting on correlations of that size is not a finding, and nothing is rested on it. The configuration this sentence describes is therefore present in one country on the specification the paper identifies from, on a count shown in Section 6.1 to be too unstable to be the headline,, and no claim about the asset side supporting growth is rested on it.

These results report the strength of the co-movement between financial assets and $r - g$. The country-level fits from which the relation of financial assets to gross debt, capital formation and the current account balance could be read are tabulated in Appendix D as description only, and no reading in this section rests on them, for the identification reason set out in Section 5.

A positive coupling between assets and debt is what crisis interventions financed by borrowing would produce. A count of the economies showing it would be taken on the scale-free criterion used elsewhere in this section — an absolute correlation above 0.3 with the same factor, of the same sign, for both ΔFA and the change in gross debt — but it is not reported on the specification the paper identifies from, and no direction is asserted for it here. The country-level version of the count is not stated either, for the identification reason set out in Section 5, although the country-level fits themselves are reported as descriptive summaries in Section 6.5 and Appendix D. Asset buffers built during a crisis may also be reversed swiftly, a possibility these estimates do not measure, leaving gross debt elevated and the balance sheet net worth potentially impaired if assets are sold at a loss. By construction, a gross-debt measure records a debt-financed acquisition of financial assets as a rise in debt whatever the acquisition does to net worth, and a joint contraction of assets and debt as a fall; that is a property of the measure, and whether it matters for sustainability is the informational question taken up in Section 6.1.

A balance-sheet reading would record both sides of these crisis expansions and of their unwinding afterwards, so that an expansion matched by assets of unchanged value is not read as a deterioration of the fiscal position, which removes the penalty gross-debt measures place on asset accumulation. Whether net worth is in fact unchanged turns on the realisability of those assets, which Section 6.6 measures descriptively and does not establish. Consequently, the focus of analysis would shift to the full trinity of debt, deficit and financial assets. Only by jointly examining all

three can the nature of fiscal adjustments may assess whether asset accumulation represents a genuine strengthening of the public balance sheet or merely a debt financed portfolio expansion with no net effects on solvency and economic growth.

Balance-sheet approaches estimate assets as primarily the present discounted value of taxes, alongside other financial and non-financial holdings, implying that traditional debt sustainability analyses can radically mis–classify countries when the financial assets side is ignored (Levy Yeyati and Sturzenegger, 2023).

However, the possibility of an abrupt unwinding after crisis events, of financial assets, debt and primary deficit consolidations, cautions that a net worth anchor must be accompanied by stress testing of assets valuations and liquidity, precisely because the reversal can be sharp and damaging. A public sector net worth anchor would capture the full cycle; whether it is a more accurate fiscal policy guide is not settled here, and Section 6.6 reports only which constraint a net-financial-liability analogue would have bound on.

Financial markets already price sovereign risk on a net-worth basis, total financial and non-financial assets less liabilities (Peppel-Srebrny, 2024), with government assets and liabilities significantly determining borrowing costs. The country-specific fiscal factors driving sovereign bond yields represent a market based sovereign bonds net worth. This has direct consequences for our estimates interpretation: any joint expansion of assets and debt that left sovereign interest rates unchanged would be better explained by net-worth pricing, whereas on the contrary gross-debt surveillance would signal a deterioration.

Balance-sheet approaches alter fiscal sustainability assessments (Levy Yeyati and Sturzenegger, 2023), with net-worth fiscal anchors outperforming debt-based anchors in theory and simulations (Chai et al., 2024). Blanchard (2019) does not take up the asset side directly, but the argument there turns on the return to what borrowing finances relative to r, which bears on the asset side by implication. Governments can borrow and invest in assets that yield at least the economic growth rate, generating a fiscal dividend that improves net worth, even without primary surpluses. The two-sided perspective implies governments are solvent if the present value of all its assets, not merely the fiscal capacity to tax, exceeds the present value of its liabilities. The corollary–that the composition of government assets matters for whether $r - g$ is favourable, translates into sustainable debt paths – as a foundation of a net-worth perspective of sustainability, not only on gross debt and primary balances–, depending on the whole balance sheet net position. Observing only liabilities risks misdiagnosing solvency when governments hold substantial financial assets (Levy Yeyati and Sturzenegger, 2023). Seiferling and Tareq (2023) argued that the widespread reliance on gross government debt and deficit/surplus has often neglected the role played by other important parts the balance sheet, including

portfolios of financial assets. Further, they find that transparent governments earn approximately 7% higher returns on their equity portfolios than opaque governments, and that governments act as investors of last resort during financial crises. Their work claims that instead of keeping risks hidden, transparency and governance are conditions for welfare-enhancing.

6.2 Is transmission common across countries?

The pooled design permits a test the country-by-country design cannot frame at all. Estimating each country separately makes cross-country heterogeneity true by construction, so it can never be evidence; estimated jointly, the hypothesis that all seventeen countries share a common loading block becomes a restriction on the pooled model. The reason is comparability. Seventeen separate models carry seventeen arbitrary rotations and seventeen score scalings, so a loading estimated for one country is not measured on the same footing as one estimated for another, and the cross-country comparisons this paper draws are not well defined under that design. Pooling places the country-specific blocks in a common factor space, which is what allows their equality to be stated as a restriction at all. Restrictions on factor loadings are themselves tested in the factor-model literature, by Breitung and Eickmeier (2011), Chen et al. (2014) and Han and Inoue (2015) among others; those procedures test instability over time rather than equality across countries, so the test used here is specific to this study and not an application of a standard one. We test it by simulating panels in which every country's loading block is the same, holding fixed the sample length and the pattern of missing quarters, re-running the identical estimation on each simulated panel, and comparing the observed dispersion of the country blocks with the null distribution so obtained. Each simulated series is written on the standardised scale directly, so that the blocks are equal after the standardisation the estimation applies, which is the form in which (23) states the restriction (Section 5). Two things the restriction leaves open are handled explicitly. The first is the behaviour of the disturbances, and the null is calibrated in two ways. One is Gaussian. The other, the dependence-respecting calibration, abandons the Gaussian draw: it resamples the fitted idiosyncratic residuals and the fitted innovations of the factor transition in moving blocks of eight quarters, on a block index common to the whole panel so that contemporaneous cross-country dependence is carried across intact, rescales the resampled innovations so that the simulated factors keep the model's own stationary variances, and re-imposes the original pattern of missing quarters on every simulated panel. Nothing in that calibration rests on the idiosyncratic covariance being diagonal or on its disturbances being Gaussian white noise, which is why it is there. The second is the size of the common block, which (23) does not identify, since equality holds whatever it is, so that plugging

in one value would choose the answer. It is swept instead over a grid of multiples κ of the restricted estimate of the common block, from 0.60 to 1.50; a 99 per cent confidence set for κ is formed from a rotation-invariant matching statistic, and the composite p-value is the supremum of the cell probabilities over that set plus the set's own level of 0.01, following Berger and Boos (1994). Two statistics enter the composite, the two that are free of the scale of the loading matrix, of which only the second, the dispersion across countries of the common covariance each block implies, is also free of its rotation; and the restriction is recorded as rejected only if the composite falls below five per cent. The form of that rule, a maximum over cells read at five per cent, predates both runs reported here, and it was left unchanged when, on an earlier run, it first failed to reject. The null it is evaluated under was rebuilt after the first run, for the reason Section 5 sets out: the original generator imposed the common block in the series' original units and standardised afterwards, so the null itself carried loading dispersion. The rebuild also made the size of the common block a nuisance parameter swept in this way, dropped a second Gaussian calibration that rescaled the idiosyncratic variances, which the standardisation makes void, and added the rescaling of the resampled factor innovations, which in the first run had inflated the simulated factor variance above the model's stationary value. The grid of κ first began at 0.80; in the rebuilt run the confidence set reached that end under both calibrations, so the grid was extended downward to 0.60, which could not lower the composite and on the panel reported here does not bear on it, the confidence set reaching no lower than 0.80 under either calibration. The first run did not reject either, at $p = 0.28$; Appendix F reports it.

On 42,000 draws, every one of which converged, the composite is $p = 0.23$, and the restriction is recorded as not rejected. The supremum is attained by the correlation statistic under the dependence-respecting calibration at $\kappa = 0.80$, the lowest level the confidence set admits. The average of the fitted country blocks sits at $\kappa = 0.93$ and the level that matches the panel's signal-to-noise ratio at 1.24, so both are inside what is swept.

The cell counts point the other way from the verdict. Nineteen of the twenty-two cells reject at five per cent. The common-covariance statistic rejects in all eleven, at probabilities between 0.010 and 0.018 under the dependence-respecting calibration and at or below 0.001 under the Gaussian one. The correlation statistic rejects in all five Gaussian cells, at or below 0.001. The three cells that do not reject are the correlation statistic under the dependence-respecting calibration at the three lowest levels the confidence set admits, where its probabilities run 0.221, 0.124 and 0.081 before falling to 0.044, 0.018 and 0.004 as the common block is enlarged. Because the rule is a maximum, those three carry the verdict, and they carry it because the dependence-respecting calibration widens the null distribution of both statistics: at

$\kappa = 1$ the 95 per cent cutoff of the correlation statistic is 21.75 under it against 5.81 under the Gaussian calibration, and that of the common-covariance statistic 14.64 against 4.99. That is the rule working as it was designed to: it was fixed to be strict, and a strict rule that fails while nineteen of its twenty-two cells reject is reporting its own strictness, not the homogeneity of the seventeen blocks. Neither reading is available from it. What the paper claims about heterogeneity rests on the coefficient tests, which reject on all seven variables, and the composite is reported as the rule defines it.

Wald tests of equality across the seventeen blocks, with a Newey–West correction at three lags and a moving-block bootstrap of 2,000 draws at block length eight, reject for all seven variables after a Holm correction, each at the same Holm-adjusted $p = 0.0035$: $r - g$, the interest rate, the current account balance, gross capital formation, the change in gross debt, the primary balance and the change in financial assets. The largest statistic is 581 for the interest rate on 48 degrees of freedom and the smallest 183 for the primary balance, against a bootstrap null mean of between 71 and 124 across the seven. On the full-sample companion the same test also rejects for all seven, the current account balance and gross capital formation least strongly, at Holm-adjusted $p = 0.027$ and $p = 0.010$. Appendix F reports the full test. The parametric bootstrap supplies the finite-sample reference distribution for the dispersion statistics and the moving-block bootstrap the one for the coefficient tests, following Horowitz (2001) and Künsch (1989) respectively. Failing a deliberately conservative rule is not the same as finding that the seventeen loading blocks are common, and this paper claims neither. What it claims is narrower than a rejection and broader than a finding of homogeneity: the evidence converges on cross-country differences in fiscal-financial transmission, while the composite criterion for declaring them established is not met.

A null is not a result until the reader is told what the test could have detected, so the size and power of the rule were measured. Its rejection frequency under the null was taken from the 42,000 draws already on disk, with no further estimation: each draw in turn is treated as the observation and run through the rule against the rest of the bank, leaving itself out. Fed the actual observation the same harness returns the reported supremum, cell and confidence sets to every digit, which is what licenses reading it as the rule. The composite rejects between 0.000 and 0.021 of the time across the eighteen null points, 0.0041 pooled over all of them and 0.0021 over the eleven inside the observed confidence set, against the nominal 0.05 at which it is read. Its power was then measured on 7,920 fresh panels in which the seventeen blocks are moved s times their fitted deviation from the common one, the rule being applied against the same frozen bank; at $s = 0$ the rejection frequency reproduces the leave-one-out frequency at every point, which is the check that the two exercises

are of one rule. At $s = 0.70$, close to the scale an attenuation correction on the original generator implied, power is 0.000 under the Gaussian calibration and at most 0.010 under the dependence-respecting one, on 400 panels apiece. At $s = 1$, the fitted dispersion itself, it runs from 0.000 to 0.083. Under the dependence-respecting calibration it first exceeds a half at $s = 1.35$, the top of the grid, at 0.525, where the exact interval is $[0.475, 0.575]$ and excludes eighty per cent; at that multiple panels can be drawn at $\kappa = 0.70$ but not at $\kappa = 0.90$, where the implied communality of a series would exceed one. So on its own null the rule rejects on average between a twelfth and a twenty-fourth as often as the level it is read at allows, and at its worst null point about two-fifths as often, and it is essentially blind to dispersion at $s = 0.70$.

What the paper says about heterogeneity rests on the coefficient tests, which are correctly sized at 0.042 under their own simulation, reject on all seven variables after the Holm correction, retain between 0.57 and 1.00 power at the scale the data shows, and say which row moved: on the alternative in which the financial-asset row alone differs across countries, that row rejects every time after Holm and the other six reject at most twice in a hundred. Appendix F reports the power curve with its Monte Carlo standard errors.

6.3 Symmetry

The third question is whether the estimated common variation is symmetric, and it is addressed by description only: no symmetry verdict is claimed. A second property is examined alongside it, that the mapping from fiscal action to $r-g$ is linear, and the two are kept apart because neither implies the other: a linear system can be driven by asymmetric shocks, and a nonlinear one can have symmetric marginals. The distinction is not decorative here. No probability is attached to the symmetry evidence below, so linearity is the only half of this pair that carries a rejection, and that rejection is evidence about the shape of the mapping and not about the shape of the shocks. The linearity half is rejected in the pooled panel under every specification, on bootstrap probabilities that are anti-conservative by an amount this design does not quantify, and country by country, on the primary first-difference specification, in thirteen of the seventeen at five per cent. The symmetry half requires more care than a normality test can supply, and we set out what the evidence does and does not establish. The normality statistics reported below are computed against a reference that assumes independent draws from a normal population, which these scores are not, so they are reported as descriptions and not as tests. The evidence we lead with is a statistic of the whole common space rather than of any factor in it, and the reason is an identification one. The identifying specification places three factors on the 79 quarters of the pre-pandemic window (Section 5), and Mardia's

multivariate skewness of that three-dimensional block of scores is $b_{1,p} = 6.83$ and its multivariate kurtosis $b_{2,p} = 25.84$. No probability is attached to either, and the reason is the reference distribution rather than the statistic. The reference available is the distribution these moments would have under independent draws from a normal population, and the scores satisfy neither condition: their first-order autocorrelations are 0.88, 0.86 and 0.76, they are estimated rather than observed, and the penalty fixing the representation is itself selected on the same data. Neither the invariance of the moment to rotation nor the fact that it is a pure third moment repairs that, because both are properties of the statistic and neither is a property of its sampling distribution. The moments stand as descriptions of the estimated common space and are not read as evidence against symmetry: in scores this persistent, sample third moments of this order need not signal an asymmetric process, and no test of symmetry whose size is established under this dependence is reported in this paper. How much of the moment the crisis episodes carry can be said exactly, and it explains why no probability is attached rather than merely qualifying the moment. Dropping the eight quarters of the global financial crisis takes $b_{1,p}$ from 6.83 to 5.52; dropping the twelve of the euro-area crisis raises it to 10.65; dropping both, twenty of the seventy-nine, leaves 1.15. No single quarter is responsible, since leaving each one out in turn moves the statistic only within 6.44 to 7.48, so what carries it is the episodes as blocks rather than an outlier. That is the same serial dependence which puts the reference distribution out of reach, and the two are one fact rather than two: a statistic generated by clustered episodes cannot be judged against a law derived for independent draws. Bai and Ng (2005) set out the corresponding corrections for moment tests on time-series data, and a valid test of this restriction would begin there. Both statistics are invariant to any non-singular linear transformation of the factor block, so each is a property of the estimated common space and not of the particular rotation the estimator happens to return, and we verified that invariance numerically under a random signed permutation and under a random orthogonal rotation rather than assuming it. The first of the two is the instrument the restriction actually calls for, because it is a pure third-moment statistic and therefore cannot be carried by heavy tails alone, whereas Jarque–Bera combines skewness with excess kurtosis and rejects for a symmetric heavy-tailed distribution as readily as for an asymmetric one. Applied one factor at a time, Jarque–Bera exceeds the one per cent point of its independent-draw reference for all three; that is the reference these scores do not satisfy, so the count is a description and not a verdict; in each of the three the excess-kurtosis term is the larger of the statistic's two terms, which is the weakness of the instrument just noted. That count is invariant to the signed permutations the sparsity penalty leaves free but not to a general rotation, so it is reported on that basis, and never factor by factor. We make no claim about the direction of

the third moment, and on these data no such claim is available. The sign of an individual factor's skewness is not identified: reversing the sign of a factor and of its loading column is an orthogonal rotation that leaves the penalised likelihood, the selected penalty, the loading pattern, the transition eigenvalues, the fitted common component and every explained-variance share numerically unchanged while reversing the skewness, and of the eight sign patterns the estimator is indifferent between, two make all three coefficients share a sign and six do not. A sentence asserting that the coefficients agree in sign would therefore be a statement about a convention rather than about the data, on this specification and on every other. Each of the seventeen country systems places six factors on that country's seven series and is identified only up to rotation, the Anderson–Rubin bound being three, and neither skewness nor excess kurtosis is invariant to rotation, so a per-factor rejection count taken across them is not a property of the data alone; those fits are reported in Section 6.5 and Appendix D as descriptive summaries only, for the identification reason set out in Section 5, and no count is quoted from them here. What this paper reports on symmetry rests on the two rotation-invariant moments above and on nothing else, and it is a description of them rather than a verdict. The pooled rotation is itself pinned by the sparsity penalty only up to a signed permutation, which is why the pooled evidence above is stated as an invariant statistic of the whole space and, where a count is given at all, as a count and not as a property of a labelled factor. Three objects should also be kept apart, because a fiscal rule is calibrated against none of the first two: the distribution of the estimated factor scores, which is what we measure; the distribution of the factor innovations; and the distribution of structural shocks. A test directed at skewness itself, rather than at normality, would be the right instrument for this restriction, and we regard it as the natural next step rather than something the present tests deliver. We stop short of reading a structural direction into the third moment, and on this specification we could not read one even if we wished to. What the multivariate moment records is the in-sample third moment of the estimated common space, which is the order of moment the symmetry restriction speaks to; the direction of those moments is exactly the part a rotation destroys, so it is not available from an estimator identified only up to one. The third moment is in addition a property of the estimated factor scores, which is the first of the three objects distinguished above and not the third, so a claim about whether contractions exceed recoveries would in any case be a claim about the wrong object. What the restriction requires is symmetry; what these data supply is an in-sample third moment of the estimated common space, whose distribution under a symmetric and dependent null is not established here, and which is therefore not evidence against the restriction. The identifying window closes in 2019Q4, so what is measured here is the third moment

of the scores over the global financial crisis and the euro-area crisis and not over the pandemic; the full-sample companion, which does contain 2020–2021, returns $b_{1,p} = 11.19$ on its four-factor block, whose score columns are also serially dependent. On either window the large excursions of the scores fall in the episodes of systemic stress rather than spreading evenly across the sample, though the quarter at which any individual factor attains its own extreme moves with the rotation and is for that reason not reported. That is the pattern Auerbach and Gorodnichenko (2012) report for government-spending multipliers that differ between recessions and expansions, and the one Adrian et al. (2019) document for the time-varying skewness of United States growth, though the resemblance is one of timing and not of direction. The symmetry evidence above does not bear on whether a linear debt rule calibrated on symmetric shocks misstates tail fiscal risk: the third moment is measured on estimated scores, no valid probability is attached to it here, and the step from a moment of the score distribution to the shocks a rule is calibrated against is the one the three objects distinguished above are meant to keep open. The full set of statistics is reported with the replication material: the pooled scores of every specification and their moments, the rotation-invariant statistics, the invariance audit, and the enumeration of the sign patterns the estimator is indifferent between.

Of the four predictions set out in Section 3, the exercise here tests the first, that transmission differs across states; the prediction that the factors align with identifiable macro-financial states is not tested, because no factor in this paper is identified well enough to be named (Section 6.5); and the remaining two are not tested with these data. Even the first is tested only in its debt-stress half: the transition variable is a lagged measure of debt stress and no institutional index enters the estimated system, so the institutional margin of that prediction is not taken to the data. Section 5 sets out a smooth-transition specification that nests the linear model, so that state dependence can be tested rather than assumed. In the pooled panel linearity is rejected in all three of the specifications considered, so the pooled result does not depend on the choice among them, and the statistic has to be named because the three do not agree on how strong the rejection is. The third-order Luukkonen–Saikkonen–Teräsvirta statistic, which is the one reported throughout this paper, returns block-bootstrap p-values of 0.001 and 0.007 on the two specifications in levels and 0.001 on the first-difference specification, so it rejects at the one per cent level in each of the three. The first-order form of the same statistic returns 0.005, 0.014 and 0.001 on the same three, which is a rejection at five per cent in all of them but at one per cent in only two. Three of those six figures sit at 0.001, the floor that 999 bootstrap draws can resolve; on the full-sample companion four do. All twelve are subject to the reservation stated in Section 5: the bootstrap draws its blocks separately within each country, so it does not carry shocks

common to several countries in the same quarter, and these pooled probabilities are anti-conservative by an amount this design does not quantify. Country by country the identifying window and the full sample reject in similar numbers of countries. On the first-difference specification the third-order statistic rejects in four of the seventeen countries at one per cent, in thirteen at five per cent and in fourteen at ten per cent; the nine that clear five per cent but not one are Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Spain and the United States, and the three that do not reject even at ten per cent are the Netherlands, Slovakia and Slovenia. The two specifications in levels reject in twelve and in thirteen of the seventeen at five per cent, the first taking the lagged change in gross debt as the transition variable, with Belgium, Finland, France, the Netherlands and Slovakia the exceptions, the second a lagged debt-level proxy, with Austria, Belgium, Italy and the Netherlands the exceptions. The full-sample companion returns twelve, thirteen and fifteen on the same three specifications at five per cent, and fifteen of seventeen at ten per cent in first differences, so the choice of window moves the country-level counts by no more than two. We report the first difference specification as the primary one because it is the one of the three whose maintained linear benchmark comes closest to a balanced regression: on the identifying window the residuals of its linear model clear an augmented Dickey–Fuller test at -3.5 in twelve of the seventeen countries, Greece, Ireland, Luxembourg, Slovenia and the United States missing the cut, and carry a median first-order autocorrelation of 0.56, against three of seventeen and a median of 0.80 in levels. A rejection against the levels benchmark is a rejection against a null that is misspecified for a reason unrelated to the transition function, so a levels count is not evidence of non-linearity on its own. This is the paper's direct evidence that fiscal-financial transmission is not linear; on whether it is common across countries, the evidence of Section 6.2 converges on differences, though the composite criterion for declaring them established is not met.

6.4 The model's own restriction

The model of Section 3 implies something sharper than state dependence in general: the association between the sovereign yield and financial-asset accumulation should differ between low- and high-debt states. That restriction is tested here on the ten-year yield, the rate the model's premium channel prices, with a specification and a decision rule fixed before it was estimated: the first-difference regression of Section 5 on the identifying window, with the transition placed on the observed lagged ratio of gross debt to GDP, its threshold common to all countries, country and quarter fixed effects, each regime required to hold at least fifteen per cent of the observations and at least five countries with eight quarters each, and inference by a bootstrap of calendar quarters, drawn jointly across countries in blocks of eight

with the null imposed, that repeats the search for the threshold and the smoothness of the transition in every one of its 999 draws. The asset regressor is the quarterly change in the four-quarter change of the assets-to-GDP ratio, so a coefficient relates a quarterly movement in the yield to a quarterly movement in the annual rate of asset accumulation, and is an association, not the effect of a purchase.

Once the regime search and common quarterly shocks are accounted for, a difference between high- and low-debt states in the yield's association with asset accumulation is not established ($p = 0.894$). The selected threshold is a debt ratio of 125.8 per cent of GDP, with 15.0 per cent of the 1,197 country-quarters above it, and the smoothness sits at the lower end of its grid, so the transition is gradual: over the observed debt ratios the weight on the high-debt coefficient runs only from 0.19 to 0.72, and the two regime coefficients are the ends of that transition rather than estimates for two observed groups. They are -0.23 basis points of the yield per percentage point of GDP in low-debt states, with a 90 per cent interval of $[-3.13, +3.00]$, and $+2.93$ in high-debt states, $[-6.54, +10.29]$, and the curve that joins them over the observed debt ratios, plotted in Figure 3, has a 90 per cent band that contains zero everywhere. Linearity of the regression as a whole is rejected, at $p = 0.027$ on the sup-F statistic with the search repeated in each draw, so the regression depends on the debt state, but the asset coefficient is not where that dependence is established. On 200 panels simulated under the null, the targeted test rejects in 8.5 per cent at its nominal five per cent, within the 0.10 the decision rule allows.

Table 4 reports the primary test beside the five secondary analyses fixed with it, none of which can change its verdict: the transition placed on debt accumulation, the lagged four-quarter change in the debt ratio, instead of its level (S-A); the consistent $r - g$ as the outcome (S-B); the full sample (S-C); the transactions measure of the asset change, on the fifteen countries for which it exists (S-D); and the conventional $r - g$ as the outcome (S-E). None rejects at five per cent on its targeted test. The smallest p is S-A's, 0.065, for a difference of $+0.100$, close to the $+0.102$ that the same transition gives when it is taken as given, with standard errors clustered by country ($p = 0.010$); that conditional result is superseded by the test above and is reported, with its qualifications, in Appendix H. In S-A and S-B the 90 per cent interval for the difference excludes zero while the targeted test does not reject: the intervals describe the spread of estimates that mostly sit on the edge of the grid, and the test is the targeted one.

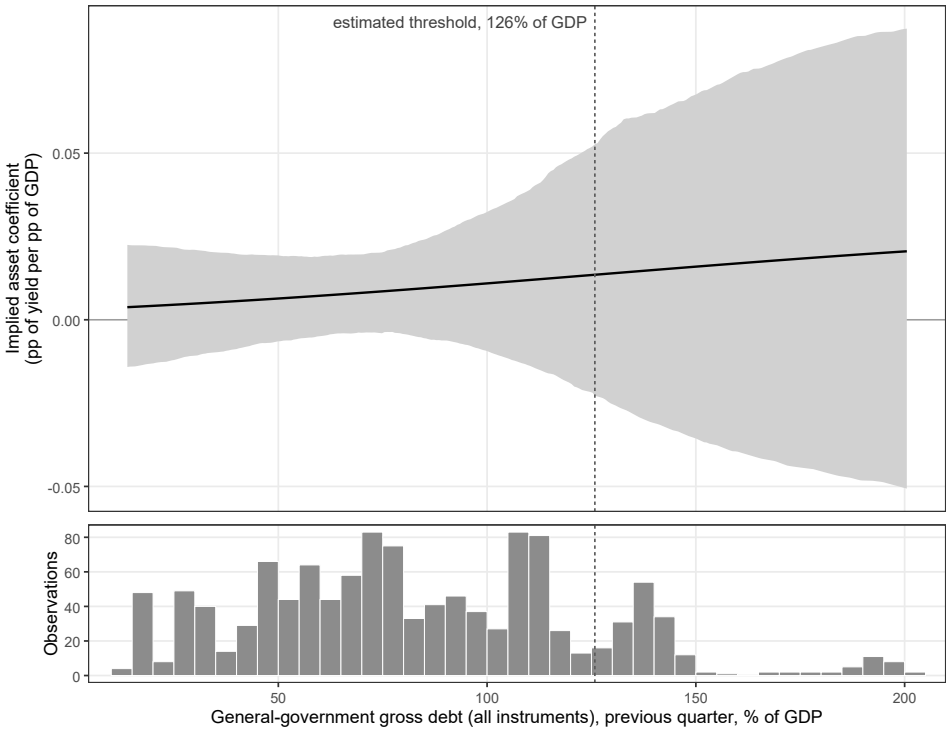
The reading is therefore this. That the reduced form depends on the state is supported: linearity is rejected in the pooled panel at one per cent under each of the three specifications of Section 6.3, on bootstrap probabilities that are anti-conservative by an amount this design does not quantify, and country by country, on

Table 4: State dependence of the asset association: the primary test and the secondary analyses

	Outcome; transition; sample	N	high	low	high	difference [90%]	p targeted	p linearity
Primary	ten-year yield; lagged debt ratio; identifying window	1,197	0.15	−0.002	+0.029	+0.032 [−0.091, +0.128]	0.894	0.027
S-A	ten-year yield; lagged change in the debt ratio	1,197	0.15	+0.005	+0.105	+0.100 [+0.053, +0.155]	0.065	0.016
S-B	consistent $r - g$; lagged debt ratio	1,197	0.85	+0.078	−0.023	−0.101 [−0.201, −0.012]	0.171	0.001
S-C	ten-year yield; lagged debt ratio; full sample	1,601	0.15	+0.005	+0.020	+0.014 [−0.048, +0.086]	0.868	0.037
S-D	ten-year yield; transactions in financial assets; 15 countries	1,053	0.18	−0.068	+0.097	+0.166 [−0.016, +0.339]	0.336	0.041
S-E	conventional $r - g$; lagged debt ratio	1,197	0.85	+0.014	−0.038	−0.053 [−0.213, +0.124]	0.550	0.035

Notes: first-difference smooth-transition regressions on the identifying window 2001Q2–2019Q4 (S-C: to 2026Q1), country and quarter fixed effects. *high*: share of country-quarters in the high state at the selected threshold; *low*, *high*: the asset coefficients of the two states, in percentage points of the outcome per percentage point of GDP; the interval is the 90 per cent percentile interval of a bootstrap without the null imposed. p targeted: the null-imposed bootstrap test of the difference, with the search for the threshold and the smoothness repeated in each of 999 draws; p linearity: the sup-F test of all regime differences, same bootstrap. The decision rule is applied to the primary row only.

Figure 3: The yield's association with asset accumulation over the debt ratio



Notes: upper panel, the asset coefficient implied by the primary estimate at each debt ratio, the high-debt and low-debt coefficients weighted by the transition, with the 90 per cent pointwise band of the bootstrap without the null imposed, each draw with its own threshold and smoothness; the dashed line is the selected threshold. Lower panel, the country-quarters at each debt ratio, in bins of five percentage points. The debt ratio is general-government gross debt on the all-instruments definition of Section 4, not the Maastricht measure, in the previous quarter.

the primary first-difference specification, in thirteen of the seventeen at five per cent, and the search-aware test above rejects linearity of the yield regression at five per cent. The restriction the model places on the asset coefficient, a difference between debt states in its association with the yield, is not established, and on $r - g$, the outcome of the accounting identity, it is not established either (S-B and S-E). It is recorded as not established rather than refuted: the power of the targeted test against a difference of the size the model implies has not been measured. Which state matters, and how sharply it separates, is not pinned down country by country, and this paper reports no per-country estimate of it.

6.5 Country analysis

This section reports the seventeen country-level fits. Each places six factors on that country's seven series, against the Anderson–Rubin bound of three set out in Section 5, so each is identified only up to a rotation of its own factor block. An individual loading, the identity of a numbered factor, the sign of a coefficient and the quarter at which a factor reaches an extreme are all properties of the basis the estimator happened to return rather than of the data, which is the same reason no factor is named anywhere in this paper. No per-country reading is built on those quantities, and what follows is restricted to statements that survive the indeterminacy. Three do, the first only under an orthogonal rotation: each series' squared-loading norm, which the fits report as a communality; which series the estimator leaves with no common component at all; and the correlation between the fitted common components of two series, which depends on the span and not on the basis chosen inside it. Appendix D tabulates the fits in full, on the database this paper adopts.

The first thing the seventeen fits show is how little room they leave. The median squared-loading norm across the 119 country-series is 0.930 and 53 of them exceed 0.95; eight fall below a third, the lowest being Luxembourg's ten-year yield at 0.051 and Slovakia's change in gross debt at 0.197. None exceeds one, though one could, because the norm is a share of variance only when the estimated factors are orthonormal, and these are not: the ratio of $\lambda_j' \Sigma_F \lambda_j$ to $\lambda_j' \lambda_j$ runs from 0.93 to 3.62 across the panel, and the two readings differ by more than 0.10 for 43 of the 119 series. The norm is invariant to an orthogonal rotation of the factor block but not to a general non-singular transformation of it, which leaves the fitted common component unchanged and moves the norm, so it is reported here as a diagnostic and not as an explained-variance share. A system that fits almost every series this closely cannot discriminate between accounts, and that, rather than any individual figure, is why this paper reports these fits as description and rests nothing on them.

The second is the one quantity of substantive interest that can be stated invariantly. The correlation between the fitted common components of $r - g$ and

the change in financial assets is defined for all seventeen countries, unlike its pooled counterpart. Its median absolute value is 0.337, it exceeds 0.3 in eleven of the seventeen, Ireland's by less than 0.001, and it runs from −0.369 in Finland to +0.635 in the United States. Thirteen of the seventeen are positive, and of the four that are not only Finland's exceeds 0.1 in absolute value. Table 5 reports it beside each country's mean squared-loading norm.

That count must not be read against the pooled one, and the difference between the two is a difference of design rather than of evidence. On the specification this paper identifies from, the same correlation is defined in fifteen of seventeen and exceeds 0.3 in absolute value in eight. One design places three factors on 119 series and the other six on seven, and the second is near-saturated for exactly that reason; the country fits also run on each country's own full sample, mostly 2001Q2–2026Q1 at about a hundred quarters, rather than on the pre-pandemic window the paper identifies from. Eleven against eight is therefore not evidence that the pooled figure is too low, and nothing in this section is offered as a correction to Section 6.1. It is offered as a description of what seventeen separately estimated systems look like, and as the reason this paper does not build on them.

Table 5: Country fits: the quantities that survive an orthogonal rotation

Country	ρ between the fitted common components of $r-g$ and ΔFA	mean squared-loading norm
United States	+0.635	0.787
Slovenia	+0.513	0.818
United Kingdom	+0.503	0.828
Spain	+0.489	0.830
Netherlands	+0.413	0.805
Belgium	+0.406	0.811
Latvia	+0.374	0.824
Finland	−0.369	0.770
Portugal	+0.337	0.833
Germany	+0.313	0.843
Ireland	+0.300	0.817
Greece	+0.244	0.757
Luxembourg	−0.099	0.593
Slovakia	−0.087	0.726
Austria	+0.057	0.799
Italy	−0.026	0.727
France	+0.018	0.811

Notes: each country system places six factors on that country's seven series and is identified only up to a rotation of its factor block. The first column is invariant to any non-singular transformation of that block and the second to an orthogonal rotation of it; no individual loading, factor label or sign is, and none is reported. ρ is the correlation between the two series' fitted common components, computed on each country's own sample. The second column averages each series' squared-loading norm over the seven series. That norm is invariant to an orthogonal rotation but not to a general one, and it is a share of variance only under orthonormal factors, which these are not, so it is reported as a diagnostic and not as an explained-variance share. Source: the country workbooks from which Appendix D is generated, on the adopted database.

6.6 Which constraint would have bound

Whether the asset side adds information once debt is observed is examined by the nested comparison of Section 6.1, which finds no evidence that it does for $r-g$ a year ahead, though it could not reliably have detected information large enough to move

a one-year debt projection by the smallest annual adjustment the EU debt safeguard demands. Neither that comparison nor the co-movement reported above says which country-quarters an anchor written on the asset side as well would have caught. This subsection answers that narrower question, and answers it by arithmetic rather than by estimation. Holding each government's debt and financial-asset stocks at their observed values, we ask which of two anchors — a sixty-per-cent gross-debt analogue, and a net-financial-liability anchor calibrated against it — would have bound on which country-quarters, across the seventeen countries of this panel from 2000Q2 to 2026Q1. The exercise runs on the full sample rather than on the identifying window of the factor model, because it is a ratio of observed stocks and the pre-2020 identification restriction that governs the estimates above has no purchase on an accounting identity; it covers 1,718 of the $17 \times 104 = 1{,}768$ country-quarters, 97.2 per cent, and every count below is taken over covered cells and never over the rectangle. What it establishes is which constraint would have bound on the balance sheets that were actually observed, and that is the whole of what it establishes. It is accounting and not behaviour. A government facing a net anchor would have faced different incentives on both sides of its balance sheet, including the incentive to acquire financial assets, which a gross anchor penalises and a net anchor does not reward, and those responses would have changed the very stocks the exercise takes as given; holding them fixed is the assumption, not an oversight. Nothing below says that any government would have borrowed more, consolidated less, or avoided any classification under a rule it never faced.

Two things have to be fixed before a count of breaches means anything, and the first is what is being counted. The liability leg is D4 gross debt (IMF–World Bank coverage), not Maastricht debt: general government gross debt on the all-instruments definition of the public sector debt statistics, at nominal value, and the asset leg is the general government stock of total financial assets; both are divided by the four-quarter sum of current-price nominal GDP, so the two anchors differ in their numerator and in nothing else. That debt definition is not the Maastricht measure, which is consolidated and confined to three instruments, and the gap between them is large and country-specific: on the definition used here the United Kingdom reads 145.68 per cent of four-quarter GDP at 2019Q4. The sixty used below is therefore a calibration device on this measure, an analogue of the Treaty reference value rather than the reference value itself, and no statement here is a statement about any country's actual excessive-deficit status. The United States, which has no domestic counterpart to a sixty-per-cent threshold at all, is carried in the arithmetic because the compositional question needs one yardstick applied to all seventeen, and is flagged wherever it appears. The second thing to fix is where the net threshold sits, and the obvious choice is the wrong one. Financial asset stocks are non-negative, so netting

them off lowers every ratio pointwise and the net measure is never above the gross one; the set of cells breaching a net sixty is therefore a subset of the set breaching a gross sixty, and a sixty-against-sixty comparison can produce exits from breach and cannot produce a single entry. On this panel it produces 529 exits and no entry whatsoever. That zero is arithmetic and not evidence — it restates the fact that governments hold financial assets — and it is reported here once, as the reason the exercise is not done that way, and used nowhere else. The net threshold is instead count-matched: it is set at the value at which the number of net breaches equals the number of gross breaches at sixty, which on the covered panel puts it at 24.07 points of GDP against 1,208 gross breaches. Entries then equal exits identically, by construction, so neither count is evidence of anything on its own. The only thing that can differ between the two anchors is which country-quarters they are, and that identity is the result.

The two anchors classify 242 of the 1,718 covered country-quarters differently, 14.1 per cent of them: 121 entries and 121 exits, equal by construction, with the anchors agreeing on the remaining 1,476 and 0.8998 of the gross breach set surviving the change of anchor. The disagreement is not spread across the panel, and its concentration rather than its size is the finding. For ten of the seventeen the two anchors agree in every covered quarter — Austria, Belgium, France, Greece, Italy, Latvia, Luxembourg, Portugal, the United Kingdom and the United States — so the whole of the reclassification sits in seven, and within them it runs in both directions. Agreeing on status is not the same as occupying the same place: a country on which the anchors never disagree can still change rank, because rank is the mean distance to the threshold, and Latvia and Luxembourg do so in Table 6. The exits are Finland's 55, Slovenia's 44 and Germany's 18, with three in Ireland and one in the Netherlands; the entries are the Netherlands' 43, Slovakia's 36 and Spain's 22, with eleven in Ireland and nine in Germany. Germany, Ireland and the Netherlands appear on both sides, at different dates. Table 6 sets out the per-country picture, and the ordering of countries by bindingness — the mean distance of each country's ratio to its own threshold — moves less than the cell-level disagreement would suggest: the two orderings correlate at 0.9240 by Spearman's measure and 0.8382 by Kendall's, and a bias-corrected and accelerated bootstrap resampling countries, at 10,000 resamples, puts the 95 per cent interval at $[0.565, 0.998]$. The null that matters here is perfect agreement rather than no agreement, and that interval excludes perfect agreement at the ten and at the five per cent level, and not at the one per cent level. Eleven of the seventeen countries do not move at all, and of the six that do, Finland moves six places less binding, Slovakia four places more binding, Latvia two places more binding, Slovenia two places less binding, and Luxembourg and the Netherlands one place more binding each; Finland's move is from eleventh to last, on an asset stock

larger than its own gross debt. What changes, then, is chiefly which country-quarters the constraint catches; the ordering of countries by bindingness also moves, though modestly, and under the mean-matched calibration the net anchor catches more cells rather than fewer. Under the alternative calibration that equalises the panel-mean distance to threshold instead of the count, more cells breach the net anchor than breach the gross one, 1,312 against 1,208. A net anchor is a different anchor and not a looser one.

Table 6: Where a net anchor and a gross anchor classify differently, by country

Country	*Exits*	*Entries*	*Gross rank*	*Net rank*	*Illiquid share*
Greece	0	0	1	1	0.433
United Kingdom	0	0	2	2	—
Italy	0	0	3	3	0.519
United States*	0	0	4	4	—
Belgium	0	0	5	5	0.596
Portugal	0	0	6	6	0.538
France	0	0	7	7	0.515
Spain	0	22	8	8	0.526
Austria	0	0	9	9	0.638
Germany	18	9	10	10	0.566
Finland	55	0	11	17	0.706
Ireland	3	11	12	12	0.473
Slovenia	44	0	13	15	0.599
Netherlands	1	43	14	13	0.501
Slovakia†	0	36	15	11	0.555
Latvia	0	0	16	14	0.596
Luxembourg	0	0	17	16	0.428
All seventeen	121	121		median illiquid share	0.538

Notes: seventeen countries over 2000Q2–2026Q1, 1,718 of 1,768 country-quarters covered; every figure is over covered cells. The gross anchor is general government gross debt on the all-instruments definition, at nominal value, over the four-quarter sum of current-price nominal GDP, with a threshold of sixty. That is not the Maastricht measure and the threshold is an analogue of the Treaty reference value rather than the reference value, so no row is a statement about any country's excessive-deficit status. The net anchor is the same denominator with total financial assets subtracted from the numerator, at a count-matched threshold of 24.07 points of GDP — the value at which the number of net breaches equals the 1,208 gross breaches — so entries equal exits by construction and what the table carries is which cells they are. An exit is a country-quarter that breaches the gross anchor and not the net one; an entry is the reverse. Rank 1 is the most binding country, ranked by mean distance to its own threshold, so a country whose net rank exceeds its gross rank binds less tightly under the net anchor than under the gross one. Illiquid share is the average share of loans and equity in general government financial assets on the Eurostat instrument split fixed before the rank movements were known; the United Kingdom and the United States are not in that EU member-state source and are left blank rather than classified, so the median is over fifteen countries. * The United States has no domestic legal counterpart to a sixty-per-cent threshold and is carried for comparability alone. † Slovakia is covered in 77 quarters, the fewest in the panel, because its debt series begins 2006Q4; every other country is covered in 99 quarters or more. 42 of the 242 status changes sit within one point of GDP of a threshold and are fragile to revision.

Three qualifications belong with those figures. First, the bootstrap is over seventeen units and the interval is correspondingly wide, at 0.432; the verdict that the two orderings differ is a five-per-cent verdict rather than a one-per-cent one, and it does not survive a change of construction. Repeating the exercise on the robustness construction, which takes both legs from the archived IFS series — it

carries no United States and no quarter after 2024Q4, and covers 89.1 per cent of the rectangle against the 95 per cent floor fixed for it in advance — returns 72 entries and 72 exits over 1,131 gross breaches — equal by the same construction, and so not an independent confirmation of it — but a rank correlation of 0.9647 whose interval excludes perfect agreement at none of the three levels. The two constructions disagree on that one point, and the disagreement is reported rather than reconciled. Second, the series are revised, and 42 of the 242 status changes sit within one point of GDP of a threshold; a revision of less than one point of GDP could reverse them, and the whole exercise is conditional on the vintages it was run on. Third, the primary construction subtracts an asset stock taken from the IMF quarterly government finance statistics balance sheet, compiled under the GFSM 2001 framework, which values financial assets at current market prices, although the extract carries no valuation flag, from gross debt at nominal value, a mismatch that is signed, time-varying and correlated with exactly the interest-rate variation this paper is about; the panel median absolute wedge between the two valuations of debt is 4.05 points of GDP against a panel median asset ratio of 35.37, so the netting the exercise measures is about 8.7 times the valuation noise it carries, which is a reason to report it and not a reason to call it clean.

Where in time the two anchors disagree is the part of the exercise that runs against intuition, which is why Figure H1 plots it rather than asserting it. A reader expecting the asset side to matter most when balance sheets move most would look for the disagreement in the financial crisis. It is not there. Of the 242 changed cells, seven fall in 2008 and 2009, against 25 in 2020 and 2021 and 26 in 2022 and 2023; by decade the split is 65, 100 and 77. Five of the eight ranked quarters in which the two anchors agree on every covered country are the consecutive run from 2008Q1 to 2009Q1 and a sixth is 2010Q1, so six of the eight sit inside the two crisis windows and the other two are 2000Q2 and 2000Q3, the first two quarters of the sample. A ninth quarter, 2026Q1, also records no disagreement, but it covers one country and is excluded from the ranking for that reason. The mean share of covered countries classified differently is 0.0515 across 2008 and 2009 and 0.0735 across 2010 to 2012, against 0.1407 over all 103 ranked quarters, and none of the ten quarters of largest disagreement falls in either window: they are 2018Q2 to 2019Q1, 2020Q2, 2021Q4, 2022Q1 and 2022Q2, and 2024Q1 and 2024Q2, the largest of them separating five of the seventeen covered countries. The quarters in which a gross anchor and a net anchor most nearly coincide are the quarters of the global financial crisis. What separates the two anchors is not a shock but the slow accumulation of financial assets in some governments and not others, which is a compositional fact about balance sheets rather than a cyclical one.

The objection a net anchor has to answer is that it treats a liquid sovereign

wealth fund and illiquid equity in a state enterprise as the same thing, and this exercise does not answer it. Saying so precisely is the point of what follows. On the instrument split fixed as the classifier before any rank movement was known — the Eurostat quarterly series of general government financial assets by instrument, with loans and equity counted illiquid and currency, deposits and debt securities counted liquid — illiquid assets exceed liquid ones in every one of the fifteen countries that source covers, at a median illiquid share of 0.538 against a median liquid share of 0.271, running from 0.428 in Luxembourg to 0.706 in Finland. The United Kingdom and the United States are not in an EU member-state database and are reported as uncovered rather than classified from impression; on a seventeen-country alternative source the same ordering holds in all seventeen. Between two-fifths and three-quarters of every covered government's financial assets are loans and equity, so the objection is about the panel and not about a few outliers. The largest single reclassification the exercise finds runs on the panel's highest illiquid share, which is Finland's at 0.706: 0.6225 of Finnish general government financial assets sit in the social security funds subsector (S1314), the highest of the fifteen, 3.83 times the median of the other fourteen and the only share in the panel above a half, and it never falls below 0.5065 in the 109 published quarters from 1999Q1. Those assets carry almost no recorded liability of their own — that subsector's recorded liabilities average 0.0295 of its assets in Finland, against 1.1205 in Spain and 1.1157 in the Netherlands — although that is a recording convention as much as an economic fact, since the European accounting standard keeps social-security pension entitlements outside the core accounts, and nothing in this source settles what stands behind the stock. Whether reclassification tracks illiquidity across the panel is the question for which the design fixed a formal test in advance, and the test cannot answer it. The rank correlation between absolute rank movement and illiquid share is 0.3170, with a permutation p-value of 0.2481 over 100,000 draws; but the smallest correlation that test would detect with eighty per cent power is 0.650 at the ten per cent level, 0.700 at five and 0.800 at one. A test with no power against a realised correlation of 0.32 has not found an absence, it has failed to look, and the non-rejection is reported as uninformative for that reason rather than as a defence. The realisability objection therefore stands unanswered by this exercise and not refuted by it, and the fact that the panel's largest reclassification attaches to the panel's highest share of loans and equity is reported at the strength of a description, which is what it is. What this section shows is that the asset side changes which country-quarters an anchor would catch; the nested comparison of Section 6.1 finds no evidence that it is informative about $r - g$ a year ahead. The claim is not that any netting is an improvement, and a rule that read the recorded balance sheet without asking what the assets are would credit the Finnish stock in full.

The exercise above is run on D4 gross debt, so its sixty per cent is an analogue of the Treaty's reference value and not the value itself. For the fifteen EU countries of the panel it is repeated on Maastricht debt, Eurostat's consolidated gross debt of general government (Eurostat, 2026b), against the reference value, with three net measures: debt less currency and deposits (N1); less currency, deposits and debt securities (N2); and less all financial assets with a haircut of one half on loans and equity (N3). The assets are taken from the non-consolidated financial accounts, so the netting also removes claims on other government units that consolidation has already removed from debt. Each net threshold is count-matched out of sample: it is fixed on 2000Q1–2009Q4, where the gross anchor binds in 253 of the 600 country-quarters, at 54.65, 52.14 and 32.75 points of GDP, and then applied unchanged to 2010Q1–2026Q1. Over those 975 country-quarters the gross anchor binds in 686. N1 and N2 flag no country-quarter that the gross anchor clears, and clear 81 and 113 that it flags; N3 flags 8, all of them Irish quarters between 2019Q3 and 2021Q3, and clears 170. The disagreement is concentrated: three countries hold 86, 85 and 71 per cent of it under N1, N2 and N3 (Slovenia, Germany and Slovakia; Finland, Germany and Slovenia; Slovenia, Finland and Austria). Table 7 asks, descriptively, whether the cells on which the anchors disagree differ in subsequent financing conditions. In the cells cleared by a net measure but flagged by the gross anchor, the change in the ten-year yield over the next four quarters does not differ from that in the cells on which the anchors agree, under any of the three measures, every 90 per cent interval for the difference containing zero; for the effective interest rate one of the nine comparisons excludes zero, N3's against the cells both anchors clear, at -0.05 points. The eight Irish cells that N3 alone flags are followed by a rise in the yield 0.70 points larger than in the cells both anchors flag, and by a fall in the effective rate 0.17 points larger; they are one country over eight quarters. The comparison shows which cells each anchor flags under a calibration fixed before the evaluation period, and how financing conditions moved in them afterwards; it does not rank the anchors.

7 Conclusions and Policy Implications

Gross debt, budget balances, government financial assets and the interest-growth differential $(r - g)$ are commonly treated as primitives governing sovereign fiscal performance. This paper treats them instead as jointly generated by latent macro-financial states.

Estimating a sparse dynamic factor model on a pooled panel of seventeen advanced economies — one system of 119 series, from which three common factors are extracted — over 2000Q2–2026Q1 and identifying on the pre-pandemic window

Table 7: Subsequent financing conditions in the cells flagged by the gross and the net anchors, 2010Q1–2026Q1

	Ten-year yield		i^{eff}	
Cells	n	Mean [90% interval]	n	Mean [90% interval]
N1: debt minus currency and deposits (threshold 54.65)				
Flagged only by gross	70	−0.11 [−0.56, 0.33]	67	−0.04 [−0.17, 0.08]
Flagged only by net	0	no cell	0	no cell
Flagged by both	571	−0.12 [−0.53, 0.35]	560	−0.12 [−0.18, −0.05]
Flagged by neither	273	−0.03 [−0.32, 0.33]	268	−0.09 [−0.16, −0.02]
Agree (both or neither)	844	−0.09 [−0.45, 0.31]	828	−0.11 [−0.17, −0.04]
Only gross − both		0.01 [−0.39, 0.37]		0.07 [−0.01, 0.16]
Only gross − neither		−0.08 [−0.43, 0.22]		0.05 [−0.03, 0.12]
Only gross − agree		−0.02 [−0.38, 0.30]		0.07 [−0.01, 0.14]
Only net − both		no cell		no cell
Only net − neither		no cell		no cell
Only net − agree		no cell		no cell
N2: debt minus currency, deposits and debt securities (threshold 52.14)				
Flagged only by gross	101	0.03 [−0.33, 0.45]	98	−0.07 [−0.16, 0.03]
Flagged only by net	0	no cell	0	no cell
Flagged by both	540	−0.15 [−0.57, 0.33]	529	−0.12 [−0.18, −0.04]
Flagged by neither	273	−0.03 [−0.32, 0.33]	268	−0.09 [−0.16, −0.02]
Agree (both or neither)	813	−0.11 [−0.46, 0.29]	797	−0.11 [−0.17, −0.04]
Only gross − both		0.18 [−0.14, 0.52]		0.05 [0.00, 0.12]
Only gross − neither		0.06 [−0.13, 0.27]		0.03 [−0.02, 0.09]
Only gross − agree		0.14 [−0.08, 0.41]		0.04 [−0.01, 0.11]
Only net − both		no cell		no cell
Only net − neither		no cell		no cell
Only net − agree		no cell		no cell
N3: debt minus financial assets, 50% haircut on loans and equity (threshold 32.75)				
Flagged only by gross	155	0.01 [−0.35, 0.44]	152	−0.13 [−0.21, −0.04]
Flagged only by net	8	0.54 [−0.16, 1.43]†	8	−0.28 [−0.34, −0.18]†
Flagged by both	486	−0.16 [−0.60, 0.34]	475	−0.10 [−0.17, −0.03]
Flagged by neither	265	−0.05 [−0.33, 0.30]	260	−0.09 [−0.15, −0.01]
Agree (both or neither)	751	−0.12 [−0.49, 0.28]	735	−0.10 [−0.16, −0.03]
Only gross − both		0.17 [−0.20, 0.55]		−0.03 [−0.08, 0.03]
Only gross − neither		0.06 [−0.10, 0.23]		−0.05 [−0.08, 0.00]
Only gross − agree		0.13 [−0.11, 0.41]		−0.04 [−0.07, 0.01]
Only net − both		0.70 [0.01, 1.57]†		−0.17 [−0.27, −0.08]†
Only net − neither		0.59 [0.03, 1.43]†		−0.19 [−0.28, −0.10]†
Only net − agree		0.66 [0.06, 1.51]†		−0.18 [−0.27, −0.09]†

Notes: Means of the subsequent four-quarter change, $x_{t+4} - x_t$, in percentage points, over the cells of 2010Q1–2026Q1 in which it is observed (last origin 2025Q1 for the yield and 2024Q4 for i^{eff}); each cell weighs equally. A cell is flagged by the gross anchor when Maastricht debt exceeds 60 per cent of GDP, and by a net anchor when the net measure is at or above its threshold, set on 2000Q1–2009Q4 so that it flags as many country-quarters as the gross anchor did there. Debt is consolidated (Maastricht); the financial assets are non-consolidated. i^{eff} is interest paid over four quarters divided by IMF–World Bank gross debt four quarters earlier. Rows "−" are differences of group means. Intervals: 90 per cent percentile intervals from a joint-time circular block bootstrap (blocks of four calendar quarters, all countries together, 999 draws, seed 20261014); draws in which a group has no cell are dropped for that group. †One country (IRL). "No cell": the group is empty.

2000Q2–2019Q4 (79 quarters) summarises latent macro-financial states in those factors, as Section 3 assumes; no factor is named, because the rotation that would license a name is not identified. Three questions bear on fiscal surveillance written on gross debt, and the paper treats them as information and risk diagnostics: whether a specified measure of government financial assets improves a specified forecast of $r - g$ beyond a baseline of gross debt and other fiscal and macroeconomic variables; whether comparable series have equal exposure to common variation across countries, or equal coefficients on the estimated factors; and whether the estimated common variation is symmetric, which is described without a verdict. They are not conditions for the adequacy of surveillance, nor restrictions that any framework in force imposes — Regulation (EU) 2024/1263 sets country-differentiated, risk-based paths — and the paper derives none of them from a statute or an optimisation problem. Pooling the seventeen economies into a single system turns each of the three from a question answered by assumption into one the data can be brought to bear on. On the first, how strongly the asset side co-moves with the differential is reported as a description; whether it adds information once debt is observed is a different question, and a nested forecast comparison fixed before it was run finds no evidence that it does for $r - g$ a year ahead, though it could not reliably have detected information large enough to move a one-year debt projection by the smallest annual adjustment the EU debt safeguard demands (Section 6.1). A second comparison, also pre-specified, on the differential as this paper measures it and with fiscal and output data lagged one quarter, leaves the question unresolved: the size of the asset information in the projected contribution of the differential to the next year's change in the debt ratio lies between 0.00 and 0.79 points of GDP in root-mean-square terms (90 per cent interval), a range that contains the 0.5 points of GDP of that smallest adjustment (Appendix K). On the second, equal exposure is not rejected by the composite rule, though all seven coefficient tests and nineteen of the twenty-two cells over which that rule is taken reject. The rule's form, the largest p-value over the cells, was fixed before any cell of the identifying specification was computed; the null generator it is evaluated under was rebuilt only after the first cells were known, because the original generator itself carried loading dispersion; and the restriction is not rejected under either generator. The rule's measured rejection frequency and power make that non-rejection a property of the rule and not evidence of homogeneity (Section 6). On the third, the paper reports the third moment of the estimated common space as a description only: the one reference distribution available for it assumes independent draws, the factor scores are strongly autocorrelated, and so no probability is attached to it and it is evidence neither for nor against symmetry. The evidence converges on cross-country differences in fiscal-financial transmission, though the composite criterion for declaring them established is not met. On the pooled specification the

paper identifies from, financial assets and $r - g$ have absolute correlations above 0.3 with the same latent factor in two of the seventeen economies — Finland and the Netherlands. That count is a summary of a continuous quantity, the strength with which financial assets and $r - g$ correlate with the same latent factor, and as such a summary it is too unstable to be read as a finding; it is that quantity rather than the count which carries the evidence. On the identifying specification its rotation-invariant companion — the correlation between the two series' fitted common components — exceeds the conventional cut of 0.3 in absolute value in eight countries and is undefined in two, because at the selected penalty the estimator leaves at least one of the two series with no common component at all (Section 6). No interval for either measure has been computed on that specification. What the data describe is how strongly the two series co-move, not how many countries fall on one side of a convention. The common component reaches the ten-year yield and $r - g$ first, and gross capital formation before any of the three fiscal flows, so on the identifying specification it is not a financial and fiscal component sitting on top of an idiosyncratic real side. Taken together, these estimates establish no macroeconomic function for government financial assets. Re-estimated country by country for Appendix G, the seventeen country-level systems sort the asset side into configurations that appendix labels buffer accumulation, crisis liquidity provision, co-movement with capital formation or the current account balance, and management orthogonal to both, with a fifth for no association, but the labels name patterns of co-movement and not what the assets do: the design identifies co-movement, not the effect or the purpose of an asset position. That classification is a descriptive reading of the estimates under a stated convention rather than a partition the data impose. How far it moves when the convention is changed is not established here, because the seventeen country-level fits on which it could be measured are reported as descriptive estimates only, unidentified for the reason given in Section 5; Appendix G sets three classifications side by side — the country-by-country one on a separate re-estimate of the seventeen country-level systems, the two pooled ones on a four-factor fit of all 119 series that is not the specification the paper identifies from — and under the pooled correlation criterion the fourth configuration has no members at all. Negative coupling of financial assets and $r - g$ — assets rising as the differential compresses — appears in one of the seventeen countries on the specification the paper identifies from, Finland, on a count too unstable to be read as a finding, and these estimates do not carry the conclusion that sovereigns can use the asset side of the balance sheet to promote economic growth. The regression of $r - g$ on the change in financial assets, the change in gross debt, the primary balance and capital formation is not linear: the linear benchmark is rejected against a smooth transition in a lagged measure of debt stress in all three pooled specifications, so the regime dependence

is tested rather than assumed, though the rejection is of the regression as a whole, does not locate the non-linearity in the coefficient on financial assets and rests on pooled bootstrap probabilities that are anti-conservative by an amount this design does not quantify; the institutional margin of the first prediction of Section 3, that transmission differs across states, is not taken to the data, because no institutional index enters the estimated system. In the framework of Section 3 an asset-side operation is a policy instrument whose effect on funding costs is regime-dependent, as liquidity provision, as shock absorption, or as support for investment and stabilization. The series estimated here is an asset-position change and not a measured purchase, its transaction, valuation and other-volume components are not separated, and the second and third predictions of Section 3 are not taken to the data, so these estimates establish no investment or stabilisation function. The theoretical model's own restriction on the asset side, that the association of the ten-year government bond yield, the rate the model's premium channel prices, with asset accumulation differs between high- and low-debt states, was tested with a specification and a decision rule fixed before it was estimated and with the transition placed on the lagged ratio of gross debt to GDP: once the regime search and common quarterly shocks are accounted for, that difference is not established ($p = 0.894$), and on $r - g$, on either measure, it is not established either. It is recorded as not established rather than refuted, because the power of the test against a difference of the size the model implies has not been measured (Section 6.4). The third moment of the factor scores is a description of the estimated common space and not a test of symmetry, so these estimates do not bear on whether a linear debt-based rule calibrated to symmetric shocks misstates tail fiscal risk.

Because the seventeen countries are estimated jointly, cross-country heterogeneity is a restriction on the model rather than an artefact of the design: had the countries been estimated one at a time it would be true by construction and could never be evidence. The restriction of common loadings is not rejected by the paper's composite rule, at $p = 0.23$, though nineteen of the twenty-two cells of the rebuilt exercise and all seven of the coefficient tests reject; the rule — which requires both dispersion statistics to reject in every cell — is a maximum, so the three that do not reject carry the verdict. The rule's form, the largest p-value over the cells, was fixed before any cell of the identifying specification was computed. The null generator it is evaluated under was rebuilt only after the first cells were known, because the original generator imposed the common block on the series in their original units and standardised them afterwards, so that the null itself carried loading dispersion; under the original generator the composite was 0.28, and the restriction is not rejected under either generator. On a generator that satisfies the restriction to machine precision, and sweeping the size of the common block because the restriction does not identify

it, the common-covariance statistic rejects in every one of its eleven cells and the correlation statistic rejects in every cell under Gaussian errors; what does not reject is the correlation statistic under the calibration that resamples the fitted residuals and transition innovations in moving blocks, and then only at the lower end of the range. That calibration carries the serial dependence, cross-sectional correlation and excess kurtosis the Gaussian null omits. It satisfies the restriction exactly, as the Gaussian one does — both impose a single loading block on all seventeen countries, and the population dispersion of the standardised blocks is of the order of 10^{-31} under either — so it is a null of (23) and not a departure from it. What it adds is structure in the idiosyncratic errors, about which (23) says nothing, and part of that structure is a residual factor with a cross-country pattern of its own. The dispersion statistics cannot tell that pattern from loading dispersion, so the null widens and the test loses much of its ability to see a violation. It is the conservative column for that reason — because it destroys power, not because it breaks the hypothesis — and it is used as one. The verdict is reported as the rule defines it. Failing a conservative rule is not evidence of homogeneity, and the individual tests converge on the opposite reading. How conservative the rule was is reported with the verdict rather than left for a reader to derive. The rule has been measured on its own null: its rejection frequency pooled over the eighteen null configurations is 0.0041, and 0.0021 over the eleven inside the observed confidence set, against the nominal 0.05 at which it is read; neither figure is the rule's size, which is the largest rejection probability over the whole null, and the largest frequency observed at any single configuration is 0.021, and its power at $s = 0.70$, close to the scale an attenuation correction on the original generator implied, is 0.000 under the Gaussian calibration and at most 0.010 under the dependence-respecting one; on its own null it therefore rejects on average between a twelfth and a twenty-fourth as often as the level it is read at allows, and at its worst null point about two-fifths as often, and it is essentially blind to dispersion on that scale, and the same conservatism is visible directly in the rebuilt cell counts. What this paper says about heterogeneity therefore rests on the coefficient tests. A test that cannot see an alternative is uninformative about it, which is neither evidence that transmission is common nor evidence that it is not. On symmetry the paper reports a moment and not a rejection. Mardia's multivariate skewness is invariant to the rotation of the factor block and is a pure third-moment statistic, and on the identifying specification it is 6.83. The only reference distribution available for it is the one it would have under independent draws from a normal, and that does not hold here: the three score columns have first-order autocorrelations of 0.88, 0.86 and 0.76, they are estimated rather than observed, and the penalty fixing the representation is selected on the same data, so no probability is attached to the statistic, no symmetry test whose size is established under that dependence has been carried out, and the

moment is evidence neither for nor against symmetry. The direction of the third moment is not identified, so none is claimed. The seventeen country systems place six factors on seven series, are identified only up to rotation and are reported in this paper as description only, and neither skewness nor excess kurtosis survives a rotation; what the paper reports on symmetry is the rotation-invariant moment alone, as a description of the estimated common space. Linearity of transmission is rejected in the pooled panel under all three specifications and at the one per cent level in each, on third-order block-bootstrap p-values running from 0.001 to 0.007 on the identifying window; the first-order form of the same statistic rejects in all three too, but at one per cent in only two of them. Country by country the evidence is not uniform: thirteen of the seventeen countries reject in first differences at five per cent and fourteen at ten per cent, and twelve and thirteen reject at five per cent on the two specifications in levels. On the full-sample companion two of the three pooled rejections sit at 0.001, the floor 999 bootstrap draws can resolve, and the third at 0.003, and the country counts differ from the window's by no more than two in either direction. Every one of these pooled probabilities is anti-conservative by an amount this design does not quantify, because the bootstrap draws its blocks separately within each country and so does not carry shocks common to several countries in the same quarter. We report the first-difference specification as the primary one because its maintained linear benchmark comes closest of the three to being well specified. That the reduced form depends on the state is supported, and pooling is what makes its cross-country heterogeneity a restriction the data can speak to rather than an assumption built into the design. If the state dependence held under a bootstrap that carries common shocks, and the heterogeneity under a criterion with power to detect it, sustainability projections that treat the mapping from fiscal action to $r - g$ as linear and common to all countries would miss how that mapping varies by country and with the pace of debt accumulation.

What remains uncertain can be stated in one place, with what follows from it in practice. First, whether the asset side carries information about the differential a year ahead beyond a forecast that already uses the current differential and country effects is unresolved: the second forecast comparison places the size of that information in a one-year debt projection between 0.00 and 0.79 points of GDP, a range that contains the 0.5 points the EU debt safeguard treats as a minimum annual adjustment. In practice this evidence gives a debt sustainability analysis no ground to add an asset term to its projection of the differential, and no ground to treat the question as closed. Second, a difference between high- and low-debt states in the yield's association with asset accumulation is not established, and the power of that test has not been measured: no differentiation by debt state of how assets enter surveillance can rest on it, and none is ruled out by it. Third, the composite rule does not reject common

loadings, but on its own null it rejects far less often than the level it is read at and it has almost no power, while all seven coefficient tests reject equality: a projection model that imposes common coefficients across countries draws no support from the rule's verdict. Fourth, every pooled estimate rests on a window that ends in 2019Q4, so none of these statements has been shown to hold under the fiscal rules that were suspended and rewritten after it, as the limitation set out below explains.

Overall, empirical results do not settle the case for a public sector net worth anchor, capturing the full cycle and assessing whether financial assets strengthen the public balance sheet or are merely a debt financed portfolio with no net effects on solvency or economic growth; Section 6.6 reports which constraint — gross debt, or a net-financial-liability proxy for such an anchor, which does not net the non-financial assets a net worth measure would include — would have bound on the balance sheets actually observed, and its calibration as an operative rule is left to future work.

What a net anchor would have done can be put with numbers rather than as a direction of travel. Section 6 asks which of two anchors — a sixty-per-cent gross-debt analogue and a net-financial-liability anchor calibrated to bind on the same number of country-quarters — would have bound on the balance sheets actually observed in these seventeen economies. The sixty is a calibration device on an all-instruments debt measure and not the Maastricht measure the Treaty reference value is written on, so no statement here bears on any country's excessive-deficit status. The exercise is accounting and not causation: it says which constraint would have bound with balance sheets held at their observed values, not what any government facing that constraint would have done. What it finds is that a net anchor is not a looser anchor but a different one: the two anchors classify 242 of the 1,718 covered country-quarters differently, 14.1 per cent of them, 121 that only the net anchor would catch and 121 that only the gross anchor would.

What the exercise does not establish matters more than what it does. Whether binding differently is binding better turns on whether the assets netted could be realised, and on that question this paper's evidence is descriptive and its test underpowered. Descriptively the composition is unfavourable to a naive net rule: assets in the loans and equity classes — which this paper calls illiquid, though the classification is by instrument and not by any measurement of realisability — exceed those held as currency, deposits and debt securities in every country the pre-specified instrument split covers, and the panel's largest single reclassification runs on its highest illiquid share. The formal test of that association rejects at none of the conventional levels, but a fifteen-country cross-section has eighty per cent power only against correlations roughly twice the realised one, so the non-rejection is uninformative rather than reassuring. The realisability objection to net anchors therefore stands unanswered by this exercise rather than refuted by it, and nothing

reported here ranks the two anchors: the exercise establishes which constraint would have bound, for these countries over these quarters. Forty-two of the status changes sit within one point of GDP of their threshold and are not resolvable against the revisions these series routinely carry. What it does settle is that a framework choosing between the two anchors is choosing which balance sheets it is willing to see.

One limitation is structural to the design rather than incidental to it, and it is set out here rather than left for a reader to find. The pooled evidence this paper identifies from is estimated on 2000Q2–2019Q4, so it stops before the pandemic and the balance-sheet interventions that came with it, before the energy shock, before the 2024 reform of the Stability and Growth Pact, before the United Kingdom's switch to public sector net financial liabilities, and before the 2025 amendment to the German debt brake. The reasons for the window are given in Section 5 and are matters of specification rather than of economics: four of the six live factor-count criteria return three factors on that space where the same battery splits on the full sample, the independent and AR(1) error specifications agree on all three dimensions there, the coefficient-equality tests reject on seven of seven variables there and on seven of seven on the full sample, and the structural-break tests are correctly sized on that space, whereas on the full-sample one the cells of the countries without a programme reject at the programme dates in 17.7 per cent of cases against 4.2 on the window, although the test keeps its nominal size on panels simulated from it, so that the excess rejections reflect instability in the full-sample data rather than a defect of the test. A factor space estimated across a break the tests cannot size is not a better answer to the question, only a longer one. But none of that offsets what the window costs, and none of it is offered as though it did. The full-sample fit, $T = 104$ quarters to 2026Q1 at four factors, and a third fit in which the eight pandemic quarters are set to missing, are both retained and reported throughout as companion specifications for exactly this purpose; no pooled number in this paper is taken from either unless it is labelled as one.

The point a referee is entitled to press hardest follows from this, and we would rather make it ourselves. This paper asks questions that bear on fiscal rules written on gross debt, and its identifying evidence ends before those rules were suspended and rewritten. We do not claim that the companion fits repair the gap: what the companion establishes is that a common factor space is recoverable on the longer sample and that the pooled design survives its inclusion, not that any finding reported above holds within the post-2019 regime. The honest statement is that the last six years are a companion specification and a limitation of this paper, not a period about which it has shown anything. Whether the heterogeneity of transmission and the asset-side co-movement described here persist into the post-reform regime is the first question we would put to an extension of this work, once that sample is long enough

to identify a factor space of its own.

Beyond that sample question, future research may look past the linear and continuously adjusting representation used here, toward models that explicitly accommodate latent state transitions, discrete balance-sheet interventions and heterogeneous macro-financial regimes.

References

Abubakar, A. B., and Mamman, S. O. (2020). Permanent and transitory effect of public debt on economic growth. *Journal of Economic Studies*, *48*(5), 1064–1083. https://doi.org/10.1108/JES-04-2020-0154

Adrian, T., Boyarchenko, N., and Giannone, D. (2019). Vulnerable growth. *American Economic Review*, *109*(4), 1263–1289. https://doi.org/10.1257/aer.20161923

Asteriou, D., Pilbeam, K., and Pratiwi, C. E. (2021). Public debt and economic growth: Panel data evidence for Asian countries. *Journal of Economics and Finance*, *45*(2), 270–287. https://doi.org/10.1007/s12197-020-09515-7

Auerbach, A. J., and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, *4*(2), 1–27. https://doi.org/10.1257/pol.4.2.1

Bai, J., and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, *70*(1), 191–221. https://doi.org/10.1111/1468-0262.00273

Bai, J., and Ng, S. (2005). Tests for skewness, kurtosis, and normality for time series data. *Journal of Business & Economic Statistics*, *23*(1), 49–60. https://doi.org/10.1198/073500104000000271

Bańbura, M., and Modugno, M. (2014). Maximum likelihood estimation of factor models on datasets with arbitrary pattern of missing data. *Journal of Applied Econometrics*, *29*(1), 133–160. https://doi.org/10.1002/jae.2306

Barigozzi, M., Cho, H., and Fryzlewicz, P. (2018). Simultaneous multiple change-point and factor analysis for high-dimensional time series. *Journal of Econometrics*, *206*(1), 187–225. https://doi.org/10.1016/j.jeconom.2018.05.003

Barro, R. J. (2023). r minus g. *Review of Economic Dynamics*, *48*, 1–17. https://doi.org/10.1016/j.red.2022.10.001

Baum, A., Checherita-Westphal, C., and Rother, P. (2013). Debt and growth: New evidence for the euro area. *Journal of International Money and Finance*, *32*, 809–821. https://doi.org/10.1016/j.jimonfin.2012.07.004

Bénassy-Quéré, A., and Borgy, V. (2026). Rethinking the policy mix: Challenges for the euro area. *Intereconomics*, *61*(3), 159–166. https://doi.org/10.2478/ie-2026-0031

Berger, R. L., and Boos, D. D. (1994). P values maximized over a confidence set for the nuisance parameter. *Journal of the American Statistical Association*, *89*(427), 1012–1016. https://doi.org/10.2307/2290928

Blanchard, O. (2019). Public debt and low interest rates. *American Economic Review*, *109*(4), 1197–1229. https://doi.org/10.1257/aer.109.4.1197

Bocola, L., and Dovis, A. (2019). Self-fulfilling debt crises: A quantitative analysis. *American Economic Review*, *109*(12), 4343–4377. https://doi.org/10.1257/aer.20161471

Bohn, H. (1998). The behavior of U. S. public debt and deficits. *The Quarterly Journal of Economics*, *113*(3), 949–963. https://doi.org/10.1162/003355398555793

Breitung, J., and Eickmeier, S. (2011). Testing for structural breaks in dynamic factor models. *Journal of Econometrics*, *163*(1), 71–84. https://doi.org/10.1016/j.jeconom.2010.11.008

Brunnermeier, M. K., and Sannikov, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, *104*(2), 379–421. https://doi.org/10.1257/aer.104.2.379

Chai, H., Harris, J., and Tieman, A. (2024). Beyond debt: Net worth fiscal anchors. *IMF Working Papers*, *2024*(137), 1. https://doi.org/10.5089/9798400280290.001

Chen, L., Dolado, J. J., and Gonzalo, J. (2014). Detecting big structural breaks in large factor models. *Journal of Econometrics*, *180*(1), 30–48. https://doi.org/10.1016/j.jeconom.2014.01.006

Clark, T. E., and West, K. D. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics*, *138*(1), 291–311.

Cole, H. L., and Kehoe, T. J. (2000). Self-fulfilling debt crises. *The Review of Economic Studies*, *67*(1), 91–116. https://doi.org/10.1111/1467-937X.00123

Corsetti, G., and Müller, G. J. (2006). Twin deficits: Squaring theory, evidence and common sense. *Economic Policy*, *21*(48), 598–638. https://doi.org/10.1111/j.1468-0327.2006.00167.x

Despois, T., and Doz, C. (2023). Identifying and interpreting the factors in factor models via sparsity: Different approaches. *Journal of Applied Econometrics*, *38*(4), 533–555. https://doi.org/10.1002/jae.2967

Dhingra, S., Huang, H., Ottaviano, G., Pessoa, J. P., Sampson, T., and van Reenen, J. (2017). The costs and benefits of leaving the EU: Trade effects. *Economic Policy*, *32*(92), 651–705. https://doi.org/10.1093/epolic/eix015

Doz, C., Giannone, D., and Reichlin, L. (2011). A two-step estimator for large approximate dynamic factor models based on Kalman filtering. *Journal of Econometrics*, *164*(1), 188–205. https://doi.org/10.1016/j.jeconom.2011.02.012

Égert, B. (2015). Public debt, economic growth and nonlinear effects: Myth or reality? *Journal of Macroeconomics*, *43*, 226–238. https://doi.org/10.1016/j.jmacro.2014.11.006

Erichson, N. B., Zheng, P., Manohar, K., Brunton, S. L., Kutz, J. N., and Aravkin, A. Y. (2020). Sparse principal component analysis via variable projection.

SIAM Journal on Applied Mathematics, *80*(2), 977–1002. https://doi.org/10.1137/18M1211350

Eurostat. (2026a, July 21). *Quarterly financial accounts for general government* (gov_10q_ggfa) [Data set]. https://doi.org/10.2908/GOV_10Q_GGFA

Eurostat. (2026b, July 21). *Quarterly government debt* (gov_10q_ggdebt) [Data set]. https://doi.org/10.2908/GOV_10Q_GGDEBT

Fan, J., Li, K., and Liao, Y. (2021). Recent developments in factor models and applications in econometric learning. *Annual Review of Financial Economics*, *13*(1), 401–430. https://doi.org/10.1146/annurev-financial-091420-011735

Fatás, A., and Summers, L. H. (2018). The permanent effects of fiscal consolidations. *Journal of International Economics*, *112*, 238–250. https://doi.org/10.1016/j.jinteco.2017.11.007

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). The generalized dynamic-factor model: Identification and estimation. *Review of Economics and Statistics*, *82*(4), 540–554. https://doi.org/10.1162/003465300559037

Forni, M., and Reichlin, L. (1998). Let's get real: A factor analytical approach to disaggregated business cycle dynamics. *Review of Economic Studies*, *65*(3), 453–473. https://doi.org/10.1111/1467-937x.00053

Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D., and Qureshi, M. S. (2013). Fiscal fatigue, fiscal space and debt sustainability in advanced economies. *The Economic Journal*, *123*(566), F4–F30. https://doi.org/10.1111/ecoj.12010

Gómez-Puig, M., Sosvilla-Rivero, S., and Martínez-Zarzoso, I. (2022). On the heterogeneous link between public debt and economic growth. *Journal of International Financial Markets, Institutions and Money*, *77*, 101528. https://doi.org/10.1016/j.intfin.2022.101528

Hadzi-Vaskov, M., and Ricci, L. A. (2016). *Does gross or net debt matter more for emerging market spreads?* (IMF Working Paper No. 16/246). International Monetary Fund.

Han, X., and Inoue, A. (2015). Tests for parameter instability in dynamic factor models. *Econometric Theory*, *31*(5), 1117–1152. https://doi.org/10.1017/S0266466614000486

Henao-Arbelaez, C., and Sobrinho, N. (2017). *Government financial assets and debt sustainability* (IMF Working Paper No. 17/173). International Monetary Fund.

Holmström, B., and Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, *106*(1), 1–40. https://doi.org/10.1086/250001

Horowitz, J. L. (2001). The bootstrap and hypothesis tests in econometrics. *Journal of Econometrics*, *100*(1), 37–40. https://doi.org/10.1016/S0304-4076(00)00051-8

Ilzetzki, E., Mendoza, E. G., and Végh, C. A. (2013). How big (small?) are fiscal multipliers? *Journal of Monetary Economics*, *60*(2), 239–254. https://doi.org/10.1016/j.jmoneco.2012.10.011

International Monetary Fund. (2025). *International financial statistics (IFS)* [Data set; legacy data portal, no longer online; the extracts used are in the replication package]. Retrieved June 2025, from https://legacydata.imf.org/?sk=4c514d48-b6ba-49ed-8ab9-52b0c1a0179b&sid=1390030341854

International Monetary Fund. (2026a). *National economic accounts (NEA), quarterly data* (IMF.STA:QNEA; Version 7.0.0) [Data set]. Retrieved September 13, 2026, from https://data.imf.org/en/datasets/IMF.STA:QNEA

International Monetary Fund. (2026b). *Quarterly government finance statistics (QGFS)* (IMF.STA:QGFS; Version 12.0.0) [Data set]. Retrieved September 17, 2026, from https://data.imf.org/en/datasets/IMF.STA:QGFS

Jordà, Ò., Schularick, M., and Taylor, A. M. (2017). Macrofinancial history and the new business cycle facts. *NBER Macroeconomics Annual*, *31*, 213–263. https://doi.org/10.1086/690241

Krugman, P. (1988). Financing vs. forgiving a debt overhang. *Journal of Development Economics*, *29*(3), 253–268. https://doi.org/10.1016/0304-3878(88)90044-2

Künsch, H. R. (1989). The jackknife and the bootstrap for general stationary observations. *The Annals of Statistics*, *17*(3), 1217–1241. https://doi.org/10.1214/aos/1176347265

Lazarus, E., Lewis, D. J., Stock, J. H., and Watson, M. W. (2018). HAR inference: Recommendations for practice. *Journal of Business & Economic Statistics*, *36*(4), 541–559.

Levy Yeyati, E., and Sturzenegger, F. (2023). A balance-sheet approach to fiscal sustainability. *Fiscal Studies*, *44*(1), 61–84. https://doi.org/10.1111/1475-5890.12319

Lim, J. J. (2019). Growth in the shadow of debt. *Journal of Banking & Finance*, *103*, 98–112. https://doi.org/10.1016/j.jbankfin.2019.04.002

Liu, Y. (2023). Government debt and risk premia. *Journal of Monetary Economics*, *136*, 18–34. https://doi.org/10.1016/j.jmoneco.2023.01.009

Mosley, L., Chan, T.-S. T., and Gibberd, A. (2024). The sparse dynamic factor model: A regularised quasi-maximum likelihood approach. *Statistics and Computing*, *34*(2), Article 68. https://doi.org/10.1007/s11222-023-10378-1

Mtar, K., and Belazreg, W. (2023). On the nexus of innovation, trade openness, financial development and economic growth in European countries: New perspective from a GMM panel VAR approach. *International Journal of Finance & Economics*, *28*(1), 766–791. https://doi.org/10.1002/ijfe.2449

Nakamura, E., and Steinsson, J. (2014). Fiscal stimulus in a monetary union: Evidence from US regions. *American Economic Review*, *104*(3), 753–792. https://doi.org/10.1257/aer.104.3.753

Obstfeld, M., and Rogoff, K. (1995). Exchange rate dynamics redux. *Journal of Political Economy*, *103*(3), 624–660. https://doi.org/10.1086/261997

OECD. (2026a). *Balance of payments* (OECD.SDD.TPS:DSD_BOP@DF_BOP; Version 1.0) [Data set]. Retrieved August 26, 2026, from https://data-explorer.oecd.org/vis?df[ds]=dsDisseminateFinalDMZ&df[id]=DSD_BOP%40DF_BOP&df[ag]=OECD.SDD.TPS

OECD. (2026b). *Financial market* (OECD.SDD.STES:DSD_STES@DF_FINMARK; Version 4.0) [Data set]. Retrieved August 20, 2026, from https://data-explorer.oecd.org/vis?df[ds]=dsDisseminateFinalDMZ&df[id]=DSD_STES%40DF_FINMARK&df[ag]=OECD.SDD.STES

Peppel-Srebrny, J. (2024). Hello from the other side: Both government liabilities and assets matter for sovereign risk. *Journal of Money, Credit and Banking*, *56*(6), 1595–1604. https://doi.org/10.1111/jmcb.13102

Philippon, T., and Schnabl, P. (2013). Efficient recapitalization. *The Journal of Finance*, *68*(1), 1–42. https://doi.org/10.1111/j.1540-6261.2012.01793.x

Ramey, V. A., and Zubairy, S. (2018). Government spending multipliers in good times and in bad: Evidence from US historical data. *Journal of Political Economy*, *126*(2), 850–901. https://doi.org/10.1086/696277

Seiferling, M., and Tareq, S. (2023). Hiding the losses: Fiscal transparency and the performance of government portfolios of financial assets. *Public Finance Review*, *51*(4), 488–512. https://doi.org/10.1177/10911421231170960

Stock, J. H., and Watson, M. W. (2002). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, *97*(460), 1167–1179. https://doi.org/10.1198/016214502388618960

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, *89*(425), 208–218. https://doi.org/10.2307/2291217

Tibshirani, R. (2011). Regression shrinkage and selection via the lasso: A retrospective. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, *73*(3), 273–282. https://doi.org/10.1111/j.1467-9868.2011.00771.x

Uematsu, Y., and Yamagata, T. (2023). Estimation of sparsity-induced weak factor models. *Journal of Business & Economic Statistics*, *41*(1), 213–227. https://doi.org/10.1080/07350015.2021.2008405

Veronesi, P., and Zingales, L. (2010). Paulson's gift. *Journal of Financial Economics*, *97*(3), 339–368. https://doi.org/10.1016/j.jfineco.2010.03.011

World Bank. (2026, August 11). *Quarterly public sector debt* [Data set]. Retrieved September 17, 2026, from https://databank.worldbank.org/source/quarterly-public-sector-debt

Yousefi, S. R. (2019). *Public sector balance sheet strength and the macro economy* (IMF Working Paper No. 19/170). International Monetary Fund.

Appendix A Mathematical derivations of Section 3

This appendix clarifies the derivation of formulas that are not explicit in section 3. For each country i, in period t, the government growth-net return $GNR_{i,t}$ defined as the difference between the interest on government debt ($i^{eff}_{i,t}$) and the GDP growth rate ($g_{i,t}$), is expressed as in (1):

$$GNR_{i,t} = i^{eff}_{i,t} - g_{i,t} \tag{1}$$

Growth-net return $GNR_{i,t}$ of government interventions may be derived from decomposing government debt dynamics. The outstanding legacy of government nominal gross debt $gd_{i,t}$ $gd_{i,t-1}$ at the end of the previous period $t-1$, will change in the present period t by the components: debt service $i^{eff}_{i,t}\, gd_{i,t-1}$; the amount of debt change (amortization or issues) $\Delta gd_{i,t}$; the nominal primary public budget balance $pb_{i,t}$; for our purposes, we include changes in financial assets holdings $\Delta fa_{i,t}$, contributing to the management of liquidity and financing government interventions in the economy; and stock flow adjustments and measurement components $sfa_{i,t}$. With these components, government debt dynamics can be decomposed as follows:

$$gd_{i,t} = (1 + i^{eff}_{i,t})gd_{i,t-1} - pb_{i,t} + \omega_{i,t}\Delta fa_{i,t} + sfa_{i,t} \tag{A.1}$$

As a function of GDP in period t, $Y_{i,t}$, this equation comes:

$$\frac{gd_{i,t}}{Y_{i,t}} = \frac{(1 + i^{eff}_{i,t})gd_{i,t-1}}{Y_{i,t}} - \frac{pb_{i,t}}{Y_{i,t}} + \frac{\omega_{i,t}\Delta fa_{i,t}}{Y_{i,t}} + \frac{sfa_{i,t}}{Y_{i,t}} \tag{A.2}$$

Adjusting for GDP in period $t-1$ and subsequent growth in period t, $g_{i,t}$:

$$\frac{gd_{i,t}}{Y_{i,t}} = \frac{(1 + i^{eff}_{i,t})gd_{i,t-1}}{(1 + g_{i,t})Y_{i,t-1}} - \frac{pb_{i,t}}{Y_{i,t}} + \frac{\omega_{i,t}\Delta fa_{i,t}}{Y_{i,t}} + \frac{sfa_{i,t}}{Y_{i,t}} \tag{A.3}$$

Writing (A.3) in ratios to GDP, $GD_{i,t} \equiv gd_{i,t}/Y_{i,t}$, $PB_{i,t} \equiv pb_{i,t}/Y_{i,t}$ and $FA_{i,t} \equiv fa_{i,t}/Y_{i,t}$, changes notation in every term but one. The asset term of (A.3) is a flow over output, while the series measured is the change in the asset ratio, $\Delta FA_{i,t} \equiv FA_{i,t} - FA_{i,t-1}$, and the two differ by a term in the lagged ratio: $\Delta fa_{i,t}/Y_{i,t} = \Delta FA_{i,t} + [g_{i,t}/(1 + g_{i,t})]FA_{i,t-1}$. Defining the stock-flow adjustment to absorb that term, $SFA_{i,t} \equiv sfa_{i,t}/Y_{i,t} + \omega_{i,t}[g_{i,t}/(1 + g_{i,t})]FA_{i,t-1}$, so that (A.3) becomes exactly

$$GD_{i,t} = \frac{(1 + i^{eff}_{i,t})GD_{i,t-1}}{1 + g_{i,t}} - PB_{i,t} + \omega_{i,t}\Delta FA_{i,t} + SFA_{i,t} \tag{A.4}$$

and noticing that summing and subtracting by $g_{i,t}$, following the standard Fisher relation applied in the Gordon Growth literature:

$$\frac{1+i_{i,t}^{eff}}{1+g_{i,t}} = \frac{(1+i_{i,t}^{eff})-g_{i,t}+g_{i,t}}{1+g_{i,t}} = \frac{(i_{i,t}^{eff}-g_{i,t})+(1+g_{i,t})}{1+g_{i,t}} = \frac{i_{i,t}^{eff}-g_{i,t}}{1+g_{i,t}}+1 \quad \text{(A.5)}$$

the debt dynamics may be expressed as:

$$GD_{i,t} = \left(\frac{i_{i,t}^{eff}-g_{i,t}}{1+g_{i,t}}+1\right) GD_{i,t-1} - PB_{i,t} + \omega_{i,t}\Delta FA_{i,t} + SFA_{i,t} \quad \text{(A.6)}$$

$$\Delta GD_{i,t} = \frac{i_{i,t}^{eff}-g_{i,t}}{1+g_{i,t}} GD_{i,t-1} - PB_{i,t} + \omega_{i,t}\Delta FA_{i,t} + SFA_{i,t} \quad (2)$$

whereas the differential between government interest rates and economic growth $i_{i,t}^{eff} - g_{i,t}$ comes finally as:

$$\frac{i_{i,t}^{eff}-g_{i,t}}{1+g_{i,t}} GD_{i,t-1} = \Delta GD_{i,t} + PB_{i,t} - \omega_{i,t}\Delta FA_{i,t} - SFA_{i,t} \quad \text{(A.7)}$$

$$GNR_{i,t} = i_{i,t}^{eff} - g_{i,t} = \frac{1+g_{i,t}}{GD_{i,t-1}} \left(\Delta GD_{i,t} + PB_{i,t} - \omega_{i,t}\Delta FA_{i,t} - SFA_{i,t}\right) \quad (3)$$

Instead of relying on $r-g$ from Equation (3), derived from the debt dynamics, we step back and model the effects of government interventions to the real economy and sovereign bond pricing depending on underlying states, which are only imperfectly observed, represented by an unobserved latent state vector assumed to load on a small number of common latent forces that a dynamic factor model estimates from the data:

$$s_{i,t} \equiv (s_{1,i,t}, \ldots, s_{m,i,t})' \quad (4)$$

These states $s_{i,t}$, that may include the country institutional quality $I_{i,t}$, government debt stress swaying risk premia δ_{GD} and other features, will jointly determine policy multipliers on growth and borrowing costs. The country institutional quality $I_{i,t}$ and government debt stress $\{S_I, \delta_{GD}\}$ are represented by logistic functional forms. Institutional quality $I_{i,t}$ is mapped into a bounded effectiveness scaler $S_I \in (0,1]$ as the distance to an institutional quality threshold $\bar{I}$, with κ_I a steepness parameter scaling a sharper or smoother effect:

$$S_I(I_{i,t}) = [1+\exp(-\kappa_I(I_{i,t}-\bar{I}))]^{-1}, \quad \kappa_I > 0 \quad (5)$$

A bounded debt-stress scaler δ_{GD} of the distance to a country-specific debt threshold GD_i^{th} captures rollover risk, fiscal fatigue and premia sensitivity, with a

hyperbolic tangent (tanh) functional form, as a standard normalized sigmoid. This smooth bounded transition functional form $\delta_{GD} \in (0,1)$ increases towards one as debt rises above the threshold, with κ_{GD} controlling steepness:

$$\delta_{GD}(GD_{i,t}) = \frac{1 + \tanh(\kappa_{GD}(GD_{i,t} - GD_i^{th}))}{2}, \quad \kappa_{GD} > 0 \tag{6}$$

On the economic growth side, $g_{i,t}$ we focus on drivers: gross capital formation ($GCF_{i,t}$), depending on its effect on growth $0 < \alpha_{1,i,t} < 1$; and current account balance ($CAB_{i,t}$), influencing growth with a coefficient $\alpha_{2,i,t}$, either positively $\alpha_{2,i,t} > 0$, or a negative constraint $\alpha_{2,i,t} < 0$, which are state $s_{i,t}$ dependent:

$$g_{i,t} = \alpha_1(s_{i,t})GCF_{i,t} + \alpha_2(s_{i,t})CAB_{i,t} + \varepsilon^g_{i,t} \tag{7}$$

We analyse government interventions concerning financial assets $\Delta FA_{i,t}$ and primary budgets $PB_{i,t}$, with impacts on the economy through transmission channels of gross capital formation ($GCF_{i,t}$) and current account balance ($CAB_{i,t}$), with multipliers $0 < \gamma_{1,i,t}(s_{i,t}) < 1$ and $0 < \tau_{1,i,t}(s_{i,t}) < 1$ on financial assets and $\gamma_{2,i,t}(s_{i,t})$ and $\tau_{2,i,t}(s_{i,t})$, unrestricted in sign, on the primary balance, contingent on states $s_{i,t}$, evidencing institutional quality $S_I(I_{i,t})$ (capturing effects from institution such as the rule of law, corruption, bureaucracy) and government debt stress $\delta_{GD}(GD_{i,t})$:

$$GCF_{i,t} = S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t}))[\gamma_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \gamma_{2,i,t}(s_{i,t})PB_{i,t}] + \varepsilon^I_{i,t} \tag{8}$$

$$CAB_{i,t} = S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t}))[\tau_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \tau_{2,i,t}(s_{i,t})PB_{i,t}] + \varepsilon^X_{i,t} \tag{9}$$

The signs of $\gamma_{2,i,t}$ and $\tau_{2,i,t}$ are not restricted: with $PB_{i,t}$ positive for a surplus, positive values would impose that a larger primary surplus raises investment and the current account balance, and nothing in this appendix or in Proposition 1, which holds $PB_{i,t}$ fixed, needs either sign. Replacing these two macroeconomic channels (8)–(9) through which government interventions are transmitted into the above economic growth equation (7), yields a reduced form for growth:

$$\begin{aligned} g_{i,t} = {} & \alpha_1(s_{i,t})\{S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t}))[\gamma_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \gamma_{2,i,t}(s_{i,t})PB_{i,t}]\} \\ & + \alpha_2(s_{i,t})\{S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t}))[\tau_{1,i,t}(s_{i,t})\Delta FA_{i,t} + \tau_{2,i,t}(s_{i,t})PB_{i,t}]\} \\ & + u^g_{i,t} \end{aligned} \tag{A.8}$$

evidencing $\Delta FA_{i,t}$ and $PB_{i,t}$ as,

$$g_{i,t} = \Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})\Delta FA_{i,t} + \Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t})PB_{i,t} + u^{g}_{i,t} \tag{10}$$

and taking multipliers from government interventions by financial assets as:

$$\Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t}) \equiv [\alpha_1(s_{i,t})\gamma_{1,i,t}(s_{i,t}) + \alpha_2(s_{i,t})\tau_{1,i,t}(s_{i,t})]S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t})) \tag{11}$$

and of primary budget as:

$$\Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t}) \equiv [\alpha_1(s_{i,t})\gamma_{2,i,t}(s_{i,t}) + \alpha_2(s_{i,t})\tau_{2,i,t}(s_{i,t})]S_I(I_{i,t})(1 - \delta_{GD}(GD_{i,t})). \tag{12}$$

The step from (12) to (15), on which Proposition 1 rests, is carried out here. Sovereign borrowing cost, the market yield $y^{10}_{i,t}$, is the sum of a global safe component r^*_t and a country-specific premium $\pi_{i,t}$:

$$y^{10}_{i,t} = r^*_t + \pi_{i,t} \tag{13}$$

and the premium is increasing in debt accumulation and in balance-sheet operations read as stress, the first scaled by debt stress and the second by debt stress and institutional weakness together:

$$\begin{aligned}\pi_{i,t} = {} & \beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})\Delta GD_{i,t} \\ & + \beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1 - S_I(I_{i,t}))(\Delta FA_{i,t} - PB_{i,t}) + \varepsilon^{\pi}_{i,t}\end{aligned} \tag{14}$$

with $\beta_{GD}(s_{i,t}) > 0$. Under the model's equality of the effective cost and the market yield, $i^{eff}_{i,t} = y^{10}_{i,t}$ (Section 3), substituting (13) into the definition (1) of the growth-net return gives

$$GNR_{i,t} = i^{eff}_{i,t} - g_{i,t} = y^{10}_{i,t} - g_{i,t} = r^*_t + \pi_{i,t} - g_{i,t} \tag{A.9}$$

which imposes nothing beyond that equality. The two behavioural equations of the model then replace $\pi_{i,t}$ and $g_{i,t}$. Collecting the three disturbances of (7)–(9), the composite growth disturbance of (A.8) is

$$u^{g}_{i,t} = \varepsilon^{g}_{i,t} + \alpha_1(s_{i,t})\varepsilon^{I}_{i,t} + \alpha_2(s_{i,t})\varepsilon^{X}_{i,t} \tag{A.10}$$

so that (10) carries the whole of the growth block, transmission channels and disturbances alike. Substituting (14) for $\pi_{i,t}$ and (10) for $g_{i,t}$ in (A.9):

$$
\begin{aligned}
GNR_{i,t} = r_t^* &+ \beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})\Delta GD_{i,t} \\
&+ \beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1 - S_I(I_{i,t}))(\Delta FA_{i,t} - PB_{i,t}) \\
&- \Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})\Delta FA_{i,t} - \Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t})PB_{i,t} \\
&+ \varepsilon_{i,t}^{\pi} - u_{i,t}^{g}
\end{aligned}
\tag{A.11}
$$

Writing the two disturbances as one,

$$
\eta_{i,t} \equiv \varepsilon_{i,t}^{\pi} - u_{i,t}^{g} = \varepsilon_{i,t}^{\pi} - \varepsilon_{i,t}^{g} - \alpha_1(s_{i,t})\varepsilon_{i,t}^{I} - \alpha_2(s_{i,t})\varepsilon_{i,t}^{X} \tag{A.12}
$$

expression (A.11) is (15), whose braces name its three groups of terms:

$$
\begin{aligned}
GNR_{i,t} = r_t^* &+ \underbrace{\beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})\Delta GD_{i,t}}_{\text{debt channel}} \\
&+ \underbrace{\beta_{FA,PB}(s_{i,t})\delta_{GD}(GD_{i,t})(1 - S_I(I_{i,t}))(\Delta FA_{i,t} - PB_{i,t})}_{\text{premium channel}} \\
&- \underbrace{\left(\Phi^{FA}(s_{i,t}, I_{i,t}, GD_{i,t})\Delta FA_{i,t} + \Phi^{PB}(s_{i,t}, I_{i,t}, GD_{i,t})PB_{i,t}\right)}_{\text{growth channel}} + \eta_{i,t}
\end{aligned}
\tag{15}
$$

Two features of this derivation bear on Proposition 1. The first is that the debt identity plays no part in it. The steps above use (1), (10), (13) and (14), with the model's equality $i_{i,t}^{eff} = y_{i,t}^{10}$, and (15) cannot be reached with (3), because (3) carries the stock-flow adjustment $SFA_{i,t}$ and the financing share $\omega_{i,t}$ and neither of them appears in (15). That the identity is unused does not make it idle. It is a second representation of the same left-hand side, so (3) and (15) hold at once and determine $GNR_{i,t}$ and $\Delta GD_{i,t}$ jointly. Equation (15) is accordingly a structural equation of a two-equation system, in which $\Delta GD_{i,t}$ enters on the right-hand side as a conditioning variable, and not a reduced form.

The second bears directly on Proposition 1, and follows from the first. Write $\delta \equiv \delta_{GD}(GD_{i,t})$, $S \equiv S_I(I_{i,t})$, $\beta_1 \equiv \beta_{GD}(s_{i,t})$, $\beta_2 \equiv \beta_{FA,PB}(s_{i,t})$, and Φ^{FA} and Φ^{PB} for the two multipliers of (11)–(12), all evaluated at (i,t). The derivative the proposition signs is the partial derivative of (15) holding $\Delta GD_{i,t}$ and $PB_{i,t}$ fixed,

$$
\frac{\partial GNR_{i,t}}{\partial \Delta FA_{i,t}} = \beta_2\delta(1 - S) - \Phi^{FA} \tag{A.13}
$$

but $\Delta GD_{i,t}$ cannot be held fixed while $\Delta FA_{i,t}$ moves, because (2) ties the two together. Substituting (2) into (15) and collecting the terms in $GNR_{i,t}$ gives

$$(1 - A_{i,t})\, GNR_{i,t} = r_t^* + \beta_1\delta\left(\omega_{i,t}\Delta FA_{i,t} - PB_{i,t} + SFA_{i,t}\right) + \beta_2\delta(1-S)(\Delta FA_{i,t} - PB_{i,t}) - \Phi^{FA}\Delta FA_{i,t} - \Phi^{PB}PB_{i,t} + \eta_{i,t} \tag{A.14}$$

where

$$A_{i,t} \equiv \beta_1\delta\frac{GD_{i,t-1}}{1+g_{i,t}} \tag{A.15}$$

so that, holding $PB_{i,t}$ and $SFA_{i,t}$ fixed, holding the two scalers fixed as (A.13) and Proposition 1 both do, letting $\Delta GD_{i,t}$ adjust as the identity requires, and provided $A_{i,t} \neq 1$,

$$\frac{\mathrm{d}GNR_{i,t}}{\mathrm{d}\Delta FA_{i,t}} = \frac{\beta_1\delta\omega_{i,t} + \beta_2\delta(1-S) - \Phi^{FA} - \beta_1\delta GD_{i,t-1}GNR_{i,t}\Phi^{FA}(1+g_{i,t})^{-2}}{1 - A_{i,t}} \tag{A.16}$$

Equation (A.16) differs from (A.13) in three places: a mechanical financing term $\beta_1\delta\omega_{i,t} \geq 0$, the premium charged on the debt issued to buy the assets; a term through growth, the last in the numerator; and a multiplier $(1-A_{i,t})^{-1}$, the feedback from the premium through the debt stock and back. Proposition 1 signs (A.13) in two limits, and both conclusions carry over to (A.16), the first unconditionally and the second under one further restriction. When institutions are strong and debt stress is absent, $S \to 1$ and $\delta \to 0$, every term carrying δ vanishes, $A_{i,t} \to 0$, and (A.16) collapses to $-\Phi^{FA}$, which is negative exactly when $\alpha_1(s_{i,t})\gamma_{1,i,t}(s_{i,t}) + \alpha_2(s_{i,t})\tau_{1,i,t}(s_{i,t}) > 0$. That inequality is not an extra assumption: by (11) it is precisely what the proposition's hypothesis that Φ^{FA} is increasing in S_I and decreasing in δ_{GD} amounts to, and it is worth making explicit only because $\alpha_2(s_{i,t})$ is permitted to be negative. When institutions are weak and debt stress is high, $S \to 0$ and $\delta \to 1$, $\Phi^{FA} \to 0$ by (11) and the numerator of (A.16) tends to $\beta_1\omega_{i,t} + \beta_2$, which is positive provided $\beta_{FA,PB}(s_{i,t}) > 0$; (14) imposes only $\beta_{GD}(s_{i,t}) > 0$, but the proposition's second hypothesis, that the premium sensitivity is increasing in δ_{GD} and in $1 - S_I$, is again exactly this restriction.

The denominator is a different matter, because nothing in the paper implies it. The sign of (A.16) is the sign of its numerator only if $A_{i,t} < 1$, that is if $\beta_{GD}(s_{i,t})\delta_{GD}(GD_{i,t})GD_{i,t-1} < 1 + g_{i,t}$. Since $A_{i,t} \geq 0$, the multiplier $(1-A_{i,t})^{-1}$ then amplifies the numerator; at $A_{i,t} > 1$ it is negative and gives (A.16) the opposite sign to its numerator. Only the high-stress conclusion is exposed to it: $A_{i,t} \to 0$ as $\delta_{GD} \to 0$, so the condition cannot fail near the first limit. Equations (5), (6) and (14) bound δ_{GD} and S_I but place no upper bound on $\beta_{GD}(s_{i,t})$, so $A_{i,t} < 1$ is

an additional restriction, and Section 3 states it with Proposition 1. At plausible magnitudes it is slack: $\delta_{GD} \leq 1$ and $GD_{i,t-1}$ is of order one in these economies, so it asks only that a unit rise in the debt ratio raise the premium by less than a unit. It is nonetheless a condition, and it is assumed and not tested anywhere in this paper. With it, and with the two hypotheses of the proposition read as the sign restrictions on (11) and (14) that they are, both conclusions hold for (A.16) as well as for (A.13). Without it, the high-stress conclusion holds only for (A.13), which conditions on a variable the identity does not leave free.

The last term in the numerator of (A.16) comes from growth. The coefficient $A_{i,t}$ of (A.15) carries $g_{i,t}$ in its denominator, and (10) ties $g_{i,t}$ to $\Delta FA_{i,t}$ through Φ^{FA}, so a derivative that lets the identity adjust must also differentiate $1/(1+g_{i,t})$; doing so contributes $-\beta_1 \delta GD_{i,t-1} GNR_{i,t} \Phi^{FA}/(1+g_{i,t})^2$. It vanishes in both limits, in the first because $\delta \to 0$ and in the second because $\Phi^{FA} \to 0$, so neither sign conclusion depends on it; between them it does not vanish, which is why it is displayed.

Equation (A.16) is the derivative that lets $\Delta GD_{i,t}$ adjust conditional on the two scalers and on $SFA_{i,t}$, and not a fully total one. Holding S_I fixed is no restriction, because it depends only on the institutional index, which the model takes as given. Holding δ_{GD} and $SFA_{i,t}$ fixed is a restriction. The debt-stress scaler is a function of the contemporaneous debt ratio, which (2) ties to $\Delta FA_{i,t}$, and by (6) its derivative is $\delta'_{GD} = \frac{1}{2}\kappa_{GD}\,\mathrm{sech}^2(\kappa_{GD}(GD_{i,t} - GD_i^{th}))$. Letting δ_{GD} respond adds terms to both the numerator and the denominator of (A.16), each proportional to δ'_{GD}, which tends to zero as δ_{GD} tends to either of its bounds. The stock-flow adjustment defined above (A.4) contains $\omega_{i,t}[g_{i,t}/(1+g_{i,t})]FA_{i,t-1}$, which moves with $g_{i,t}$ and so with $\Delta FA_{i,t}$; letting it move replaces $GD_{i,t-1}GNR_{i,t}$ by $GD_{i,t-1}GNR_{i,t} - \omega_{i,t}FA_{i,t-1}$ in the last term of the numerator, which still vanishes in both limits. The fully total derivative therefore has the same two limits as (A.16), and neither sign conclusion turns on what (A.16) holds fixed. Between the limits the omitted terms need not vanish and (A.16) is not the whole derivative, a further reason to read Proposition 1 as a statement about its two limits and not as a comparative static along the path between them.

Appendix B Variables description and computations

Table B1: Variables description and computations

Acronym	*Variable name*	*Computation*	*Source/Description*
GNR	Gov. growth net returns (% per year)	$r - g$	Government debt growth net returns: the effective interest rate on government debt less the growth rate of nominal GDP. The effective rate is the four-quarter sum of general government interest expense over the gross debt stock four quarters earlier, built from the interest and debt sources of the PB and ΔGD rows; nominal GDP growth is the growth of the four-quarter GDP total of the g row. It is not the yield of the y^{10} row.
g	GDP growth rate	Year-on-year % change of the four-quarter GDP total defined below, the g of the GNR row: $g[Q] = (\mathrm{GDP}_Y[Q]/\mathrm{GDP}_Y[Q-4] - 1) \times 100$; the conventional measure, kept as a labelled alternative, takes the year-on-year change of single-quarter GDP, $\left(\mathrm{GDP}_Q[Q]/\mathrm{GDP}_Q[Q-4] - 1\right) \times 100$; Four-quarter total, the denominator of every ratio in this table: $\mathrm{GDP}_Y[Q] = \sum_{k=0}^{3} \mathrm{GDP}_Q[Q-k]$. The two do not coincide, and the difference is material: exact accounting for a debt ratio built on GDP_Y requires the growth of GDP_Y itself, which for Austria at 2010Q1 is -0.85 per cent where the growth of GDP_Q is $+1.52$ per cent. Both series are nominal, so this is a question of temporal aggregation. It is why the $r - g$ of this paper uses the growth of the four-quarter total, which the accounting of the debt ratio this table defines requires; the paragraph after the table sizes the difference.	IMF Quarterly National Economic Accounts, dataflow `IMF.STA:QNEA(7.0.0)`, series key {`ISO3`}`.B1GQ.V.SA.XDC.Q`. Retrieved 10 September 2026. Current prices, seasonally adjusted, domestic currency.

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Table B1 – continued from the previous page

Acronym	*Variable name*	*Computation*	*Source/Description*
y^{10}	Government bond yield (marginal)	Long-term interest rates: government bonds (10 years). Determined by financial markets; yield on sovereign debt bonds issued by central governments, as a benchmark for long-term credit conditions. This is the marginal rate on new issuance and is not the rate entering $r - g$, which is the effective rate of the GNR row.	OECD financial market statistics, agency `OECD.SDD.STES`, dataflow `DSD_STES@DF_FINMARK(4.0)`, measure `IRLT` (long-term interest rates), per cent per annum, quarterly. Central government bonds (excludes regional/local debt). Retrieved 23 June 2025 and in August 2026. For Greece, the 43 quarters from 2015Q3 to 2026Q1 carry the value the OECD reports for that quarter. The August 2026 extract has no Greek observation for 2015Q3, which is taken instead from the June 2025 retrieval; that gap had led the compilations of the database drawn from that extract to hold the following quarter's value in each of the 43, a difference of up to 1.73 percentage points. The nested forecast comparison of Appendix J, whose window ends in 2019Q4, re-dated the affected quarters itself and so already used these values.
CAB	Current account balance	Annual % over GDP. Nominal annualized: sum of the quarter [Q] with previous three quarters [Q-1, Q-2, Q-3]: $\mathrm{CAB}_Y[Q] = \sum_{k=0}^{3} \mathrm{CAB}_Q[Q-k]$; Annual over GDP: $\frac{\mathrm{CAB}_Y}{\mathrm{GDP}_Y} \times 100$	OECD. Total economy current account to the rest of the world. Balance (revenue minus expenditure). Entry: Transactions. Frequency: Quarterly. Agency `OECD.SDD.TPS`, balance-of-payments dataflow `DSD_BOP@DF_BOP(1.0)`, measure `CA`; national currency, calendar and seasonally adjusted. Retrieved in August 2026.

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Table B1 – continued from the previous page

Acronym	*Variable name*	*Computation*	*Source/Description*
GCF	Gross capital formation / Investment	Annual % over GDP. Nominal annualized: sum of the actual quarter [Q] with the previous three quarters [Q-1, Q-2, Q-3]: $\mathrm{GCF}_Y[Q] = \sum_{k=0}^{3} \mathrm{GCF}_Q[Q-k]$; Annual over GDP: $\frac{\mathrm{GCF}_Y}{\mathrm{GDP}_Y} \times 100$	IMF Quarterly National Economic Accounts, dataflow `IMF.STA:QNEA(7.0.0)`, series key `{ISO3}.P5.V.SA.XDC.Q`. Retrieved 13 September 2026. Current prices, seasonally adjusted, domestic currency, over GDP on the same basis, so that this ratio is nominal over nominal like the other ratios of this table. Gross capital formation: acquisitions less disposals of fixed capital, inventories and valuables.
ΔGD	Change in government gross debt	Nominal annual over GDP: $\frac{\mathrm{GD}_Q}{\mathrm{GDP}} = \frac{\mathrm{GD}_Q}{\mathrm{GDP}_Y} \times 100$ Annual change over GDP: $\Delta\mathrm{GD}_Y[Q] = \left(\frac{\mathrm{GD}_Q[Q]}{\mathrm{GDP}_Y[Q]} - \frac{\mathrm{GD}_Q[Q-4]}{\mathrm{GDP}_Y[Q-4]}\right) \times 100$	Worldbank Quarterly Public Sector Debt. Gross PSD, General Gov., All maturities, All Instruments, Nominal Value, National Currency [DP.DOD.DECT.CR.GG]. Corresponds to the D4 debt-coverage category in the IMF–World Bank Public Sector Debt Statistics (PSDS). Includes: Debt securities; Loans; Currency and deposits; Insurance, pensions, and standardized guarantee schemes; Other accounts payable; Special Drawing Rights (SDRs). Database last updated 11 August 2026; retrieved 30 August 2026. See IMF (2011), *Public sector debt statistics: Guide for compilers and users*, International Monetary Fund, DOI: 10.5089/9781616351564.069.

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Table B1 – continued from the previous page

Acronym	*Variable name*	*Computation*	*Source/Description*
ΔFA	Change in government assets holdings	Nominal annual over GDP: $\frac{\text{FA}_Q}{\text{GDP}} = \frac{\text{FA}_Q}{\text{GDP}_Y} \times 100$; Annual change in government assets holdings over GDP: $\Delta\text{FA}_Y[Q] = \left(\frac{\text{FA}_Q[Q]}{\text{GDP}_Y[Q]} - \frac{\text{FA}_Q[Q-4]}{\text{GDP}_Y[Q-4]}\right) \times 100$	IMF Quarterly Government Finance Statistics, dataflow `IMF.STA:QGFS(12.0.0)`, indicator `FA_FA_SP_XDC`: general government, balance sheet, financial assets, stock positions; domestic currency, quarterly, compiled on the *Government Finance Statistics Manual 2001* (GFSM 2001) methodology. Retrieved 25 August 2026. Only the total is used, so currency and deposits, debt securities, loans and equity are not separated in the factor model. The extract carries no valuation flag; the GFSM 2001 framework values financial assets at current market prices, whereas the gross debt of the ΔGD row is at nominal value. The series is a stock of financial assets and so contains no contingent liabilities, guarantees included. The workbook the ratios are built from carried Ireland's 1999Q1 asset stock forward unchanged to 2000Q2; those five quarters are taken instead from this series, which agrees with the workbook exactly in 1999Q1 and from 2000Q3 on. Only the net-anchor exercise of Section 6.6 uses Ireland's asset stock at 2000Q2; the factor model's Ireland asset series begins in 2001Q3.
PB	Primary budget balance	Annual % over GDP: $\text{PB}_Q = \text{NLB}_Q + \text{Interest}_Q$ Annual: sum of the actual quarter [Q] with the previous three quarters [Q-1, Q-2, Q-3]: $\text{PB}_Y[Q] = \sum_{k=0}^{3} \text{PB}_Q[Q-k]$; Annual over GDP: $\frac{\text{PB}_Y}{\text{GDP}_Y} \times 100$	IMF Quarterly Government Finance Statistics, dataflow `IMF.STA:QGFS(12.0.0)`: general government, statement of operations; domestic currency, quarterly, GFSM 2001 methodology. Net lending (+)/net borrowing (−), transactions [NLB], indicator `GNLB_T_XDC`, retrieved 14 August 2026; interest payable, transactions, retrieved in August 2026, which is also the interest expense of the GNR row.

The difference between the four-quarter growth of the g row and the single-quarter growth of the conventional measure can be quantified without re-estimating anything. Substituting the growth of the four-quarter GDP total for the growth of the single quarter, holding the effective-interest term and the coverage fixed, changes the sign of 132 of the 1,233 observed $r - g$ cells dated through 2019Q4, with a mean absolute change of 1.24 points of the differential and a maximum of 24.57; across all 1,637 observed cells the figures are 168 and 1.56 points. The effect is largest in the six economies where a single quarter's year-on-year growth and the growth of the four-quarter total diverge most: through 2019Q4, Ireland moves by 3.12 points on average, Latvia by 3.00, Luxembourg by 2.00, Slovakia by 1.73, Greece by 1.31 and Slovenia by 1.30. These are comparisons of constructed data and not of re-estimated results. S14, a secondary analysis of the forecast test of Appendix J, uses the substituted measure and is reported in Table J1. This paper adopts the four-quarter measure as its $r - g$ (Section 4) and keeps the single-quarter one as a labelled alternative; the forecast comparison of Appendix J, fixed before that choice, stays on the single-quarter measure.

Appendix C Estimation period and stationarity

Table C1 reports Augmented Dickey–Fuller tests for every variable and country, on the series in levels, first differences and second differences, together with the window over which each series is tested and the resulting order of integration.

Table C1: Augmented Dickey–Fuller stationarity tests, by variable and country

Country	*Start*	*End*	*Level*	Δ	Δ^2	*Order*
Panel A: GNR — government growth-net return ($r-g$)						
Austria	2001Q4	2025Q4	−3.042	−5.170***	−6.319***	I(1)
Belgium	2001Q1	2025Q4	−2.650	−4.124***	−6.819***	I(1)
Finland	2001Q1	2025Q4	−2.605	−5.075***	−7.145***	I(1)
France	2001Q1	2025Q4	−2.295	−4.181***	−8.336***	I(1)
Germany	2002Q4	2025Q4	−2.895	−3.898**	−6.669***	I(1)
Greece	2001Q1	2024Q4	−1.278	−3.744**	−6.731***	I(1)
Ireland	2002Q4	2025Q4	−1.915	−4.126***	−7.003***	I(1)
Italy	2001Q1	2025Q4	−2.301	−4.034**	−7.364***	I(1)
Latvia	2001Q1	2025Q3	−2.599	−3.420*	−4.797***	I(2)
Luxembourg	2002Q4	2025Q4	−2.170	−3.617**	−6.318***	I(1)
Netherlands	2001Q1	2025Q4	−2.296	−3.990**	−6.791***	I(1)
Portugal	2001Q1	2025Q4	−2.086	−4.222***	−7.260***	I(1)
Slovakia	2007Q4	2025Q4	−4.001**	−4.492***	−6.727***	I(0)
Slovenia	2002Q2	2025Q4	−1.915	−3.266*	−6.549***	I(2)
Spain	2001Q1	2025Q4	−1.719	−4.064***	−7.374***	I(1)
United Kingdom	2001Q1	2026Q1	−2.335	−4.838***	−8.355***	I(1)
United States	2001Q4	2025Q4	−2.357	−3.792**	−6.731***	I(1)
Panel B: y^{10} — government bond yield (marginal)						
Austria	2001Q1	2026Q1	−1.156	−4.301***	−6.408***	I(1)
Belgium	2001Q1	2026Q1	−0.998	−4.392***	−6.279***	I(1)
Finland	2001Q1	2026Q1	−0.892	−4.219***	−6.407***	I(1)
France	2000Q1	2026Q1	−0.669	−4.150***	−7.095***	I(1)
Germany	2001Q1	2026Q1	−0.619	−4.223***	−7.001***	I(1)
Greece	2001Q1	2026Q1	−2.196	−3.683**	−6.282***	I(1)
Ireland	2001Q1	2026Q1	−1.810	−4.273***	−6.337***	I(1)
Italy	2001Q1	2026Q1	−2.371	−4.274***	−7.009***	I(1)
Latvia	2001Q1	2026Q1	−2.234	−5.288***	−5.940***	I(1)
Luxembourg	2001Q1	2026Q1	−1.804	−3.801**	−6.155***	I(1)
Netherlands	2001Q1	2026Q1	−1.006	−4.187***	−6.615***	I(1)
Portugal	2001Q1	2025Q4	−2.232	−3.970**	−6.285***	I(1)
Slovakia	2001Q1	2026Q1	−1.712	−4.315***	−6.474***	I(1)
Slovenia	2002Q2	2026Q1	−2.204	−3.333*	−5.898***	I(2)
Spain	2001Q1	2026Q1	−2.012	−3.259*	−6.710***	I(2)
United Kingdom	2001Q1	2026Q1	−0.998	−4.034**	−6.764***	I(1)
United States	2001Q1	2026Q1	−1.713	−3.826**	−7.506***	I(1)

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Table C1 – continued from the previous page

Country	*Start*	*End*	*Level*	Δ	Δ^2	*Order*
Panel C: CAB — current account balance						
Austria	2001Q1	2026Q1	−2.906	−4.525***	−8.125***	I(1)
Belgium	2003Q4	2026Q1	−3.418*	−7.160***	−8.163***	I(1)
Finland	2001Q1	2026Q1	−0.678	−6.060***	−9.805***	I(1)
France	2000Q3	2026Q1	−2.112	−6.996***	−8.558***	I(1)
Germany	2001Q1	2026Q1	−1.701	−4.489***	−7.082***	I(1)
Greece	2002Q4	2026Q1	−1.388	−3.321*	−8.403***	I(2)
Ireland	2002Q4	2026Q1	−2.850	−5.089***	−5.725***	I(1)
Italy	2001Q1	2026Q1	−1.596	−4.508***	−7.408***	I(1)
Latvia	2001Q1	2026Q1	−2.278	−3.340*	−5.258***	I(2)
Luxembourg	2001Q1	2026Q1	−4.122***	−6.957***	−8.018***	I(0)
Netherlands	2004Q1	2026Q1	−2.783	−5.468***	−7.194***	I(1)
Portugal	2001Q1	2025Q4	−1.760	−3.053	−5.219***	I(2)
Slovakia	2004Q4	2026Q1	−1.777	−5.106***	−6.064***	I(1)
Slovenia	2001Q1	2026Q1	−1.576	−4.184***	−6.631***	I(1)
Spain	2001Q1	2026Q1	−1.768	−2.442	−6.266***	I(2)
United Kingdom	2001Q1	2026Q1	−1.562	−4.236***	−9.227***	I(1)
United States	2001Q1	2026Q1	−1.538	−2.852	−6.170***	I(2)
Panel D: GCF — gross capital formation						
Austria	2001Q1	2026Q1	−2.216	−4.230***	−6.061***	I(1)
Belgium	2001Q1	2026Q1	−2.882	−4.480***	−6.402***	I(1)
Finland	2001Q1	2026Q1	−2.520	−4.523***	−6.684***	I(1)
France	2000Q3	2026Q1	−2.458	−3.391*	−5.208***	I(2)
Germany	2001Q1	2026Q1	−3.408*	−4.378***	−6.178***	I(1)
Greece	2001Q1	2026Q1	−0.749	−3.386*	−6.331***	I(2)
Ireland	2001Q1	2026Q1	−2.165	−5.117***	−6.147***	I(1)
Italy	2001Q1	2026Q1	−1.265	−4.064***	−5.258***	I(1)
Latvia	2001Q1	2026Q1	−2.589	−3.158*	−5.496***	I(2)
Luxembourg	2001Q1	2026Q1	−2.232	−6.883***	−10.561***	I(1)
Netherlands	2001Q1	2026Q1	−2.268	−5.213***	−8.540***	I(1)
Portugal	2001Q1	2025Q4	−1.396	−3.680**	−5.876***	I(1)
Slovakia	2001Q1	2026Q1	−2.811	−5.481***	−6.527***	I(1)
Slovenia	2001Q1	2026Q1	−1.485	−3.277*	−5.214***	I(2)
Spain	2001Q1	2026Q1	−1.325	−2.288	−5.498***	I(2)
United Kingdom	2001Q1	2026Q1	−1.785	−4.439***	−7.513***	I(1)
United States	2001Q1	2026Q1	−2.194	−2.372	−5.105***	I(2)

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Table C1 – continued from the previous page

Country	*Start*	*End*	*Level*	Δ	Δ^2	*Order*
Panel E: ΔGD — change in government gross debt						
Austria	2001Q3	2026Q1	-3.563^{**}	-6.152^{***}	-7.430^{***}	I(0)
Belgium	2001Q3	2026Q1	-3.115	-5.513^{***}	-7.157^{***}	I(1)
Finland	2001Q3	2026Q1	-3.625^{**}	-6.162^{***}	-6.121^{***}	I(0)
France	2001Q3	2026Q1	-3.058	-5.714^{***}	-8.463^{***}	I(1)
Germany	2001Q3	2026Q1	-2.137	-4.248^{***}	-8.808^{***}	I(1)
Greece	2001Q3	2026Q1	-2.658	-6.561^{***}	-7.771^{***}	I(1)
Ireland	2001Q3	2026Q1	-2.004	-4.358^{***}	-7.501^{***}	I(1)
Italy	2001Q1	2026Q1	-2.922	-4.746^{***}	-7.217^{***}	I(1)
Latvia	2001Q3	2026Q1	-2.554	-4.442^{***}	-6.772^{***}	I(1)
Luxembourg	2001Q3	2026Q1	-3.086	-7.728^{***}	-10.573^{***}	I(1)
Netherlands	2001Q3	2026Q1	-2.570	-5.074^{***}	-7.968^{***}	I(1)
Portugal	2001Q3	2026Q1	-2.205	-4.665^{***}	-7.326^{***}	I(1)
Slovakia	2007Q4	2026Q1	-3.302^{*}	-4.800^{***}	-6.656^{***}	I(1)
Slovenia	2001Q3	2026Q1	-2.484	-5.847^{***}	-8.321^{***}	I(1)
Spain	2001Q3	2026Q1	-1.815	-4.805^{***}	-7.738^{***}	I(1)
United Kingdom	2001Q3	2026Q1	-3.125	-5.754^{***}	-7.097^{***}	I(1)
United States	2001Q3	2026Q1	-2.672	-4.729^{***}	-8.187^{***}	I(1)
Panel F: ΔFA — change in government financial assets						
Austria	2001Q3	2025Q4	-3.836^{**}	-6.242^{***}	-8.299^{***}	I(0)
Belgium	2001Q3	2025Q4	-2.737	-5.211^{***}	-7.397^{***}	I(1)
Finland	2001Q3	2025Q4	-4.096^{***}	-5.762^{***}	-7.188^{***}	I(0)
France	2001Q3	2025Q4	-3.685^{**}	-5.628^{***}	-7.783^{***}	I(0)
Germany	2001Q3	2025Q4	-2.173	-5.529^{***}	-10.458^{***}	I(1)
Greece	2001Q3	2024Q4	-2.650	-4.703^{***}	-8.575^{***}	I(1)
Ireland	2001Q3	2025Q4	-2.663	-5.821^{***}	-8.207^{***}	I(1)
Italy	2001Q1	2025Q4	-3.467^{**}	-5.377^{***}	-8.116^{***}	I(0)
Latvia	2001Q3	2025Q3	-2.615	-4.635^{***}	-7.849^{***}	I(1)
Luxembourg	2001Q3	2025Q4	-3.173^{*}	-7.606^{***}	-9.190^{***}	I(1)
Netherlands	2001Q3	2025Q4	-3.271^{*}	-5.075^{***}	-7.365^{***}	I(1)
Portugal	2001Q3	2025Q4	-2.908	-5.091^{***}	-6.957^{***}	I(1)
Slovakia	2001Q3	2025Q4	-2.797	-4.894^{***}	-7.883^{***}	I(1)
Slovenia	2001Q3	2025Q4	-2.673	-5.770^{***}	-9.130^{***}	I(1)
Spain	2001Q3	2025Q4	-2.569	-4.956^{***}	-8.080^{***}	I(1)
United Kingdom	2001Q3	2026Q1	-2.653	-4.672^{***}	-7.975^{***}	I(1)
United States	2002Q1	2025Q4	-2.827	-5.450^{***}	-8.355^{***}	I(1)

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Table C1 – continued from the previous page

Country	*Start*	*End*	*Level*	Δ	Δ^2	*Order*
Panel G: PB — primary budget balance						
Austria	2001Q4	2025Q4	−2.283	−5.141***	−7.866***	I(1)
Belgium	2001Q1	2025Q4	−2.622	−4.177***	−7.056***	I(1)
Finland	2001Q1	2025Q4	−2.363	−4.413***	−5.552***	I(1)
France	2000Q3	2025Q4	−3.031	−5.123***	−5.759***	I(1)
Germany	2002Q4	2025Q4	−1.966	−4.456***	−5.906***	I(1)
Greece	2001Q1	2024Q4	−2.286	−4.746***	−6.674***	I(1)
Ireland	2002Q4	2025Q4	−2.297	−3.626**	−4.880***	I(1)
Italy	2001Q1	2025Q4	−2.133	−3.621**	−6.902***	I(1)
Latvia	2001Q1	2025Q3	−2.620	−3.515**	−6.040***	I(1)
Luxembourg	2002Q4	2025Q4	−2.553	−5.268***	−6.029***	I(1)
Netherlands	2001Q1	2025Q4	−2.640	−4.570***	−7.078***	I(1)
Portugal	2001Q1	2025Q4	−2.828	−4.387***	−6.816***	I(1)
Slovakia	2001Q1	2025Q4	−2.559	−4.082***	−6.352***	I(1)
Slovenia	2001Q1	2025Q4	−2.324	−4.931***	−6.699***	I(1)
Spain	2001Q1	2025Q4	−1.666	−3.874**	−5.917***	I(1)
United Kingdom	2001Q1	2026Q1	−2.931	−5.387***	−7.342***	I(1)
United States	2001Q4	2025Q4	−2.141	−4.962***	−6.798***	I(1)

Notes: Augmented Dickey–Fuller tests (`tseries::adf.test` in R), alternative hypothesis stationary, regression with constant and linear trend, lag order $k = \lfloor (T-1)^{1/3} \rfloor = 4$ throughout. *Level*, Δ and Δ^2 report the ADF t-statistic on the series in levels, first differences and second differences. ***, ** and * denote rejection of the unit-root null at the 1%, 5% and 10% level. `adf.test` interpolates p-values from the Dickey–Fuller table and censors them at 0.01, so *** marks $p \leq 0.01$. *Order* is the order of integration at the 5% level: I(0) if the null is rejected in levels, I(1) if in first differences, I(2) if only in second differences. *Start* and *End* are per series, not per country: see the final paragraph of these notes. PB is the primary *balance* on the surplus-positive convention of Table B1: a positive value is a primary surplus and a negative value a primary deficit. The heading `Gov.PrimaryDeficit` that the database column carries is a legacy misnomer and records no change of sign. The ADF t-statistic is in any case invariant to the sign of the series, so nothing in this table turns on the point. Every series is tested on its whole observed span. That is the sample the full-sample companion uses; it is not the sample the pooled specification this paper identifies from uses, and this paragraph closes on that difference. Each country enters with every quarter the database offers it, and the missing cells at the edges of that window are absorbed by the Kalman filter rather than removed by dropping quarters, so the panel is ragged and the seven series of a country need not share one window. *Start* and *End* are accordingly the observed span of the individual series, which runs from 73 to 105 quarters and is shorter than its country's own window for 76 of the 119 series. The span is contiguous in every case: the missingness in the database is edge-only, with no interior gaps, and the script that produces this table stops rather than test a series stitched across a hole. The window meant in what follows is the full observed window its country enters with — the span of that country's own data, which for every country runs past 2019Q4 — and not the 2000Q2–2019Q4 window the identifying specification is estimated on. *End* coincides with the end of that observed window wherever the series runs to the end of it, and *Start* precedes the beginning of that same observed window by one quarter, and by two where the country's recipe takes a second difference, because differencing costs leading observations. Every series is nonetheless tested to the end of its own span, while the pooled specification this paper identifies from stops at 2019Q4, so on that specification the sample the tests use and the sample the estimation uses do not coincide. The order of integration is determined on the full span, the transformation it implies is applied to the untruncated series, and the panel is cut at 2019Q4 only afterwards: the transformation each series enters with is therefore selected partly on quarters the identifying specification never sees. All 119 series end after 2019Q4, by between 20 and 25 quarters and by 25 for the median series, and 2,909 of the 11,729 observations the tests use — 24.8 per cent — lie beyond the identifying window. On the full-sample companion, whose panel runs to the end of the same spans, there is no mismatch. The recipe has not been re-derived on 2000Q2–2019Q4 alone, and we therefore do not report how many of the 119 entries such a re-derivation would move. Holding the transformations fixed across the specifications is deliberate, because it is what makes the three fits of Section 5 comparable to one another at all and the canonical correlations reported there between their factor spaces would not otherwise be interpretable; but the choice has this cost, and it is stated here rather than left to be inferred from the dates in the table.

Table C2 sets the transformation actually applied to each series against the order of integration its tests imply. The operative rule is rejection of the augmented Dickey–Fuller null at the ten per cent level: a series enters at the order at which the null is first rejected at that level. The transformations are derived from that rule, so the applied transformation matches the test verdict for all 119 series and none is marked †; the table is the check that the two have not drifted apart. Because the choice of level is itself a judgement, the stricter five per cent classification is reported alongside as a complementary check: 15 series agree at ten per cent but not at five, marked ‡, so agreement at the stricter level is 104 of 119. That set is listed beneath the table.

Table C2: Transformation applied to each series, against its estimated order of integration

Country	GNR	y^{10}	CAB	GCF	ΔGD	ΔFA	PB
Austria	Δy	Δy	Δy	Δy	y	y	Δy
Belgium	Δy	Δy	$y^{\ddagger}$	Δy	Δy	Δy	Δy
Finland	Δy	Δy	Δy	Δy	y	y	Δy
France	Δy	Δy	Δy	$\Delta y^{\ddagger}$	Δy	y	Δy
Germany	Δy	Δy	Δy	$y^{\ddagger}$	Δy	Δy	Δy
Greece	Δy	Δy	$\Delta y^{\ddagger}$	$\Delta y^{\ddagger}$	Δy	Δy	Δy
Ireland	Δy	Δy	Δy	Δy	Δy	Δy	Δy
Italy	Δy	Δy	Δy	Δy	Δy	y	Δy
Latvia	$\Delta y^{\ddagger}$	Δy	$\Delta y^{\ddagger}$	$\Delta y^{\ddagger}$	Δy	Δy	Δy
Luxembourg	Δy	Δy	y	Δy	Δy	$y^{\ddagger}$	Δy
Netherlands	Δy	Δy	Δy	Δy	Δy	$y^{\ddagger}$	Δy
Portugal	Δy	Δy	$\Delta^2 y$	Δy	Δy	Δy	Δy
Slovakia	y	Δy	Δy	Δy	$y^{\ddagger}$	Δy	Δy
Slovenia	$\Delta y^{\ddagger}$	$\Delta y^{\ddagger}$	Δy	$\Delta y^{\ddagger}$	Δy	Δy	Δy
Spain	Δy	$\Delta y^{\ddagger}$	$\Delta^2 y$	$\Delta^2 y$	Δy	Δy	Δy
United Kingdom	Δy	Δy	Δy	Δy	Δy	Δy	Δy
United States	Δy	Δy	$\Delta^2 y$	$\Delta^2 y$	Δy	Δy	Δy

Notes: y denotes the series in levels, Δy in first differences and $\Delta^2 y$ in second differences, as actually supplied to the estimator. The operative rule is the order at which the unit-root null is first rejected at the ten per cent level. † marks a series whose transformation differs from that verdict; ‡ marks one that agrees at ten per cent but not at the five per cent level reported in Table C1. No series is marked †. Series marked ‡: Belgium CAB (applied y, tests I(0) at 10%, I(1) at 5%); France GCF (applied Δy, tests I(1) at 10%, I(2) at 5%); Germany GCF (applied y, tests I(0) at 10%, I(1) at 5%); Greece CAB (applied Δy, tests I(1) at 10%, I(2) at 5%); Greece GCF (applied Δy, tests I(1) at 10%, I(2) at 5%); Latvia GNR (applied Δy, tests I(1) at 10%, I(2) at 5%); Latvia CAB (applied Δy, tests I(1) at 10%, I(2) at 5%); Latvia GCF (applied Δy, tests I(1) at 10%, I(2) at 5%); Luxembourg ΔFA (applied y, tests I(0) at 10%, I(1) at 5%); Netherlands ΔFA (applied y, tests I(0) at 10%, I(1) at 5%); Slovakia ΔGD (applied y, tests I(0) at 10%, I(1) at 5%); Slovenia GNR (applied Δy, tests I(1) at 10%, I(2) at 5%); Slovenia y^{10} (applied Δy, tests I(1) at 10%, I(2) at 5%); Slovenia GCF (applied Δy, tests I(1) at 10%, I(2) at 5%); Spain y^{10} (applied Δy, tests I(1) at 10%, I(2) at 5%). Under-differencing leaves a non-stationary regressor in the system; over-differencing induces a non-invertible moving-average unit root in the residuals, which is one candidate explanation for the Ljung–Box rejections reported in Appendix E.

Appendix D Factor loadings and scores

This appendix reports, country by country, the sparse dynamic factor model estimated on the seven series described in Appendix B. Each country is fitted separately with $r = 6$ factors and a single lag in the factor transition, using the EM-sparse algorithm of Mosley et al. (2024) with i.i.d. idiosyncratic errors. The transformation applied before estimation is country-specific: it follows each country's own stationarity results and spans levels, first differences and second differences. Table C2 in Appendix C records, for every country and every series, the transformation actually supplied to the estimator. Estimation windows differ across countries because coverage of the underlying series differs; each subsection states the window used. Units and definitions are those of Table B1 in Appendix B; the seven estimated series appear here under abbreviated names. $r - g$ is government growth net returns (GNR in that table), in per cent per year, and y^{10} the long-term (ten-year) government bond yield; Ext. Trade is the current account balance (CAB), CapitalFormation gross capital formation (GCF) and Primary balance the primary budget balance (PB), these three in per cent of GDP, the last on the surplus-positive convention of Table B1, recorded in Appendix C: a positive value is a primary surplus and a negative value a primary deficit; and ΔGov.Debt and ΔFinAssets are the annual changes in the ratios to GDP of gross government debt (ΔGD) and of government financial assets (ΔFA), which are therefore in percentage points of GDP.

D.1 Austria

Austria is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D1 shows the loading of each series on each factor and Figure D2 the same loadings as a heat map; Table D1 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.525); Factor 2, CapitalFormation (0.636); Factor 3, $r - g$ (-0.593); Factor 4, y^{10} (0.871); Factor 5, Primary balance (0.641); Factor 6, CapitalFormation (0.633). The sum of squared loadings is largest for y^{10} (0.993) and smallest for ΔFinAssets (0.298); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D1: Austria: factor loadings by series

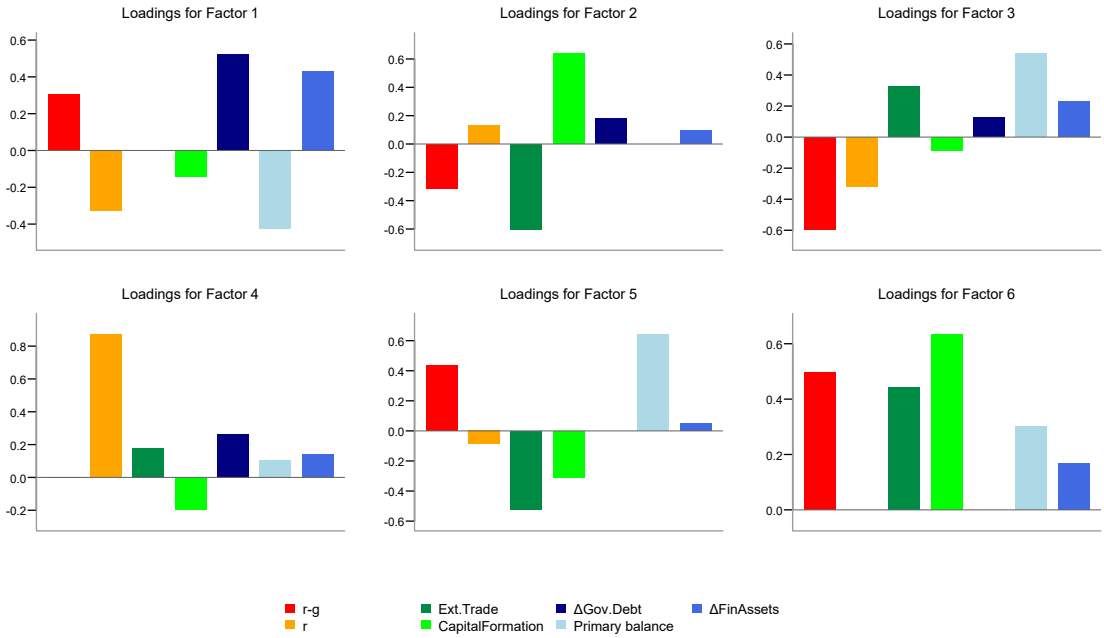


Figure D2: Austria: loading heat map

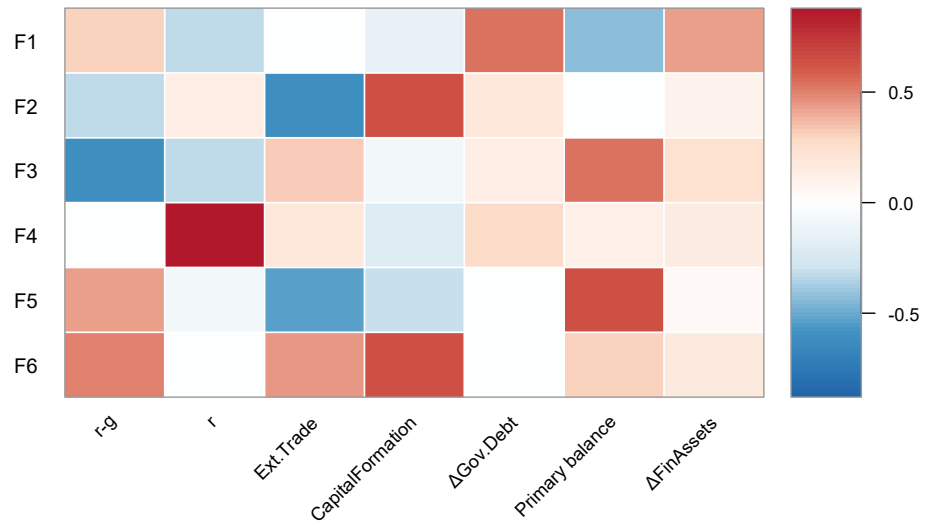


Table D1: Austria: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.305	-0.316	-0.593	-0.002	0.433	0.498	0.980
y^{10}	-0.329	0.132	-0.319	0.871	-0.087	0.000	0.993
Ext. Trade	0.000	-0.601	0.328	0.178	-0.525	0.443	0.972
CapitalFormation	-0.144	0.636	-0.086	-0.198	-0.310	0.633	0.969
ΔGov.Debt	0.525	0.178	0.128	0.264	0.000	0.000	0.393
Primary balance	-0.428	0.000	0.536	0.108	0.641	0.301	0.984
ΔFinAssets	0.430	0.093	0.228	0.145	0.048	0.169	0.298
$\sum \lambda^2$	0.866	0.924	0.925	0.931	0.980	0.964	5.590
Prop. of common var.	0.155	0.165	0.165	0.167	0.175	0.173	1.000

Figure D3 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D4 shows the seven underlying series. Table D2 lists the factor estimates quarter by quarter.

Figure D3: Austria: standardised factor estimates and standardised data

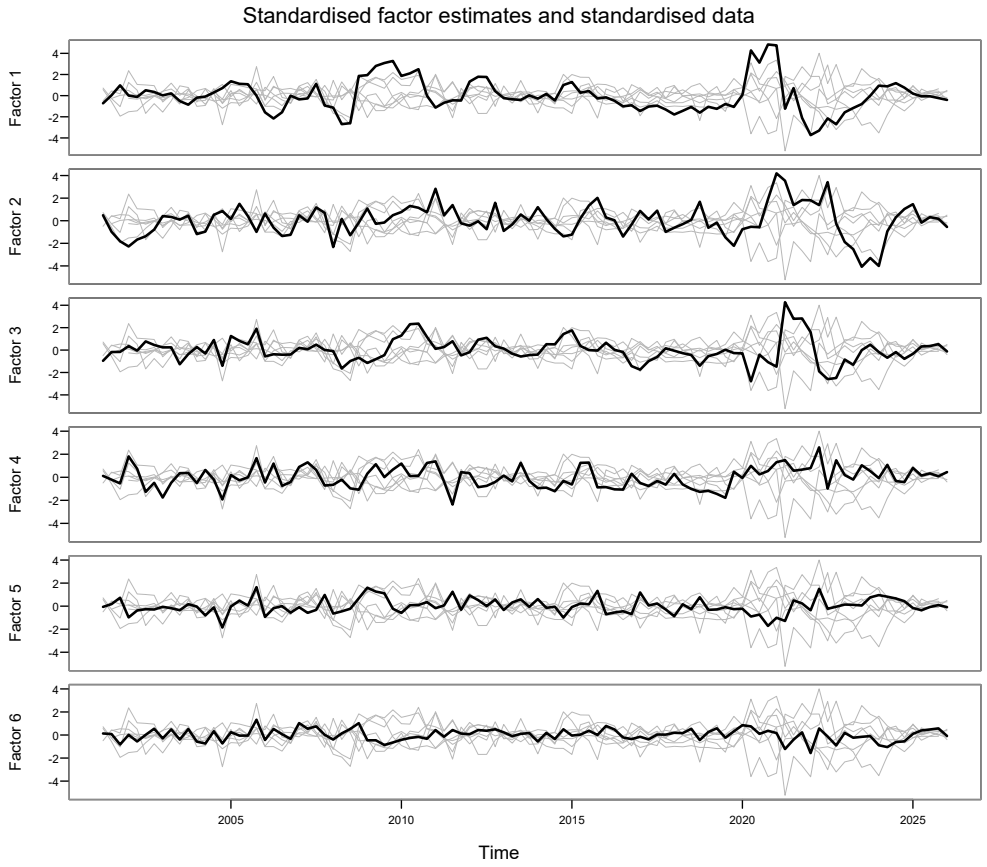


Figure D4: Austria: input series over the estimation window

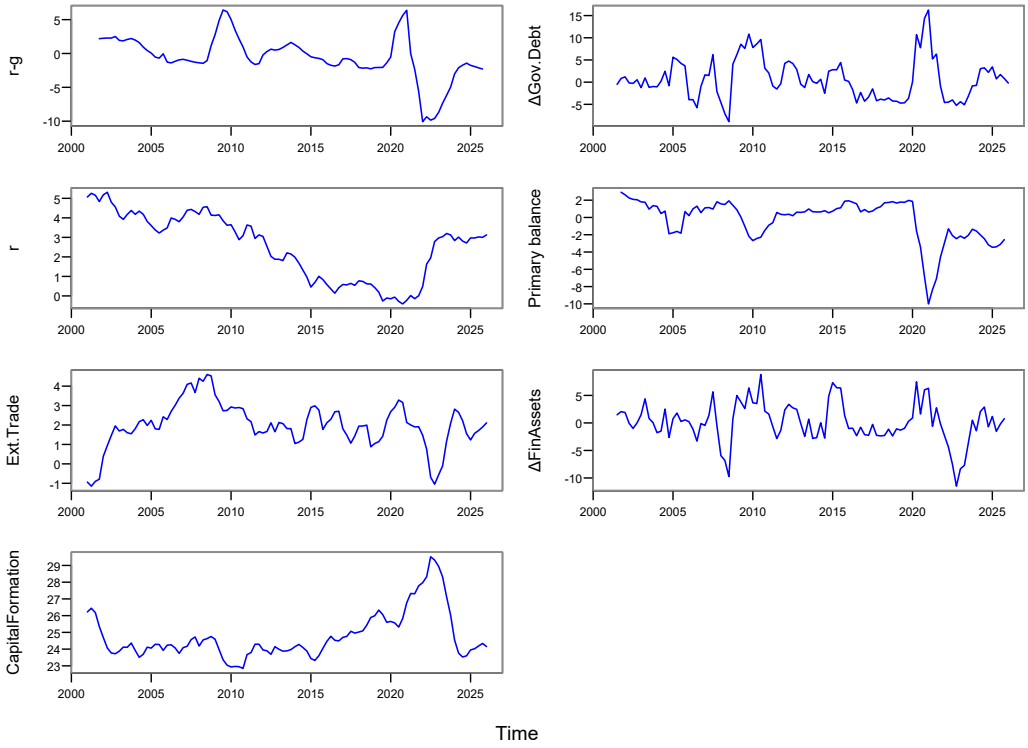


Table D2: Austria: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-0.723	0.482	-0.959	0.119	-0.063	0.133	2013Q4	0.033	0.001	-0.450	-0.320	-0.060	0.173
2001Q3	0.041	-0.981	-0.174	-0.207	0.181	0.083	2014Q1	-0.288	1.200	-0.403	-0.943	0.614	-0.568
2001Q4	0.976	-1.818	-0.164	-0.531	0.731	-0.823	2014Q2	0.168	0.158	0.523	-0.902	-0.149	0.188
2002Q1	0.005	-2.278	0.356	1.812	-0.978	0.013	2014Q3	-0.462	-0.712	0.512	-1.213	-0.046	-0.345
2002Q2	-0.123	-1.669	-0.050	0.740	-0.376	-0.558	2014Q4	1.002	-1.385	1.429	-0.320	-0.998	0.496
2002Q3	0.508	-1.375	0.773	-1.268	-0.265	0.016	2015Q1	1.284	-1.232	1.759	-0.626	-0.078	-0.050
2002Q4	0.348	-0.785	0.455	-0.479	-0.291	0.547	2015Q2	0.275	0.204	0.362	1.251	0.227	0.036
2003Q1	0.023	0.412	0.240	-1.758	-0.057	-0.291	2015Q3	0.437	1.353	-0.002	1.276	0.159	0.373
2003Q2	0.217	0.335	0.243	-0.473	-0.180	0.491	2015Q4	-0.272	2.025	-0.044	-0.876	1.327	0.003
2003Q3	-0.515	0.102	-1.266	0.356	-0.352	-0.395	2016Q1	-0.163	0.285	0.657	-0.855	-0.700	0.791
2003Q4	-0.856	0.446	-0.361	0.376	0.172	0.520	2016Q2	-0.487	0.040	0.022	-1.047	-0.551	0.477
2004Q1	-0.192	-1.183	0.273	-0.501	-0.036	-0.576	2016Q3	-1.011	-1.402	-0.179	-1.068	-0.443	-0.211
2004Q2	-0.086	-0.962	-0.308	0.636	-0.804	-0.737	2016Q4	-0.913	-0.335	-1.433	0.306	-0.690	-0.355
2004Q3	0.300	0.522	0.899	-0.196	-0.107	0.330	2017Q1	-1.455	0.872	-1.747	-0.491	1.188	-0.151
2004Q4	0.716	0.857	-1.407	-1.928	-1.856	-0.734	2017Q2	-1.032	0.121	-0.985	-0.760	0.061	-0.381
2005Q1	1.362	0.153	1.257	0.194	-0.028	0.248	2017Q3	-0.947	0.892	-0.620	-0.321	0.224	0.044
2005Q2	1.118	1.496	0.818	-0.266	0.471	-0.053	2017Q4	-1.314	-0.983	0.161	-0.637	-0.286	0.035
2005Q3	1.076	0.405	0.514	-0.002	0.048	-0.049	2018Q1	-1.786	-0.622	-0.058	0.298	-0.876	0.194
2005Q4	0.000	-0.989	1.909	1.663	1.649	1.325	2018Q2	-1.437	-0.301	-0.277	-0.623	0.161	0.162
2006Q1	-1.578	0.633	-0.565	-0.455	-0.932	-0.433	2018Q3	-1.067	0.075	-0.441	-1.011	-0.268	0.542
2006Q2	-2.162	-0.607	-0.374	1.193	-0.182	0.526	2018Q4	-1.609	1.680	-1.401	-1.264	0.774	-0.440
2006Q3	-1.570	-1.354	-0.413	-0.739	0.006	0.069	2019Q1	-1.053	-0.626	-0.542	-1.175	-0.322	0.258
2006Q4	-0.002	-1.245	-0.399	-0.417	-0.567	-0.335	2019Q2	-1.249	-0.160	-0.347	-1.441	-0.283	0.584
2007Q1	-0.354	0.483	0.197	0.896	-0.092	1.035	2019Q3	-0.790	-1.465	0.033	-1.783	-0.090	-0.233
2007Q2	-0.265	-0.088	0.072	1.296	-0.582	0.542	2019Q4	-1.066	-2.224	-0.269	0.478	-0.262	0.317
2007Q3	1.115	1.180	0.485	0.644	-0.326	0.753	2020Q1	0.069	-0.750	-0.293	-0.059	-0.221	0.854
2007Q4	-0.931	0.698	0.011	-0.718	0.972	0.017	2020Q2	4.275	-0.536	-2.789	0.994	-0.901	0.748
2008Q1	-1.135	-2.330	-0.092	-0.650	-0.646	-0.393	2020Q3	3.121	-0.574	-0.405	0.251	-0.732	0.097
2008Q2	-2.696	0.163	-1.647	-0.204	-0.452	0.142	2020Q4	4.838	1.908	-1.075	0.545	-1.717	0.365
2008Q3	-2.601	-1.286	-0.981	-0.962	-0.227	0.525	2021Q1	4.753	4.194	-1.482	1.311	-1.006	0.171
2008Q4	1.860	-0.168	-0.682	-1.079	0.679	1.029	2021Q2	-1.240	3.542	4.271	1.489	-1.286	-1.218
2009Q1	1.939	1.084	-1.151	0.312	1.602	-0.459	2021Q3	0.715	1.382	2.790	0.565	0.498	-0.399
2009Q2	2.807	-0.274	-0.812	1.136	1.264	-0.458	2021Q4	-2.104	1.834	2.821	0.666	0.236	0.234
2009Q3	3.086	-0.158	-0.454	0.010	1.116	-0.865	2022Q1	-3.733	1.808	1.598	0.793	-0.361	-1.567
2009Q4	3.279	0.488	0.973	0.671	-0.229	-0.646	2022Q2	-3.303	1.384	-1.900	2.597	1.505	0.566
2010Q1	1.863	0.773	1.297	1.192	-0.588	-0.415	2022Q3	-2.151	3.416	-2.594	-0.983	-0.229	-0.130
2010Q2	2.106	1.311	2.309	0.103	0.082	-0.256	2022Q4	-2.711	-0.254	-2.491	1.473	-0.050	-0.909
2010Q3	2.501	1.148	2.360	0.124	0.095	-0.162	2023Q1	-1.579	-1.882	-0.844	0.194	0.138	0.189
2010Q4	-0.006	0.749	1.168	1.245	0.354	-0.331	2023Q2	-1.167	-2.515	-1.312	-0.198	0.116	-0.235
2011Q1	-1.131	2.830	0.109	1.382	-0.176	0.430	2023Q3	-0.795	-4.076	-0.006	1.043	0.054	-0.156
2011Q2	-0.676	0.481	0.270	-0.373	0.060	-0.149	2023Q4	-0.002	-3.301	0.484	0.536	0.763	-0.093
2011Q3	-0.442	1.387	0.790	-2.367	1.246	0.435	2024Q1	0.945	-4.004	-0.175	-0.070	0.958	-0.889
2011Q4	-0.464	-0.230	-0.475	0.444	-0.316	0.127	2024Q2	0.879	-0.938	-0.685	1.081	0.820	-1.047
2012Q1	1.326	-0.436	-0.189	0.353	0.926	0.072	2024Q3	1.184	0.303	-0.179	-0.322	0.665	-0.613
2012Q2	1.788	-0.045	0.926	-0.859	0.527	0.433	2024Q4	0.756	1.037	-0.784	-0.424	0.432	-0.549
2012Q3	1.764	-0.749	1.097	-0.745	0.004	0.372	2025Q1	0.193	1.465	-0.341	0.808	-0.163	0.099
2012Q4	0.423	1.589	0.337	-0.407	0.578	0.497	2025Q2	-0.064	-0.173	0.325	0.150	-0.354	0.397
2013Q1	-0.228	-0.903	0.137	0.147	-0.334	0.234	2025Q3	-0.059	0.319	0.358	0.336	-0.072	0.481
2013Q2	-0.346	-0.329	-0.322	-0.355	0.320	-0.099	2025Q4	-0.230	0.174	0.531	0.088	0.094	0.573
2013Q3	-0.404	0.553	-0.576	1.247	0.592	0.100	2026Q1	-0.402	-0.551	-0.120	0.451	-0.082	-0.081

D.2 Belgium

Belgium is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D5 shows the loading of each series on each factor and Figure D6 the same loadings as a heat map; Table D3 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.537); Factor 2, CapitalFormation (0.604); Factor 3, y^{10} (-0.671); Factor 4, Ext. Trade (0.570); Factor 5, CapitalFormation (-0.647); Factor 6, $r - g$ (0.778). The sum of squared loadings is largest for CapitalFormation (0.977) and smallest for ΔGov.Debt (0.372); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D5: Belgium: factor loadings by series

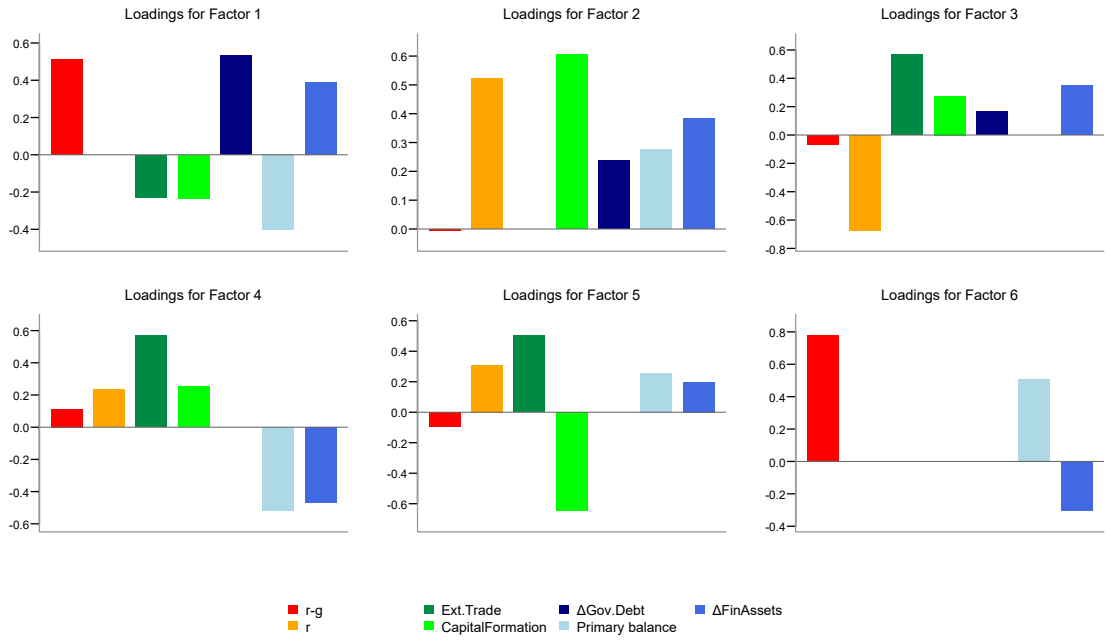


Figure D6: Belgium: loading heat map

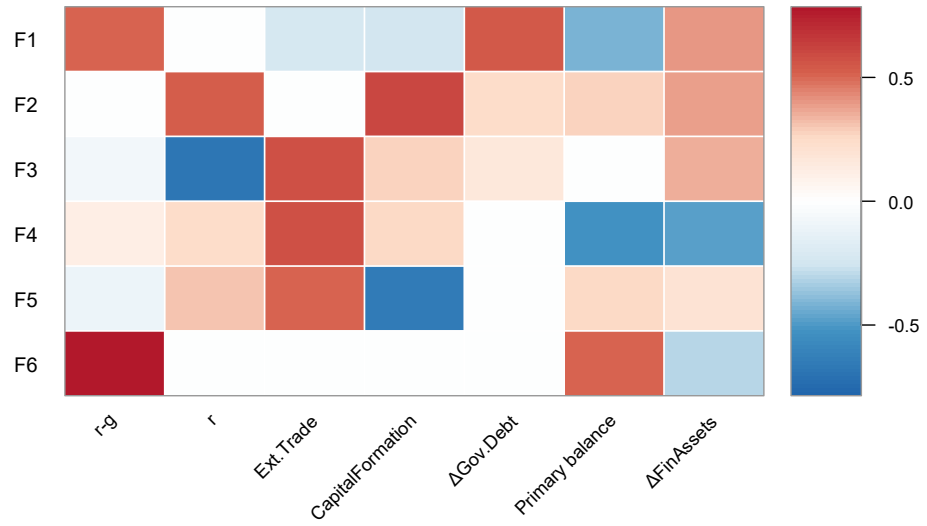


Table D3: Belgium: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.513	-0.004	-0.064	0.114	-0.093	0.778	0.893
y^{10}	0.000	0.523	-0.671	0.233	0.309	0.000	0.874
Ext. Trade	-0.229	0.000	0.566	0.570	0.503	0.000	0.951
CapitalFormation	-0.234	0.604	0.275	0.251	-0.647	0.000	0.977
ΔGov.Debt	0.537	0.237	0.167	0.000	0.000	0.000	0.372
Primary balance	-0.405	0.275	0.000	-0.520	0.257	0.508	0.834
ΔFinAssets	0.391	0.385	0.351	-0.472	0.198	-0.305	0.779
$\sum \lambda^2$	0.974	0.918	1.002	0.948	0.881	0.956	5.680
Prop. of common var.	0.172	0.162	0.176	0.167	0.155	0.168	1.000

Figure D7 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D8 shows the seven underlying series. Table D4 lists the factor estimates quarter by quarter.

Figure D7: Belgium: standardised factor estimates and standardised data

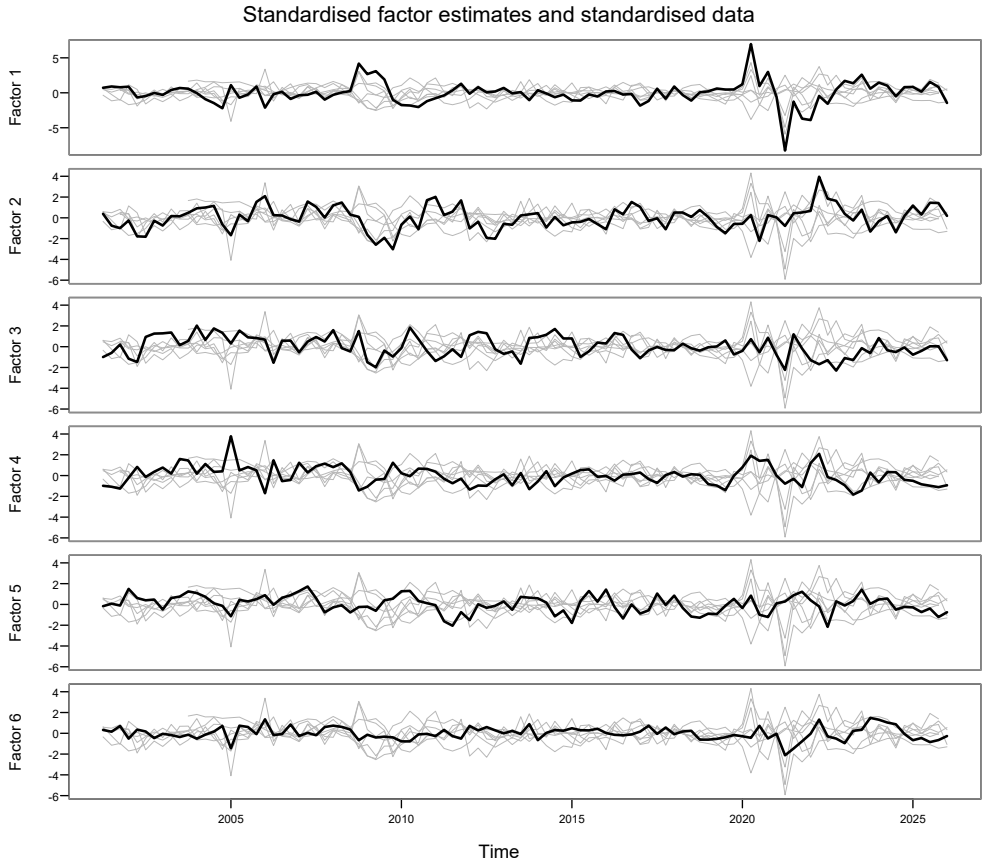


Figure D8: Belgium: input series over the estimation window

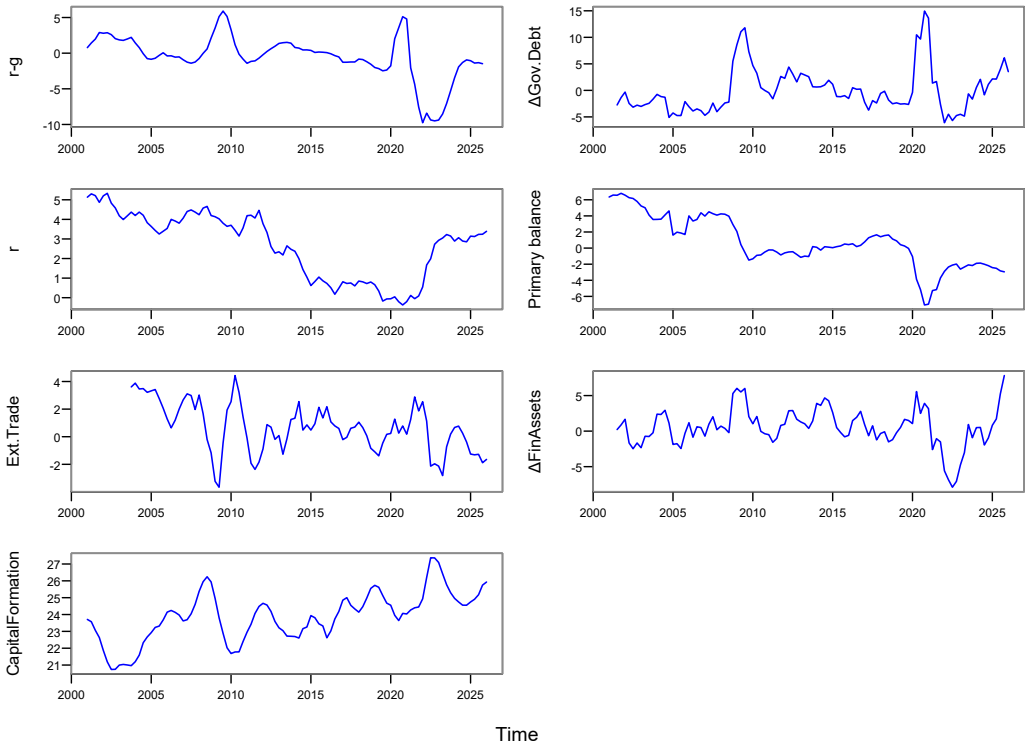


Table D4: Belgium: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.720	0.384	-0.997	-0.984	-0.170	0.321	2013Q4	-1.052	0.336	0.839	-1.312	0.649	0.885
2001Q3	0.903	-0.754	-0.596	-1.078	0.072	0.154	2014Q1	0.369	0.453	0.916	-0.570	0.588	-0.662
2001Q4	0.791	-0.992	0.211	-1.257	-0.108	0.702	2014Q2	-0.097	-0.937	1.150	0.423	0.181	0.042
2002Q1	0.874	-0.253	-1.156	-0.219	1.496	-0.518	2014Q3	-0.632	0.080	1.706	-1.023	-1.147	0.315
2002Q2	-0.661	-1.782	-1.477	0.834	0.616	0.358	2014Q4	-0.260	-0.719	0.793	-0.188	-0.574	0.248
2002Q3	-0.470	-1.820	0.941	-0.132	0.396	0.174	2015Q1	-1.089	-0.441	0.797	0.188	-1.784	0.485
2002Q4	-0.022	-0.278	1.262	0.366	0.471	-0.463	2015Q2	-1.108	-0.387	-0.992	0.529	0.315	0.299
2003Q1	-0.305	-0.743	1.285	0.770	-0.495	-0.047	2015Q3	-0.228	-0.112	-0.441	0.597	1.292	0.278
2003Q2	0.404	0.172	1.367	0.189	0.613	-0.174	2015Q4	-0.537	-0.605	0.398	-0.123	0.257	0.438
2003Q3	0.687	0.148	0.151	1.593	0.756	-0.358	2016Q1	0.194	-1.102	0.303	-0.034	1.431	0.029
2003Q4	0.581	0.490	0.554	1.452	1.255	-0.134	2016Q2	0.252	0.819	1.316	-0.495	-0.251	-0.145
2004Q1	-0.044	0.912	2.031	0.172	1.094	-0.524	2016Q3	-0.251	0.339	1.137	0.089	-1.358	-0.192
2004Q2	-0.912	0.994	0.652	1.115	0.736	-0.129	2016Q4	-0.174	1.527	-0.277	0.135	0.006	-0.109
2004Q3	-1.468	1.153	1.757	0.352	0.109	0.159	2017Q1	-1.830	1.050	-1.113	0.293	-0.877	0.163
2004Q4	-2.212	-0.641	1.336	0.409	-0.136	0.713	2017Q2	-1.168	-0.301	-0.358	-0.342	-0.565	0.730
2005Q1	1.098	-1.674	0.307	3.784	-1.142	-1.446	2017Q3	0.579	-0.044	-0.029	-0.709	1.048	-0.058
2005Q2	-0.707	0.309	1.550	0.496	0.434	0.733	2017Q4	-0.848	-1.100	-0.314	0.046	-0.048	0.556
2005Q3	-0.240	-0.326	0.900	0.822	0.289	0.610	2018Q1	0.902	0.507	-0.334	0.344	0.849	-0.082
2005Q4	0.897	1.536	0.831	0.497	0.508	-0.082	2018Q2	-0.320	0.512	0.287	-0.103	-0.338	0.174
2006Q1	-2.125	2.089	0.678	-1.702	0.884	1.346	2018Q3	-1.117	0.084	-0.148	0.128	-1.167	0.244
2006Q2	-0.201	0.255	-1.547	1.466	-0.036	-0.170	2018Q4	0.061	0.760	-0.383	0.073	-1.281	-0.611
2006Q3	0.145	0.225	0.575	-0.535	0.639	-0.073	2019Q1	0.207	0.078	-0.058	-0.828	-0.886	-0.610
2006Q4	-0.883	-0.132	0.585	-0.424	0.889	0.849	2019Q2	0.611	-0.871	0.026	-0.961	-0.934	-0.536
2007Q1	-0.357	-0.365	-0.543	1.234	1.277	-0.254	2019Q3	0.461	-1.479	0.602	-1.410	-0.133	-0.384
2007Q2	-0.280	1.571	0.489	0.298	1.725	0.042	2019Q4	0.476	-0.589	-0.749	-0.020	0.536	-0.164
2007Q3	0.142	1.053	0.934	0.902	0.587	-0.186	2020Q1	1.218	-0.575	-0.400	0.795	-0.348	-0.284
2007Q4	-0.974	0.023	0.497	1.148	-0.767	0.602	2020Q2	6.971	0.256	0.728	1.916	0.847	-0.434
2008Q1	-0.292	1.194	1.595	0.810	-0.283	0.737	2020Q3	0.958	-2.225	-0.535	1.413	-0.993	0.720
2008Q2	0.041	1.474	-0.133	1.176	-0.078	0.615	2020Q4	2.987	0.263	0.867	1.515	-1.213	-0.498
2008Q3	0.252	0.307	-0.460	0.352	-0.771	0.362	2021Q1	-0.587	0.041	-0.786	-0.009	0.103	-0.040
2008Q4	4.177	0.096	1.505	-1.435	-0.250	-0.668	2021Q2	-8.256	-0.780	-2.236	-0.781	0.281	-2.119
2009Q1	2.673	-1.630	-1.492	-1.070	-0.215	-0.137	2021Q3	-1.256	0.437	1.190	-0.311	0.884	-1.466
2009Q2	3.095	-2.591	-1.980	-0.395	-0.614	-0.405	2021Q4	-3.699	0.519	-0.175	-1.113	1.223	-0.768
2009Q3	1.909	-1.920	-0.369	-0.334	0.397	-0.324	2022Q1	-3.899	0.669	-1.289	1.208	0.402	-0.040
2009Q4	-1.000	-3.027	-0.954	1.237	0.548	-0.394	2022Q2	-0.450	3.957	-1.692	2.092	-0.189	1.321
2010Q1	-1.756	-0.661	-0.133	0.229	1.267	-0.776	2022Q3	-1.573	1.837	-1.295	-0.149	-2.158	-0.301
2010Q2	-1.817	0.131	1.845	-0.066	1.303	-0.779	2022Q4	0.471	1.636	-2.305	-0.425	0.300	-0.510
2010Q3	-2.041	-1.096	0.730	0.671	0.334	-0.110	2023Q1	1.710	0.385	-1.089	-0.933	-0.107	-0.951
2010Q4	-1.198	1.700	-0.424	0.644	0.120	-0.052	2023Q2	1.361	-0.257	-1.285	-1.832	0.326	0.238
2011Q1	-0.778	2.009	-1.379	0.370	-0.070	-0.270	2023Q3	2.596	0.786	-0.139	-1.445	1.419	0.337
2011Q2	-0.371	0.271	-0.926	-0.310	-1.621	0.329	2023Q4	0.589	-1.324	-0.621	0.288	0.059	1.483
2011Q3	0.361	0.592	-0.250	-0.744	-2.050	-0.268	2024Q1	1.457	-0.320	0.834	-0.652	0.479	1.310
2011Q4	1.297	1.680	-0.983	-0.295	-0.740	-0.514	2024Q2	1.020	0.178	-0.342	0.338	0.566	1.037
2012Q1	-0.132	-1.021	1.100	-1.353	-1.512	0.718	2024Q3	-0.514	-1.397	-0.517	0.323	-0.506	0.867
2012Q2	0.800	-0.378	1.429	-0.960	0.012	0.275	2024Q4	0.810	0.174	-0.068	-0.412	-0.239	-0.059
2012Q3	0.058	-1.932	1.294	-0.987	-0.324	0.601	2025Q1	0.837	1.185	-0.773	-0.505	-0.284	-0.668
2012Q4	0.186	-2.016	-0.248	-0.366	-0.131	0.286	2025Q2	0.165	0.308	-0.405	-0.839	-0.738	-0.454
2013Q1	0.701	-0.574	-0.690	0.118	0.289	-0.007	2025Q3	1.554	1.452	0.039	-0.978	-0.390	-0.847
2013Q2	-0.083	-0.670	-0.459	-0.945	-0.517	0.232	2025Q4	0.839	1.425	0.040	-1.115	-1.215	-0.652
2013Q3	0.104	0.226	-1.639	0.244	0.730	-0.073	2026Q1	-1.451	0.189	-1.302	-0.929	-0.751	-0.265

D.3 Finland

Finland is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D9 shows the loading of each series on each factor and Figure D10 the same loadings as a heat map; Table D5 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, CapitalFormation (-0.448); Factor 2, ΔFinAssets (0.595); Factor 3, y^{10} (0.715); Factor 4, y^{10} (0.589); Factor 5, CapitalFormation (0.734); Factor 6, ΔFinAssets (0.648). The sum of squared loadings is largest for Ext. Trade (0.989) and smallest for Primary balance (0.239); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D9: Finland: factor loadings by series

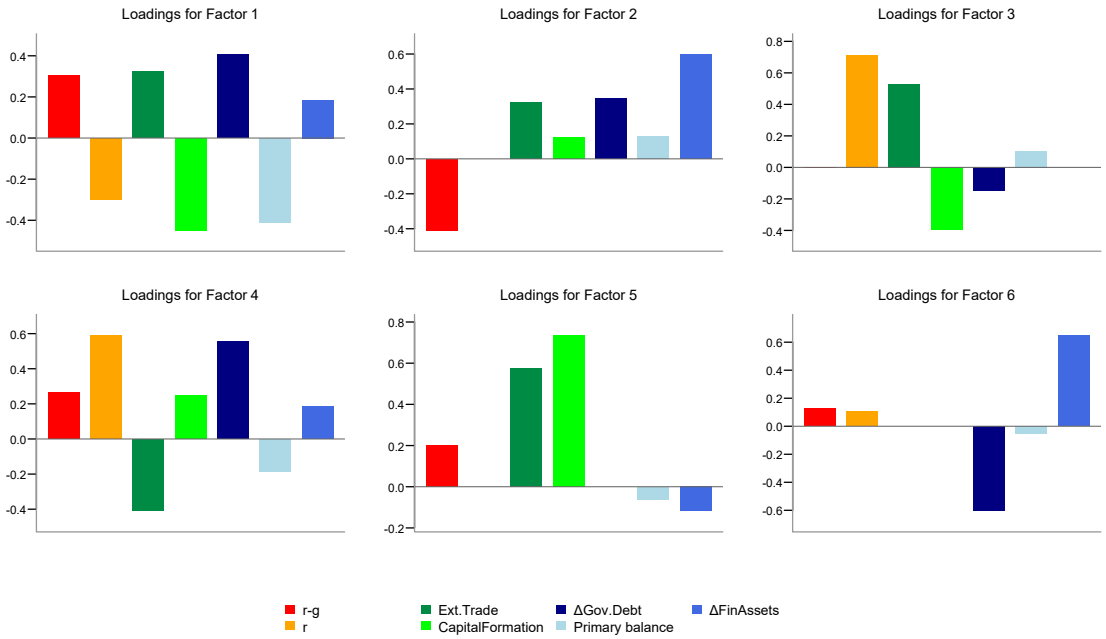


Figure D10: Finland: loading heat map

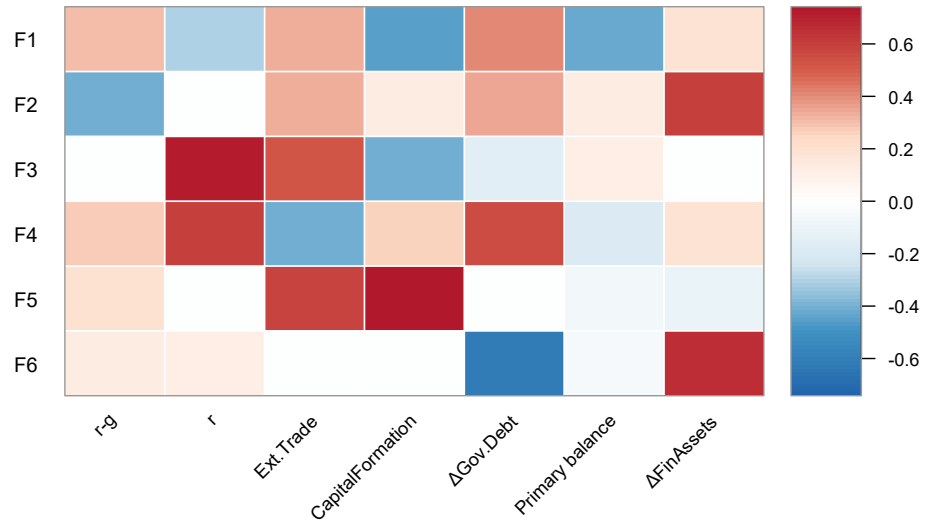


Table D5: Finland: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.306	-0.408	-0.005	0.265	0.204	0.126	0.388
y^{10}	-0.299	0.000	0.715	0.589	0.000	0.107	0.959
Ext. Trade	0.325	0.326	0.527	-0.409	0.576	0.000	0.989
CapitalFormation	-0.448	0.124	-0.398	0.250	0.734	0.000	0.977
ΔGov.Debt	0.405	0.346	-0.149	0.556	0.000	-0.604	0.980
Primary balance	-0.413	0.129	0.104	-0.184	-0.061	-0.053	0.239
ΔFinAssets	0.185	0.595	0.000	0.186	-0.117	0.648	0.857
$\sum \lambda^2$	0.858	0.780	0.980	1.026	0.930	0.815	5.388
Prop. of common var.	0.159	0.145	0.182	0.190	0.173	0.151	1.000

Figure D11 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D12 shows the seven underlying series. Table D6 lists the factor estimates quarter by quarter.

Figure D11: Finland: standardised factor estimates and standardised data

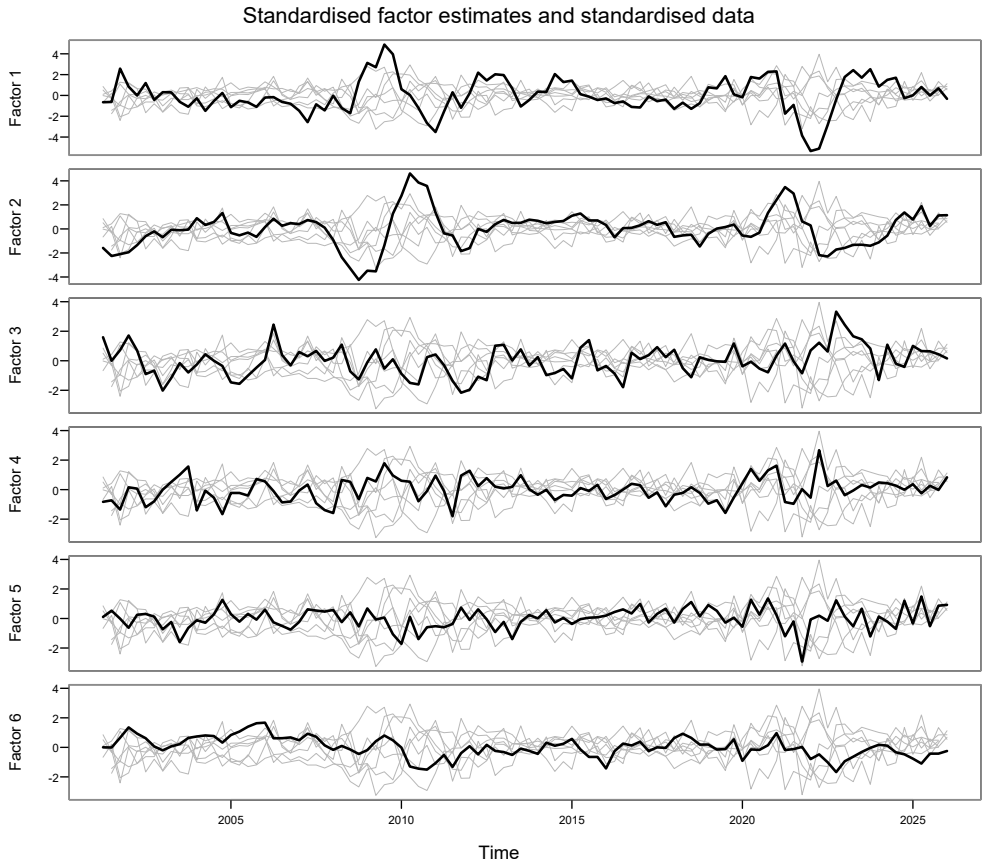


Figure D12: Finland: input series over the estimation window

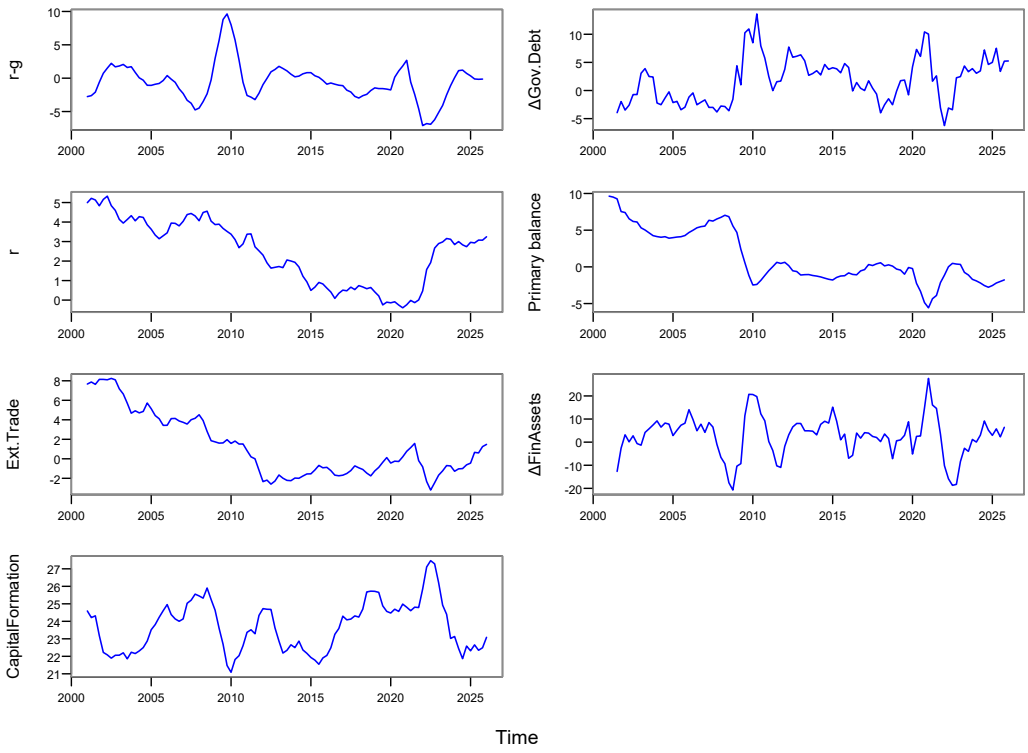


Table D6: Finland: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-0.654	-1.584	1.591	-0.827	0.106	0.003	2013Q4	-0.434	0.769	-0.334	0.027	0.235	-0.240
2001Q3	-0.626	-2.255	-0.001	-0.725	0.530	-0.011	2014Q1	0.368	0.678	0.252	-0.348	0.007	-0.441
2001Q4	2.576	-2.097	0.721	-1.350	-0.012	0.652	2014Q2	0.327	0.471	-0.974	-0.014	0.557	0.316
2002Q1	0.845	-1.940	1.710	0.156	-0.631	1.352	2014Q3	2.056	0.582	-0.828	-0.711	-0.270	0.110
2002Q2	-0.013	-1.346	0.693	0.068	0.254	0.928	2014Q4	1.281	0.661	-0.556	-0.361	0.042	0.241
2002Q3	1.191	-0.599	-0.903	-1.193	0.307	0.619	2015Q1	1.427	1.094	-1.194	-0.411	-0.371	0.574
2002Q4	-0.431	-0.204	-0.668	-0.792	0.127	0.063	2015Q2	0.116	1.286	0.884	0.107	-0.043	-0.139
2003Q1	0.310	-0.693	-2.020	-0.005	-0.736	-0.203	2015Q3	-0.113	0.706	1.400	-0.088	0.052	-0.649
2003Q2	0.296	-0.069	-1.165	0.517	-0.234	0.065	2015Q4	-0.438	0.706	-0.650	0.325	0.081	-0.648
2003Q3	-0.619	-0.107	-0.158	1.011	-1.601	0.219	2016Q1	-0.289	0.327	-0.353	-0.635	0.197	-1.428
2003Q4	-1.090	-0.057	-0.793	1.566	-0.639	0.642	2016Q2	-0.742	-0.699	-0.868	-0.367	0.440	-0.259
2004Q1	-0.269	0.891	-0.255	-1.410	-0.119	0.737	2016Q3	-0.578	0.061	-1.791	0.005	0.618	0.256
2004Q2	-1.485	0.327	0.437	-0.070	-0.289	0.807	2016Q4	-1.106	0.090	0.549	0.395	0.340	0.126
2004Q3	-0.540	0.575	0.002	-0.543	0.276	0.765	2017Q1	-1.162	0.301	0.109	0.301	0.990	0.382
2004Q4	0.240	1.325	-0.351	-1.661	1.265	0.322	2017Q2	-0.103	0.647	0.378	-0.550	-0.265	-0.251
2005Q1	-1.130	-0.329	-1.466	-0.226	0.296	0.836	2017Q3	-0.555	0.341	0.922	-0.185	0.314	0.001
2005Q2	-0.514	-0.535	-1.564	-0.226	-0.217	1.070	2017Q4	-0.402	0.560	0.243	-1.131	0.666	-0.038
2005Q3	-0.670	-0.305	-1.008	-0.403	0.315	1.400	2018Q1	-1.287	-0.661	0.745	-0.345	-0.279	0.631
2005Q4	-1.095	-0.660	-0.464	0.721	-0.078	1.633	2018Q2	-0.693	-0.542	-0.499	-0.223	0.643	0.925
2006Q1	-0.203	0.154	0.057	0.554	0.598	1.673	2018Q3	-1.289	-0.478	-1.118	0.160	1.116	0.641
2006Q2	-0.164	0.850	2.450	-0.100	-0.261	0.615	2018Q4	-0.744	-1.462	0.235	-0.223	0.144	0.186
2006Q3	-0.628	0.260	0.390	-0.865	-0.504	0.619	2019Q1	0.778	-0.406	0.060	-0.952	0.910	0.192
2006Q4	-0.799	0.479	-0.312	-0.808	-0.757	0.674	2019Q2	0.679	0.028	-0.038	-0.721	0.520	-0.147
2007Q1	-1.466	0.404	0.595	-0.065	-0.237	0.480	2019Q3	1.852	0.158	-0.074	-1.580	-0.281	-0.119
2007Q2	-2.568	0.740	0.302	0.343	0.619	0.927	2019Q4	0.095	0.364	1.172	-0.525	0.057	0.544
2007Q3	-0.853	0.572	0.668	-0.921	0.538	0.721	2020Q1	-0.180	-0.550	-0.379	0.422	-0.561	-0.912
2007Q4	-1.440	0.104	-0.011	-1.388	0.469	0.136	2020Q2	1.768	-0.659	-0.063	1.405	1.261	-0.141
2008Q1	-0.022	-0.914	0.213	-1.586	0.580	-0.170	2020Q3	1.621	-0.307	-0.532	0.593	0.259	-0.170
2008Q2	-1.216	-2.352	1.094	0.656	-0.248	0.097	2020Q4	2.268	1.336	-0.784	1.303	1.359	0.146
2008Q3	-1.713	-3.293	-0.718	0.530	0.419	-0.146	2021Q1	2.309	2.417	0.345	1.625	0.228	0.967
2008Q4	1.317	-4.245	-1.267	-0.647	-0.540	-0.452	2021Q2	-1.752	3.482	1.166	-0.842	-1.206	-0.182
2009Q1	3.126	-3.477	-0.107	0.792	0.680	-0.185	2021Q3	-0.909	2.950	-0.041	-0.961	-0.200	-0.118
2009Q2	2.710	-3.530	0.771	0.552	-0.069	0.413	2021Q4	-3.856	0.624	-0.849	0.031	-2.923	0.030
2009Q3	4.886	-1.372	-0.535	1.786	0.064	0.802	2022Q1	-5.348	0.292	0.697	-0.545	-0.065	-0.793
2009Q4	3.958	1.269	0.103	0.958	-1.065	0.479	2022Q2	-5.114	-2.181	1.225	2.672	0.190	-0.476
2010Q1	0.596	2.754	-0.879	0.600	-1.721	-0.014	2022Q3	-2.846	-2.292	0.626	0.241	-0.154	-1.012
2010Q2	0.039	4.616	-1.500	0.536	0.111	-1.299	2022Q4	-0.396	-1.716	3.325	0.612	1.230	-1.676
2010Q3	-1.203	3.861	-1.602	-0.788	-1.397	-1.442	2023Q1	1.774	-1.583	2.436	-0.379	0.137	-0.942
2010Q4	-2.662	3.575	0.242	-0.100	-0.592	-1.508	2023Q2	2.431	-1.326	1.657	-0.060	-0.529	-0.633
2011Q1	-3.519	1.370	0.428	0.949	-0.522	-1.069	2023Q3	1.704	-1.315	1.453	0.317	0.664	-0.313
2011Q2	-1.545	-0.367	-0.302	-0.097	-0.593	-0.515	2023Q4	2.522	-1.402	0.827	0.139	-1.217	-0.037
2011Q3	0.301	-0.540	-1.356	-1.805	-0.373	-1.327	2024Q1	0.843	-1.119	-1.309	0.486	0.129	0.176
2011Q4	-1.191	-1.847	-2.166	0.952	0.739	-0.392	2024Q2	1.495	-0.558	1.089	0.432	-0.212	0.115
2012Q1	0.204	-1.602	-1.961	1.283	-0.101	0.077	2024Q3	1.702	0.742	-0.199	0.273	-0.705	-0.336
2012Q2	2.191	0.000	-1.078	0.229	0.620	-0.477	2024Q4	-0.268	1.369	-0.417	-0.010	1.202	-0.473
2012Q3	1.438	-0.246	-1.318	0.777	-0.053	0.171	2025Q1	0.008	0.779	1.008	0.367	-0.338	-0.765
2012Q4	2.032	0.390	1.016	0.197	-0.920	-0.240	2025Q2	0.810	1.891	0.655	-0.252	1.491	-1.098
2013Q1	1.955	0.751	1.081	0.081	-0.232	-0.319	2025Q3	-0.006	0.254	0.642	0.265	-0.521	-0.426
2013Q2	0.667	0.496	0.014	0.188	-1.397	-0.515	2025Q4	0.684	1.124	0.440	-0.023	0.868	-0.425
2013Q3	-1.069	0.508	0.771	0.978	-0.223	-0.083	2026Q1	-0.327	1.144	0.158	0.831	0.926	-0.247

D.4 France

France is estimated over 2000Q2–2026Q1, that is 104 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D13 shows the loading of each series on each factor and Figure D14 the same loadings as a heat map; Table D7 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.560); Factor 2, y^{10} (0.599); Factor 3, CapitalFormation (0.757); Factor 4, y^{10} (0.733); Factor 5, Ext. Trade (0.548); Factor 6, Primary balance (0.746). The sum of squared loadings is largest for ΔFinAssets (0.985) and smallest for $r - g$ (0.364); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D13: France: factor loadings by series

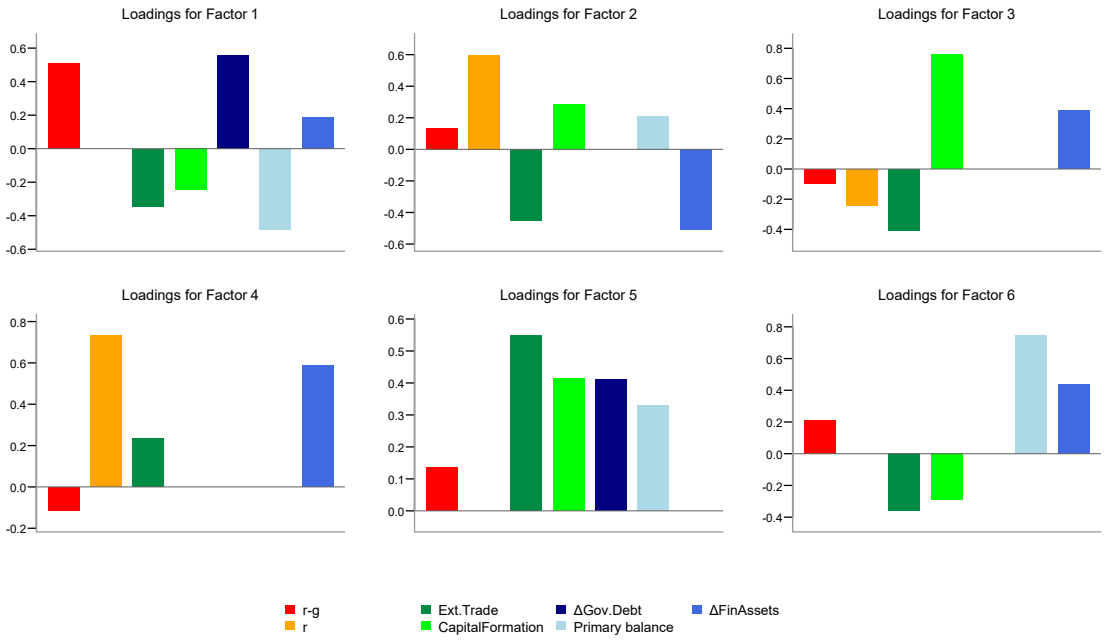


Figure D14: France: loading heat map

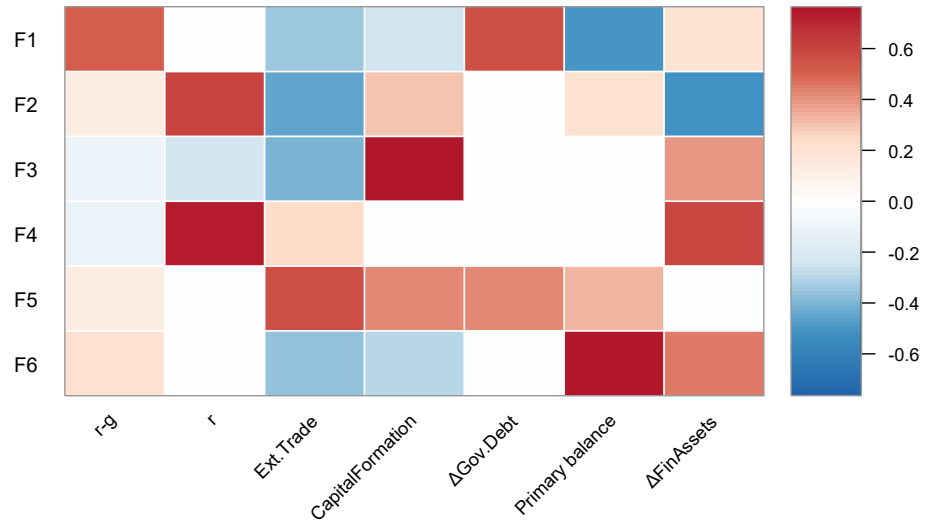


Table D7: France: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.510	0.136	-0.097	-0.116	0.136	0.209	0.364
y^{10}	0.000	0.599	-0.242	0.733	0.000	0.000	0.954
Ext. Trade	-0.344	-0.452	-0.405	0.237	0.548	-0.360	0.973
CapitalFormation	-0.246	0.288	0.757	0.000	0.414	-0.291	0.972
ΔGov.Debt	0.560	0.000	0.000	0.000	0.412	0.000	0.483
Primary balance	-0.486	0.210	0.000	0.000	0.328	0.746	0.944
ΔFinAssets	0.186	-0.512	0.389	0.586	0.000	0.440	0.985
$\sum \lambda^2$	1.023	0.971	0.956	0.949	0.769	1.008	5.676
Prop. of common var.	0.180	0.171	0.169	0.167	0.135	0.178	1.000

Figure D15 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D16 shows the seven underlying series. Table D8 lists the factor estimates quarter by quarter.

Figure D15: France: standardised factor estimates and standardised data

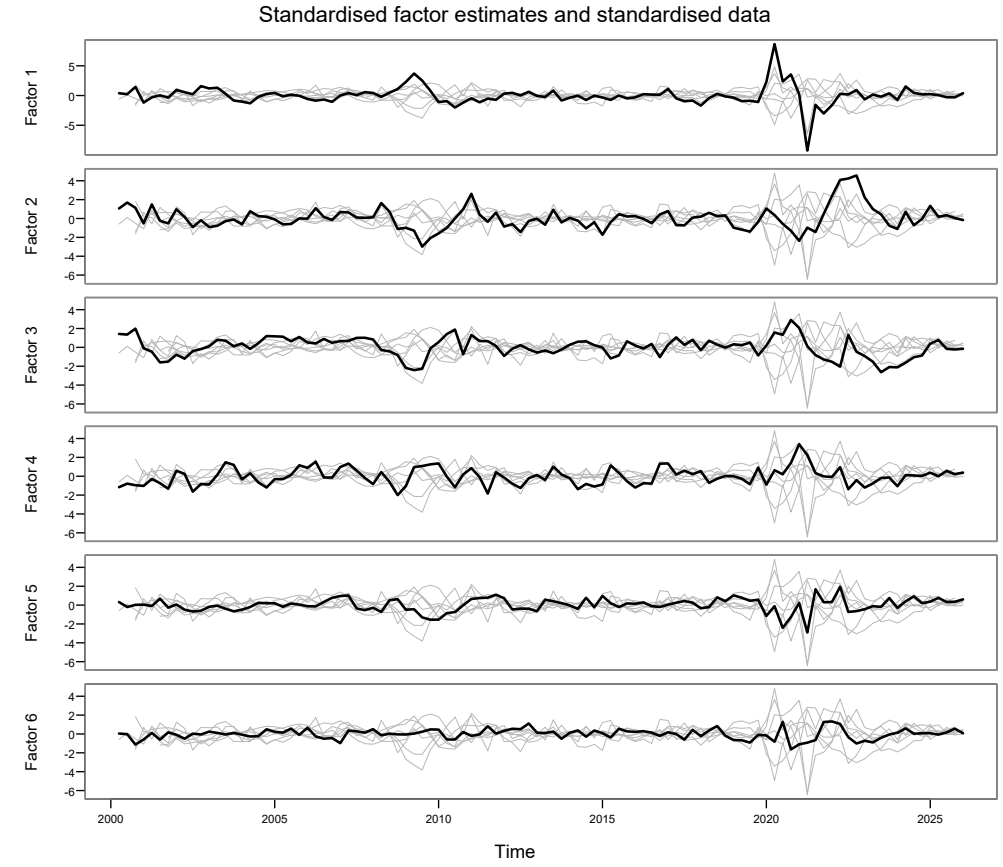


Figure D16: France: input series over the estimation window

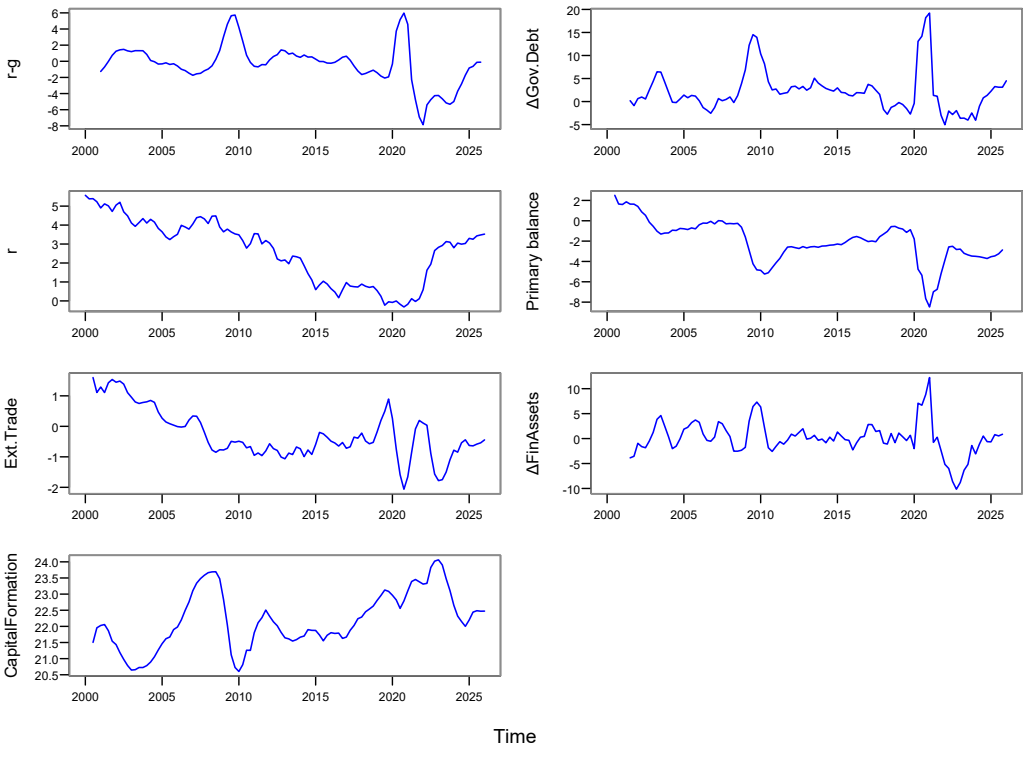


Table D8: France: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2000Q2	0.386	1.064	1.425	-1.162	0.321	0.045	2013Q2	-0.242	-0.665	-0.338	-0.394	0.570	0.089
2000Q3	0.226	1.691	1.356	-0.785	-0.204	-0.027	2013Q3	0.840	0.928	-0.620	1.019	0.403	0.257
2000Q4	1.447	1.130	1.987	-0.955	0.025	-1.124	2013Q4	-0.831	-0.416	-0.238	0.189	0.209	-0.512
2001Q1	-1.193	-0.523	-0.095	-1.029	0.046	-0.591	2014Q1	-0.374	0.074	0.274	-0.217	-0.028	0.121
2001Q2	-0.305	1.492	-0.462	-0.288	-0.101	0.112	2014Q2	-0.022	-0.231	0.599	-1.378	-0.409	0.378
2001Q3	0.036	-0.246	-1.595	-0.735	0.684	-0.603	2014Q3	-0.754	-1.052	0.635	-0.819	0.775	-0.288
2001Q4	-0.362	-0.516	-1.513	-1.330	-0.271	0.175	2014Q4	-0.030	-0.392	0.238	-1.091	-0.222	0.368
2002Q1	0.926	0.980	-0.772	0.583	0.067	-0.086	2015Q1	-0.332	-1.732	-0.017	-0.879	0.972	0.083
2002Q2	0.560	0.161	-1.201	0.251	-0.487	-0.504	2015Q2	-0.744	-0.385	-1.177	1.148	0.205	-0.374
2002Q3	0.215	-0.920	-0.391	-1.639	-0.682	0.029	2015Q3	-0.001	0.457	-0.861	0.368	-0.212	0.556
2002Q4	1.583	-0.191	-0.183	-0.824	-0.594	-0.052	2015Q4	-0.493	0.215	0.641	-0.543	0.178	0.274
2003Q1	1.188	-0.912	0.117	-0.887	-0.187	0.251	2016Q1	-0.302	0.247	0.196	-1.202	0.143	0.221
2003Q2	1.317	-0.772	0.806	0.159	-0.063	0.109	2016Q2	0.229	-0.039	-0.128	-0.746	0.289	0.295
2003Q3	0.291	-0.284	0.709	1.479	-0.388	-0.056	2016Q3	0.160	-0.500	0.364	-0.778	-0.115	0.142
2003Q4	-0.847	-0.111	0.114	1.198	-0.658	0.107	2016Q4	0.125	0.429	-1.040	1.328	-0.201	-0.209
2004Q1	-1.025	-0.620	0.417	-0.336	-0.488	-0.075	2017Q1	1.120	0.788	0.287	1.352	0.044	0.184
2004Q2	-1.311	0.771	-0.165	0.309	-0.205	-0.265	2017Q2	-0.473	-0.708	1.058	0.170	0.254	-0.010
2004Q3	-0.227	0.255	0.436	-0.657	0.244	-0.239	2017Q3	-0.973	-0.729	0.253	0.594	0.430	-0.602
2004Q4	0.269	0.181	1.197	-1.197	0.195	0.501	2017Q4	-0.846	0.093	0.801	0.216	0.252	0.443
2005Q1	0.438	-0.100	1.170	-0.305	0.195	0.234	2018Q1	-1.696	0.197	-0.296	0.545	-0.337	-0.234
2005Q2	-0.128	-0.621	1.131	-0.302	-0.181	0.142	2018Q2	-0.444	0.607	0.703	-0.699	-0.210	0.360
2005Q3	0.129	-0.578	0.638	0.134	0.166	0.572	2018Q3	0.268	0.246	0.290	-0.266	0.816	0.842
2005Q4	-0.047	0.007	1.103	1.162	0.055	-0.099	2018Q4	-0.152	0.330	-0.033	0.019	0.412	-0.199
2006Q1	-0.583	-0.017	0.519	0.892	-0.113	0.659	2019Q1	-0.361	-0.978	0.328	-0.001	1.035	-0.641
2006Q2	-0.863	1.095	0.415	1.538	-0.144	-0.259	2019Q2	-0.944	-1.177	0.239	-0.274	0.779	-0.669
2006Q3	-0.664	0.154	0.899	-0.128	0.341	-0.504	2019Q3	-0.883	-1.387	0.507	-0.853	0.458	-0.919
2006Q4	-1.067	-0.150	0.484	-0.141	0.773	-0.431	2019Q4	-1.090	-0.361	-0.858	0.920	0.566	-0.097
2007Q1	0.008	0.691	0.665	0.973	0.959	-0.974	2020Q1	2.294	1.064	0.198	-0.898	-1.150	-0.157
2007Q2	0.401	0.647	0.684	1.345	1.018	0.360	2020Q2	8.656	0.362	1.579	0.634	-0.116	-0.825
2007Q3	0.074	0.101	1.009	0.600	-0.366	0.281	2020Q3	2.359	-0.567	1.329	0.204	-2.415	1.287
2007Q4	0.572	0.067	1.016	-0.177	-0.532	0.138	2020Q4	3.547	-1.300	2.913	1.412	-1.268	-1.642
2008Q1	0.434	0.124	0.836	-0.837	-0.286	0.512	2021Q1	0.261	-2.369	2.065	3.393	0.230	-1.102
2008Q2	-0.220	1.639	-0.274	0.446	-0.725	-0.149	2021Q2	-9.250	-0.969	0.127	2.237	-2.908	-0.923
2008Q3	0.469	0.734	-0.424	-0.595	0.503	0.025	2021Q3	-1.566	-1.438	-0.820	0.326	1.681	-0.644
2008Q4	1.090	-1.113	-0.818	-2.001	0.636	-0.044	2021Q4	-3.026	0.480	-1.278	-0.017	0.301	1.268
2009Q1	2.284	-0.989	-2.176	-1.059	-0.492	-0.058	2022Q1	-1.634	2.371	-1.524	-0.082	0.319	1.328
2009Q2	3.699	-1.283	-2.412	0.965	-0.435	0.037	2022Q2	0.282	4.087	-2.051	0.959	1.938	1.062
2009Q3	2.508	-2.985	-2.273	1.081	-1.306	0.240	2022Q3	0.174	4.240	1.317	-1.381	-0.725	-0.340
2009Q4	0.794	-2.072	-0.110	1.258	-1.548	0.483	2022Q4	0.915	4.563	-0.466	-0.415	-0.639	-1.014
2010Q1	-1.092	-1.583	0.555	1.362	-1.541	0.463	2023Q1	-0.649	2.234	-0.921	-1.203	-0.435	-0.718
2010Q2	-0.939	-0.999	1.407	-0.038	-0.855	-0.591	2023Q2	0.158	0.999	-1.523	-0.766	-0.116	-0.898
2010Q3	-2.018	0.002	1.888	-1.171	-0.738	-0.589	2023Q3	-0.194	0.441	-2.636	-0.184	-0.192	-0.404
2010Q4	-1.197	0.984	-0.723	0.187	-0.020	0.211	2023Q4	0.365	-0.757	-2.088	-0.129	0.755	-0.060
2011Q1	-0.460	2.616	1.314	0.855	0.651	-0.201	2024Q1	-0.731	-1.113	-2.117	-1.061	-0.296	0.132
2011Q2	-1.140	0.416	0.671	-0.085	0.746	-0.042	2024Q2	1.529	0.699	-1.624	0.126	0.400	0.595
2011Q3	-0.526	-0.351	0.657	-1.838	0.790	0.824	2024Q3	0.438	-0.716	-1.046	0.083	0.927	0.029
2011Q4	-0.721	0.632	0.153	0.420	1.106	0.039	2024Q4	0.192	-0.048	-0.866	0.031	0.201	0.074
2012Q1	0.318	-0.859	-0.908	-0.139	0.719	0.340	2025Q1	0.223	1.343	0.391	0.377	0.368	0.080
2012Q2	0.452	-0.590	-0.147	-0.796	-0.471	0.551	2025Q2	0.069	0.182	0.774	-0.006	0.766	-0.069
2012Q3	0.017	-1.441	0.233	-1.254	-0.373	0.505	2025Q3	-0.295	0.344	-0.159	0.571	0.325	0.162
2012Q4	0.635	-0.267	-0.211	-0.219	-0.379	1.113	2025Q4	-0.305	0.050	-0.226	0.222	0.359	0.567
2013Q1	-0.057	-0.018	-0.556	0.134	-0.656	0.140	2026Q1	0.386	-0.165	-0.150	0.384	0.606	0.077

D.5 Germany

Germany is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D17 shows the loading of each series on each factor and Figure D18 the same loadings as a heat map; Table D9 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.615); Factor 2, y^{10} (0.592); Factor 3, Primary balance (0.630); Factor 4, CapitalFormation (0.706); Factor 5, Ext. Trade (0.723); Factor 6, $r - g$ (0.791). The sum of squared loadings is largest for CapitalFormation (0.999) and smallest for ΔGov.Debt (0.412); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D17: Germany: factor loadings by series

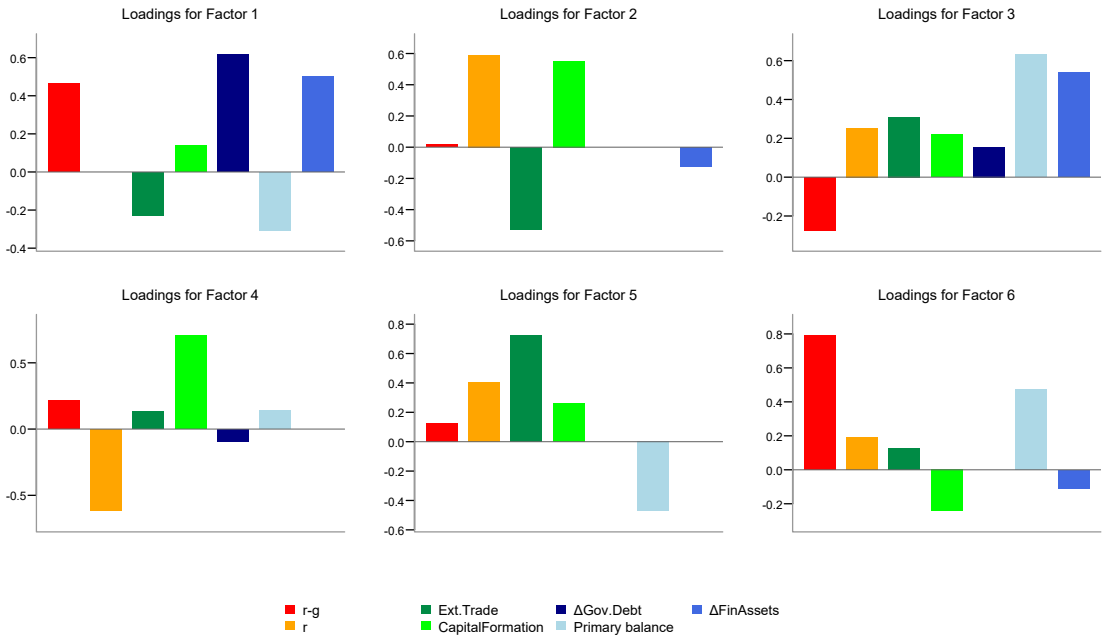


Figure D18: Germany: loading heat map

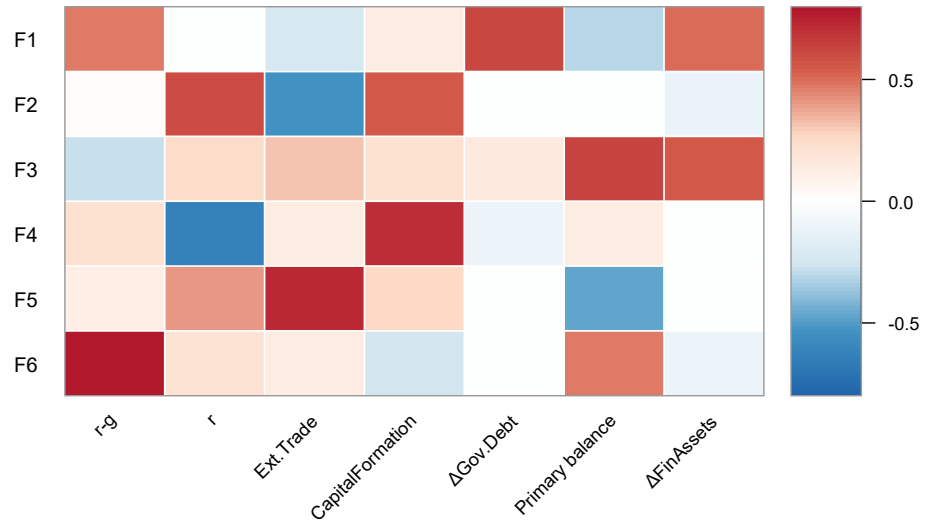


Table D9: Germany: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.463	0.017	-0.273	0.217	0.125	0.791	0.977
y^{10}	0.000	0.592	0.253	-0.617	0.404	0.194	0.996
Ext. Trade	-0.230	-0.532	0.310	0.134	0.723	0.129	0.989
CapitalFormation	0.137	0.553	0.220	0.706	0.262	-0.242	0.999
ΔGov.Debt	0.615	0.000	0.156	-0.098	0.000	0.000	0.412
Primary balance	-0.306	0.000	0.630	0.139	-0.471	0.471	0.953
ΔFinAssets	0.504	-0.122	0.541	0.000	0.000	-0.111	0.573
$\sum \lambda^2$	1.011	0.954	0.997	0.973	0.992	0.973	5.899
Prop. of common var.	0.171	0.162	0.169	0.165	0.168	0.165	1.000

Figure D19 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D20 shows the seven underlying series. Table D10 lists the factor estimates quarter by quarter.

Figure D19: Germany: standardised factor estimates and standardised data

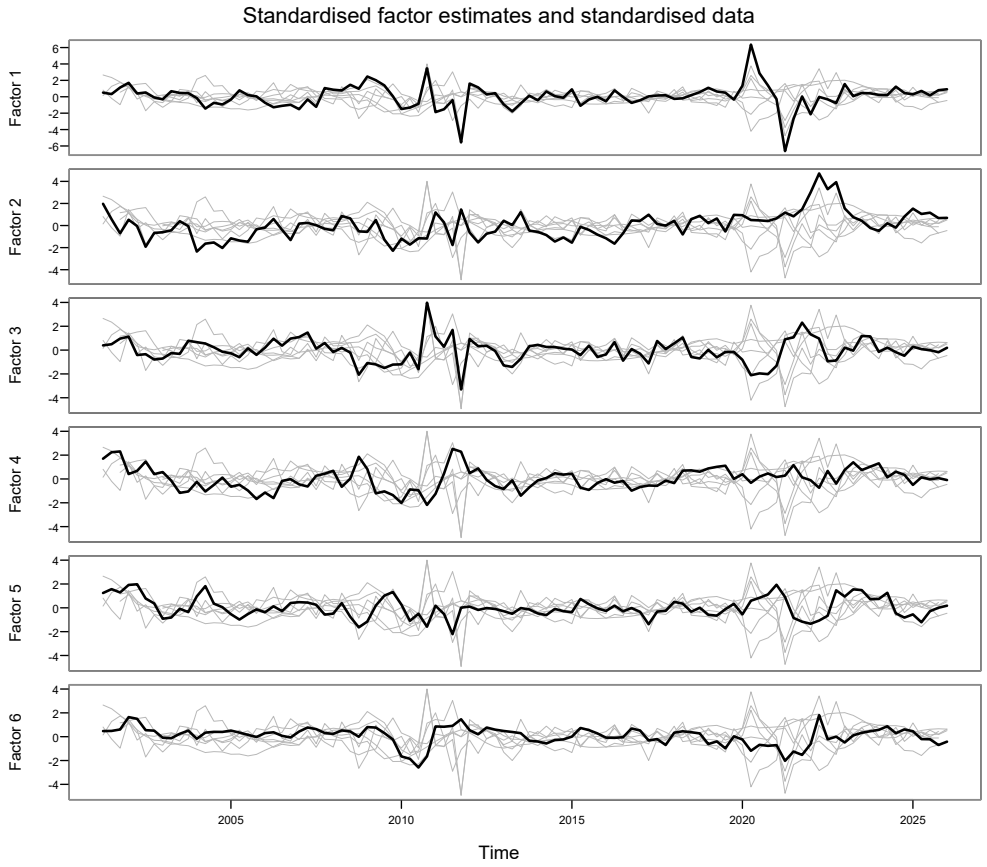


Figure D20: Germany: input series over the estimation window

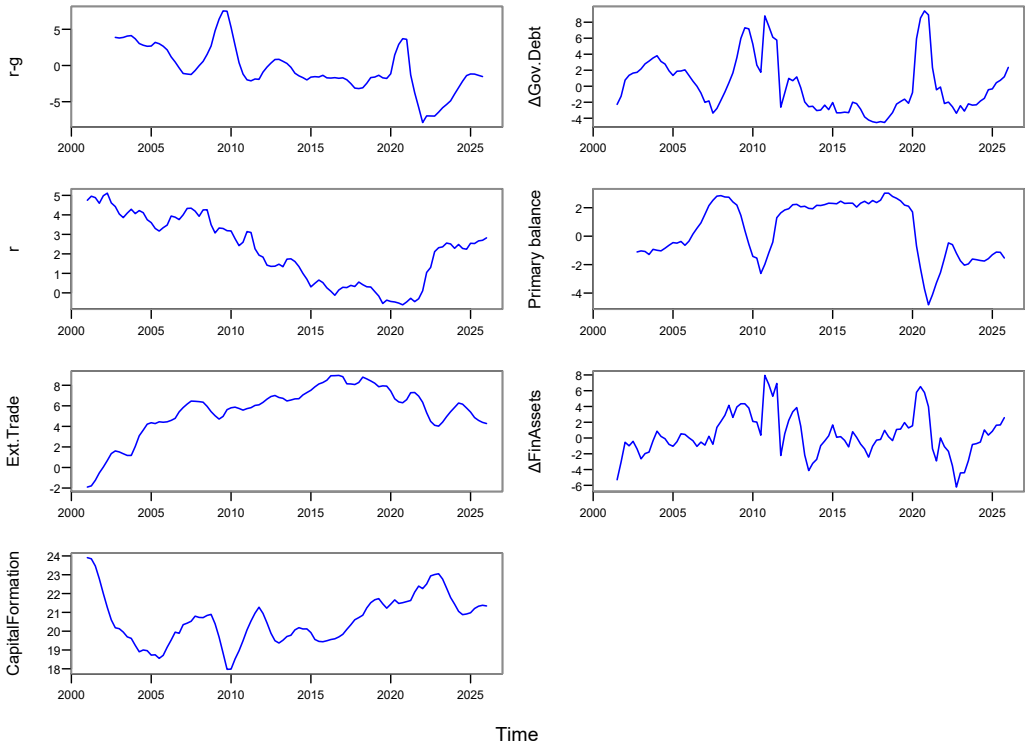


Table D10: Germany: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.507	1.968	0.394	1.699	1.243	0.471	2013Q4	0.136	-0.461	0.322	-0.710	-0.135	-0.337
2001Q3	0.324	0.572	0.494	2.244	1.568	0.484	2014Q1	-0.435	-0.574	0.440	-0.123	-0.472	-0.405
2001Q4	1.123	-0.703	0.963	2.310	1.288	0.606	2014Q2	0.648	-0.862	0.264	0.078	-0.611	-0.569
2002Q1	1.701	0.526	1.128	0.411	1.921	1.643	2014Q3	0.072	-1.449	0.254	0.481	-0.089	-0.280
2002Q2	0.397	-0.048	-0.415	0.680	1.977	1.501	2014Q4	-0.122	-1.080	0.121	0.362	-0.280	-0.228
2002Q3	0.524	-1.922	-0.344	1.461	0.782	0.546	2015Q1	0.909	-1.552	0.046	0.443	-0.365	0.006
2002Q4	-0.085	-0.676	-0.794	0.404	0.370	0.507	2015Q2	-1.098	-0.111	-0.423	-0.728	0.743	0.732
2003Q1	-0.314	-0.604	-0.709	0.580	-0.916	-0.088	2015Q3	-0.335	-0.396	0.376	-0.922	0.308	0.552
2003Q2	0.678	-0.437	-0.252	-0.140	-0.815	-0.129	2015Q4	0.000	-0.826	-0.587	-0.334	-0.045	0.272
2003Q3	0.469	0.397	-0.319	-1.168	-0.078	0.242	2016Q1	-0.557	-1.155	-0.359	-0.020	-0.262	-0.081
2003Q4	0.435	-0.054	0.785	-1.050	-0.359	0.508	2016Q2	0.775	-1.644	0.669	-0.315	0.185	-0.097
2004Q1	-0.160	-2.359	0.665	-0.240	0.964	-0.174	2016Q3	-0.019	-0.704	-0.889	-0.169	-0.296	-0.044
2004Q2	-1.442	-1.640	0.552	-1.052	1.823	0.340	2016Q4	-0.750	0.461	0.088	-0.982	-0.016	0.692
2004Q3	-0.739	-1.534	0.237	-0.496	0.349	0.406	2017Q1	-0.426	0.427	-0.302	-0.677	-0.341	0.519
2004Q4	-0.939	-2.032	-0.140	0.115	0.055	0.400	2017Q2	0.048	0.982	-1.120	-0.549	-1.379	-0.322
2005Q1	-0.344	-1.159	-0.268	-0.655	-0.534	0.512	2017Q3	0.149	0.143	0.763	-0.577	-0.316	-0.203
2005Q2	0.783	-1.359	-0.599	-0.485	-0.986	0.346	2017Q4	0.183	-0.006	0.083	-0.141	-0.254	-0.696
2005Q3	0.203	-1.468	0.146	-0.972	-0.522	0.155	2018Q1	-0.257	0.433	0.563	-0.345	0.503	0.334
2005Q4	0.046	-0.337	-0.399	-1.682	-0.128	-0.026	2018Q2	-0.192	-0.807	1.060	0.700	0.352	0.452
2006Q1	-0.728	-0.188	0.202	-1.140	-0.374	0.303	2018Q3	0.192	0.575	-0.577	0.722	-0.328	0.376
2006Q2	-1.283	0.593	0.959	-1.607	0.127	0.370	2018Q4	0.565	0.881	-0.703	0.607	0.011	0.273
2006Q3	-1.104	-0.388	0.373	-0.165	-0.277	0.072	2019Q1	1.096	0.224	0.026	0.881	-0.583	-0.620
2006Q4	-0.974	-1.316	0.967	-0.016	0.405	-0.067	2019Q2	0.644	0.639	-0.610	1.018	-0.655	-0.396
2007Q1	-1.514	0.175	1.093	-0.454	0.475	0.432	2019Q3	0.501	-0.538	-0.160	1.112	-0.118	-0.979
2007Q2	-0.322	0.241	1.473	-0.633	0.440	0.770	2019Q4	-0.346	0.967	-0.159	0.008	0.341	0.032
2007Q3	-1.229	0.032	0.120	0.268	0.257	0.636	2020Q1	1.338	0.935	-0.820	0.397	-0.551	-0.237
2007Q4	1.059	-0.296	0.596	0.438	-0.572	0.311	2020Q2	6.362	0.500	-2.109	-0.325	0.596	-1.183
2008Q1	0.860	-0.432	-0.165	0.687	-0.488	0.234	2020Q3	2.869	0.467	-1.956	0.195	0.840	-0.682
2008Q2	0.767	0.863	0.187	-0.660	0.397	0.526	2020Q4	1.376	0.408	-2.019	0.463	1.123	-0.766
2008Q3	1.464	0.647	-0.211	0.025	-0.749	0.437	2021Q1	-0.293	0.648	-1.314	0.160	1.938	-0.708
2008Q4	0.973	-0.527	-2.070	1.863	-1.641	-0.007	2021Q2	-6.621	1.172	0.907	0.279	0.911	-2.027
2009Q1	2.469	-0.574	-1.086	0.829	-1.152	0.807	2021Q3	-2.692	0.834	1.068	1.163	-0.849	-1.244
2009Q2	2.018	0.500	-1.203	-1.209	0.192	0.766	2021Q4	0.026	1.469	2.305	0.137	-1.163	-1.528
2009Q3	1.386	-1.271	-1.503	-1.041	1.024	0.306	2022Q1	-2.138	2.998	1.321	-0.107	-1.338	-0.620
2009Q4	0.064	-2.292	-1.217	-1.357	1.325	-0.233	2022Q2	-0.024	4.717	0.969	-0.743	-1.069	1.807
2010Q1	-1.496	-1.187	-1.196	-2.013	0.270	-1.658	2022Q3	-0.316	3.289	-0.938	0.675	-0.667	-0.219
2010Q2	-1.315	-1.728	-0.215	-0.879	-1.094	-1.857	2022Q4	-0.784	3.929	-0.876	-0.390	1.469	0.002
2010Q3	-0.868	-1.153	-1.606	-0.952	-0.492	-2.588	2023Q1	1.550	1.552	0.205	0.759	0.945	-0.486
2010Q4	3.469	-1.176	3.977	-2.180	-1.585	-1.646	2023Q2	0.071	0.763	-0.049	1.398	1.551	0.112
2011Q1	-1.870	1.190	1.145	-1.250	0.175	0.859	2023Q3	0.460	0.431	1.183	0.743	1.486	0.328
2011Q2	-1.521	0.282	0.278	0.452	-0.522	0.836	2023Q4	0.415	-0.220	1.151	1.031	0.725	0.457
2011Q3	-0.413	-1.768	1.694	2.526	-2.210	0.910	2024Q1	0.236	-0.486	-0.154	1.312	0.744	0.573
2011Q4	-5.564	1.454	-3.306	2.286	0.015	1.463	2024Q2	0.204	0.203	0.210	0.138	1.263	0.875
2012Q1	1.612	-0.631	0.928	0.486	0.104	0.546	2024Q3	1.233	-0.189	-0.175	0.607	-0.463	0.294
2012Q2	1.140	-1.532	0.329	0.882	-0.177	0.202	2024Q4	0.461	0.810	-0.477	0.351	-0.822	0.614
2012Q3	0.276	-0.737	0.360	-0.079	-0.019	0.769	2025Q1	0.296	1.532	0.279	-0.498	-0.550	0.458
2012Q4	0.439	-0.553	-0.075	-0.602	-0.097	0.590	2025Q2	0.711	1.048	0.083	0.141	-1.213	-0.201
2013Q1	-0.934	0.436	-1.285	-0.843	-0.283	0.479	2025Q3	0.171	1.164	-0.005	-0.028	-0.284	-0.215
2013Q2	-1.785	0.042	-1.414	-0.109	-0.500	0.395	2025Q4	0.801	0.675	-0.181	0.071	0.007	-0.699
2013Q3	-0.816	1.208	-0.730	-1.407	-0.020	0.295	2026Q1	0.914	0.696	0.194	-0.092	0.180	-0.416

D.6 Greece

Greece is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D21 shows the loading of each series on each factor and Figure D22 the same loadings as a heat map; Table D11 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.483); Factor 2, CapitalFormation (-0.578); Factor 3, Primary balance (0.526); Factor 4, ΔFinAssets (0.551); Factor 5, ΔFinAssets (-0.621); Factor 6, Ext. Trade (0.575). The sum of squared loadings is largest for Primary balance (0.995) and smallest for ΔGov.Debt (0.385); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D21: Greece: factor loadings by series

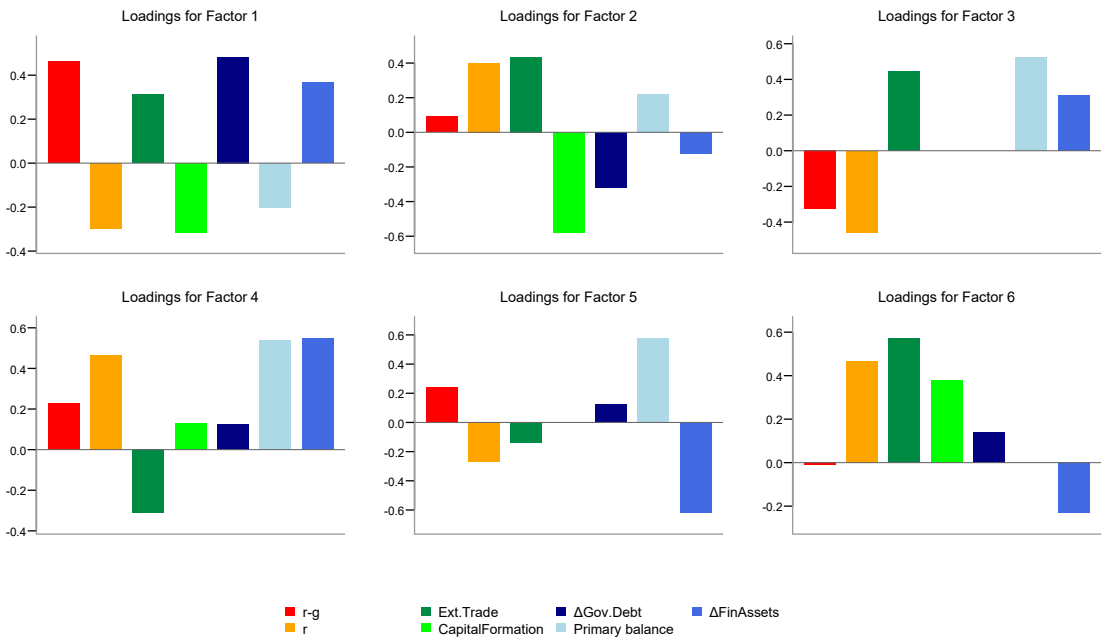


Figure D22: Greece: loading heat map

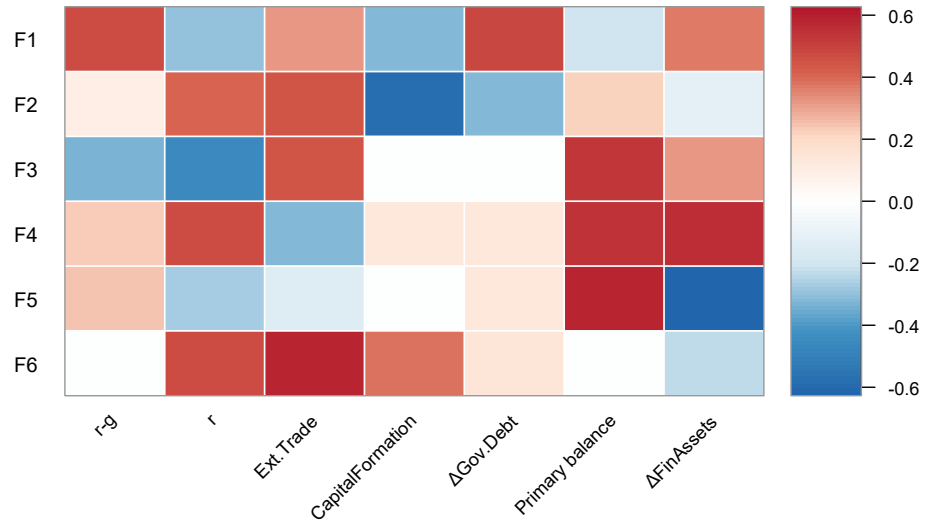


Table D11: Greece: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.462	0.092	-0.327	0.228	0.243	-0.012	0.440
y^{10}	-0.296	0.401	-0.458	0.466	-0.273	0.466	0.967
Ext. Trade	0.312	0.435	0.445	-0.312	-0.140	0.575	0.931
CapitalFormation	-0.315	-0.578	0.000	0.129	0.000	0.378	0.592
ΔGov.Debt	0.483	-0.319	0.000	0.125	0.125	0.139	0.385
Primary balance	-0.200	0.220	0.526	0.541	0.581	0.000	0.995
ΔFinAssets	0.367	-0.124	0.312	0.551	-0.621	-0.232	0.991
$\sum \lambda^2$	0.905	0.858	0.888	0.996	0.892	0.763	5.302
Prop. of common var.	0.171	0.162	0.167	0.188	0.168	0.144	1.000

Figure D23 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D24 shows the seven underlying series. Table D12 lists the factor estimates quarter by quarter.

Figure D23: Greece: standardised factor estimates and standardised data

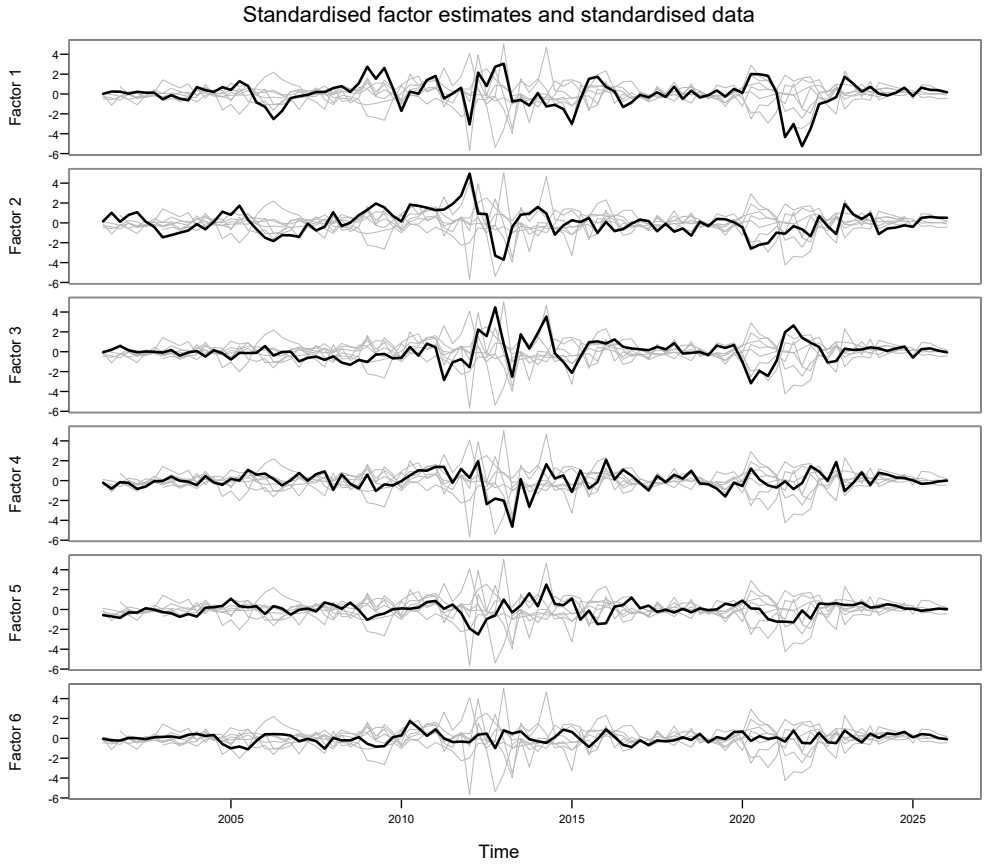


Figure D24: Greece: input series over the estimation window

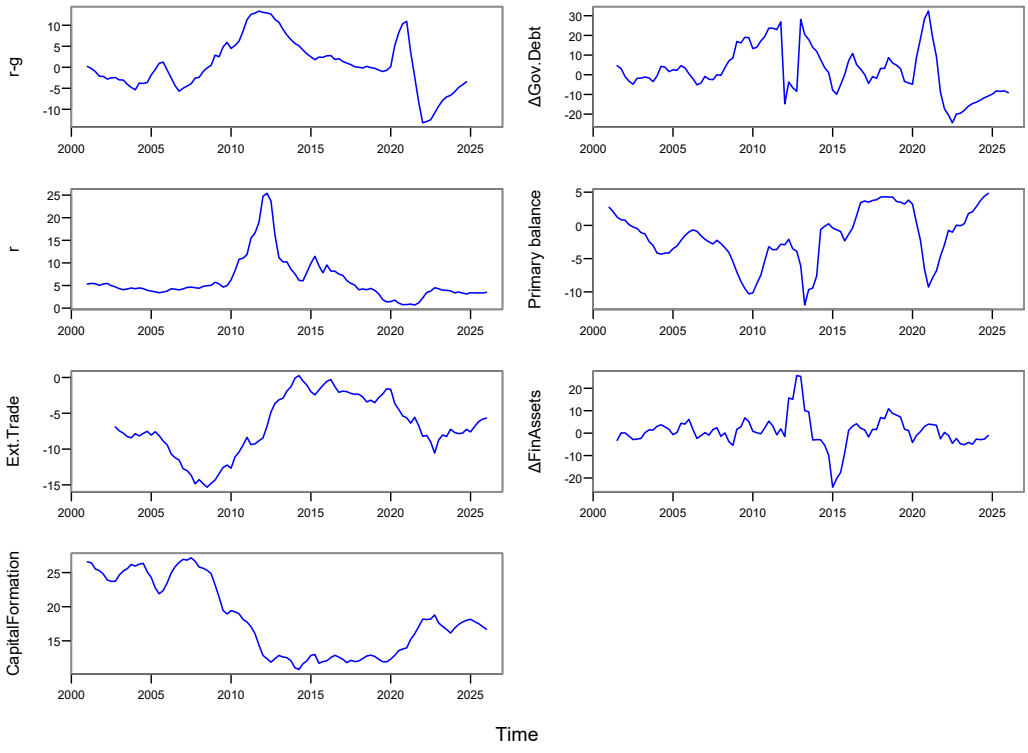


Table D12: Greece: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.018	0.152	-0.059	-0.221	-0.573	-0.046	2013Q4	-1.162	0.936	0.329	-2.618	1.628	-0.087
2001Q3	0.249	1.014	0.208	-0.823	-0.699	-0.199	2014Q1	0.097	1.604	1.889	-0.503	0.314	-0.299
2001Q4	0.235	0.107	0.584	-0.148	-0.844	-0.226	2014Q2	-1.248	0.945	3.538	1.648	2.507	-0.453
2002Q1	0.033	0.803	0.144	-0.241	-0.298	0.041	2014Q3	-1.091	-1.189	-0.150	0.208	0.564	0.124
2002Q2	0.225	1.073	-0.055	-0.839	-0.310	0.011	2014Q4	-1.516	-0.257	-1.028	0.526	0.428	0.877
2002Q3	0.131	0.130	0.020	-0.603	0.118	-0.071	2015Q1	-3.001	0.261	-2.122	-1.146	1.094	0.645
2002Q4	0.136	-0.409	-0.008	-0.062	-0.024	0.107	2015Q2	-0.508	0.084	-0.516	1.016	-1.032	-0.125
2003Q1	-0.529	-1.455	-0.086	-0.044	-0.257	0.148	2015Q3	1.531	0.494	0.957	-0.801	-0.109	-0.871
2003Q2	-0.119	-1.231	0.175	0.405	-0.383	0.192	2015Q4	1.740	-1.012	1.042	-0.147	-1.469	-0.092
2003Q3	-0.463	-0.997	-0.387	-0.022	-0.744	0.059	2016Q1	0.725	0.097	0.831	2.089	-1.389	0.894
2003Q4	-0.635	-0.768	-0.074	-0.110	-0.461	0.366	2016Q2	0.254	-0.825	1.236	0.112	0.287	0.150
2004Q1	0.695	-0.120	0.070	-0.461	-0.710	0.449	2016Q3	-1.317	-0.625	0.497	1.095	0.446	-0.654
2004Q2	0.386	-0.650	-0.488	0.456	0.177	0.250	2016Q4	-0.837	-0.074	0.326	0.501	1.199	-0.879
2004Q3	0.207	0.108	0.153	-0.199	0.234	0.331	2017Q1	-0.078	0.333	0.244	-0.257	0.121	-0.192
2004Q4	0.691	1.125	-0.157	-0.424	0.360	-0.575	2017Q2	-0.340	0.173	0.153	-0.996	0.389	-0.685
2005Q1	0.404	0.811	-0.787	0.176	1.082	-0.999	2017Q3	0.167	-0.847	0.495	0.459	-0.237	-0.210
2005Q2	1.293	1.733	-0.114	0.010	0.332	-0.818	2017Q4	-0.298	-0.089	0.243	-0.163	-0.025	-0.307
2005Q3	0.815	0.321	-0.135	1.080	0.221	-1.104	2018Q1	0.748	-0.887	0.860	0.586	-0.307	-0.203
2005Q4	-0.845	-0.637	-0.113	0.600	0.325	-0.242	2018Q2	-0.501	-0.563	-0.190	0.201	0.067	0.102
2006Q1	-1.283	-1.494	0.572	0.707	-0.453	0.407	2018Q3	0.338	-1.287	-0.117	0.990	-0.290	-0.169
2006Q2	-2.531	-1.836	-0.384	0.167	0.343	0.425	2018Q4	-0.349	0.113	-0.027	-0.286	0.106	0.451
2006Q3	-1.712	-1.238	-0.024	-0.486	0.101	0.405	2019Q1	-0.122	-0.322	-0.334	-0.372	-0.067	-0.403
2006Q4	-0.415	-1.254	0.018	0.025	-0.516	0.289	2019Q2	0.358	0.397	0.619	-0.806	0.011	0.128
2007Q1	-0.222	-1.407	-0.970	0.768	-0.029	-0.292	2019Q3	-0.255	0.363	0.382	-1.601	0.577	-0.068
2007Q2	-0.082	-0.069	-0.623	-0.002	0.089	-0.044	2019Q4	0.515	0.065	0.673	-0.219	0.407	0.629
2007Q3	0.201	-0.769	-0.487	0.650	-0.194	-0.283	2020Q1	0.115	-0.450	-1.012	-0.549	0.895	0.692
2007Q4	0.199	-0.386	-0.819	0.925	0.703	-1.041	2020Q2	2.010	-2.596	-3.170	1.212	0.110	-0.257
2008Q1	0.594	1.070	-0.469	-0.921	0.479	0.041	2020Q3	1.989	-2.205	-1.924	0.117	0.065	0.218
2008Q2	0.789	-0.322	-1.090	0.621	0.066	-0.211	2020Q4	1.831	-2.039	-2.443	-0.478	-0.969	-0.060
2008Q3	0.214	0.017	-1.318	-0.308	0.691	-0.202	2021Q1	0.150	-0.982	-0.929	-0.707	-1.216	0.094
2008Q4	1.071	0.772	-0.818	-0.792	-0.060	0.116	2021Q2	-4.341	-1.095	1.969	-0.079	-1.230	-0.333
2009Q1	2.743	1.306	-1.032	0.603	-1.046	-0.545	2021Q3	-3.013	-0.328	2.644	-0.851	-1.292	0.774
2009Q2	1.532	1.949	-0.286	-1.043	-0.609	-0.840	2021Q4	-5.256	-0.644	1.415	-0.249	-0.095	-0.464
2009Q3	2.619	1.556	-0.223	-0.382	-0.406	-0.788	2022Q1	-3.503	-1.344	0.908	1.451	-0.919	-0.495
2009Q4	0.598	0.705	-0.665	-0.487	0.020	0.110	2022Q2	-1.007	0.673	0.488	0.916	0.601	0.578
2010Q1	-1.702	0.143	-0.608	-0.053	0.109	0.300	2022Q3	-0.744	-0.337	-1.081	-0.024	0.535	-0.390
2010Q2	0.233	1.833	0.479	0.521	0.070	1.730	2022Q4	-0.315	-1.131	-0.939	1.872	0.621	-0.485
2010Q3	0.032	1.717	-0.399	1.036	0.191	1.047	2023Q1	1.738	1.897	0.311	-1.057	0.453	0.790
2010Q4	1.410	1.524	0.818	0.999	0.753	0.253	2023Q2	1.044	0.859	0.177	-0.225	0.443	0.139
2011Q1	1.817	1.283	0.445	1.385	0.855	0.901	2023Q3	0.241	0.394	0.226	0.825	0.687	-0.408
2011Q2	-0.451	1.339	-2.848	1.374	0.076	-0.006	2023Q4	0.722	0.935	0.444	-0.447	0.185	0.484
2011Q3	0.046	1.900	-1.070	-0.221	0.496	-0.375	2024Q1	0.035	-1.124	0.332	0.794	0.274	0.036
2011Q4	0.627	2.734	-0.747	1.179	-0.350	-0.327	2024Q2	-0.156	-0.569	0.082	0.610	0.527	0.493
2012Q1	-3.065	4.957	-1.561	0.274	-1.913	-0.389	2024Q3	0.121	-0.470	0.328	0.315	0.381	0.400
2012Q2	2.151	0.929	2.239	1.949	-2.519	0.375	2024Q4	0.643	-0.249	0.520	0.257	0.093	0.655
2012Q3	0.798	0.882	1.578	-2.365	-0.973	0.472	2025Q1	-0.203	-0.402	-0.593	0.023	0.058	0.102
2012Q4	2.744	-3.304	4.468	-1.812	-0.590	-0.988	2025Q2	0.643	0.505	0.262	-0.320	-0.136	0.428
2013Q1	3.049	-3.702	0.812	-2.018	0.997	0.804	2025Q3	0.421	0.602	0.340	-0.259	-0.037	0.355
2013Q2	-0.747	-0.412	-2.535	-4.645	-0.306	0.488	2025Q4	0.376	0.526	0.110	-0.089	0.086	0.021
2013Q3	-0.594	0.819	1.752	0.151	0.411	0.705	2026Q1	0.173	0.511	-0.058	0.000	0.037	-0.094

D.7 Ireland

Ireland is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D25 shows the loading of each series on each factor and Figure D26 the same loadings as a heat map; Table D13 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.605); Factor 2, CapitalFormation (0.624); Factor 3, y^{10} (-0.679); Factor 4, y^{10} (0.552); Factor 5, $r - g$ (0.693); Factor 6, ΔGov.Debt (0.642). The sum of squared loadings is largest for ΔFinAssets (1.000) and smallest for Ext. Trade (0.436); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D25: Ireland: factor loadings by series

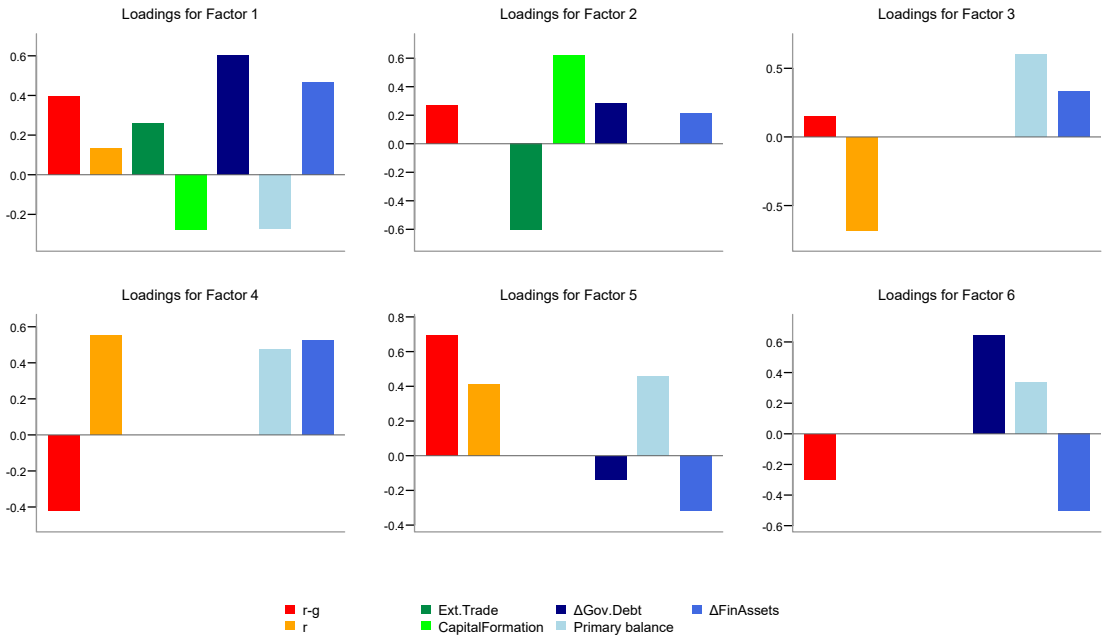


Figure D26: Ireland: loading heat map

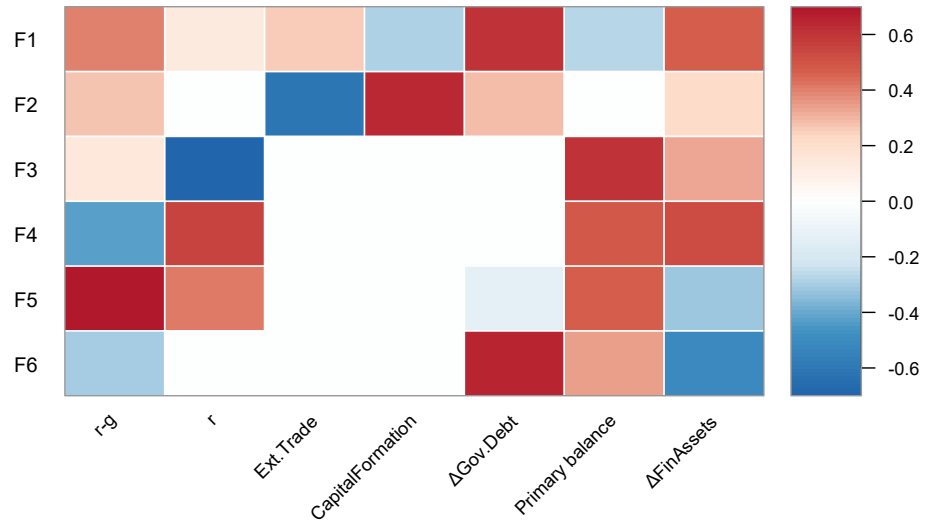


Table D13: Ireland: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.396	0.271	0.152	-0.421	0.693	-0.296	0.998
y^{10}	0.135	0.000	-0.679	0.552	0.413	0.000	0.954
Ext. Trade	0.262	-0.606	0.000	0.000	0.000	0.000	0.436
CapitalFormation	-0.280	0.624	0.000	0.000	0.000	0.000	0.467
ΔGov.Debt	0.605	0.280	0.000	0.000	-0.137	0.642	0.876
Primary balance	-0.273	0.000	0.598	0.477	0.460	0.339	0.986
ΔFinAssets	0.466	0.213	0.330	0.524	-0.318	-0.503	1.000
$\sum \lambda^2$	0.979	0.953	0.951	0.984	0.982	0.868	5.717
Prop. of common var.	0.171	0.167	0.166	0.172	0.172	0.152	1.000

Figure D27 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D28 shows the seven underlying series. Table D14 lists the factor estimates quarter by quarter.

Figure D27: Ireland: standardised factor estimates and standardised data

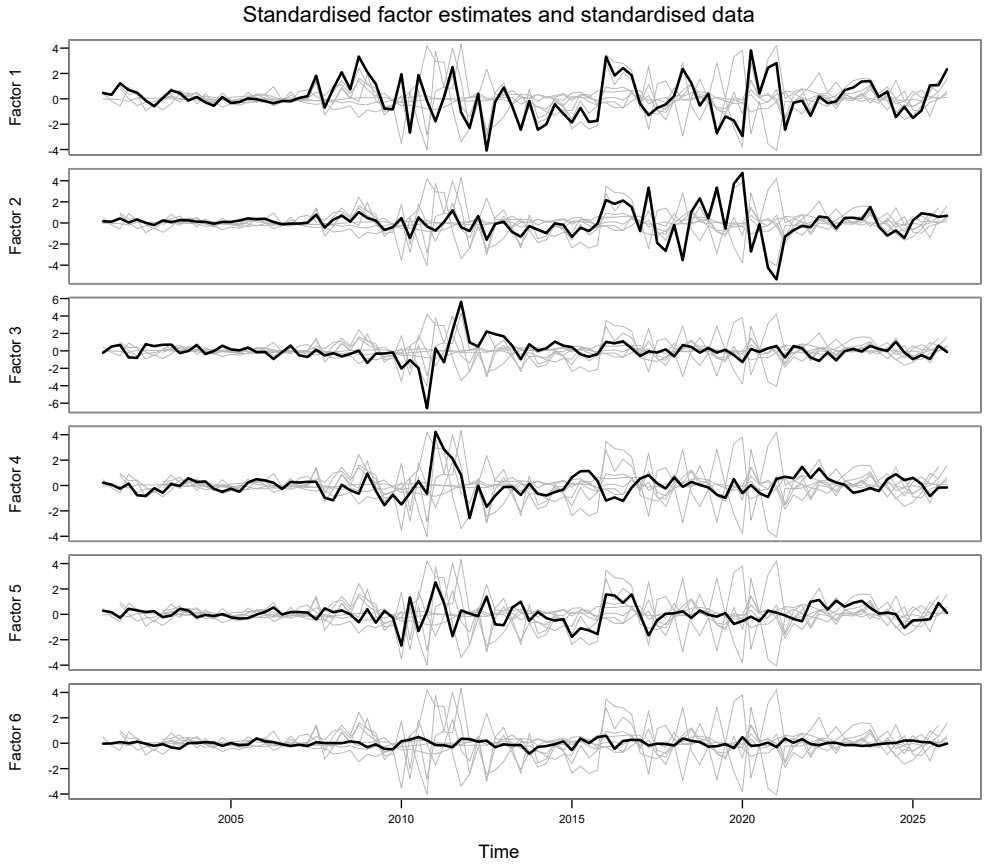


Figure D28: Ireland: input series over the estimation window

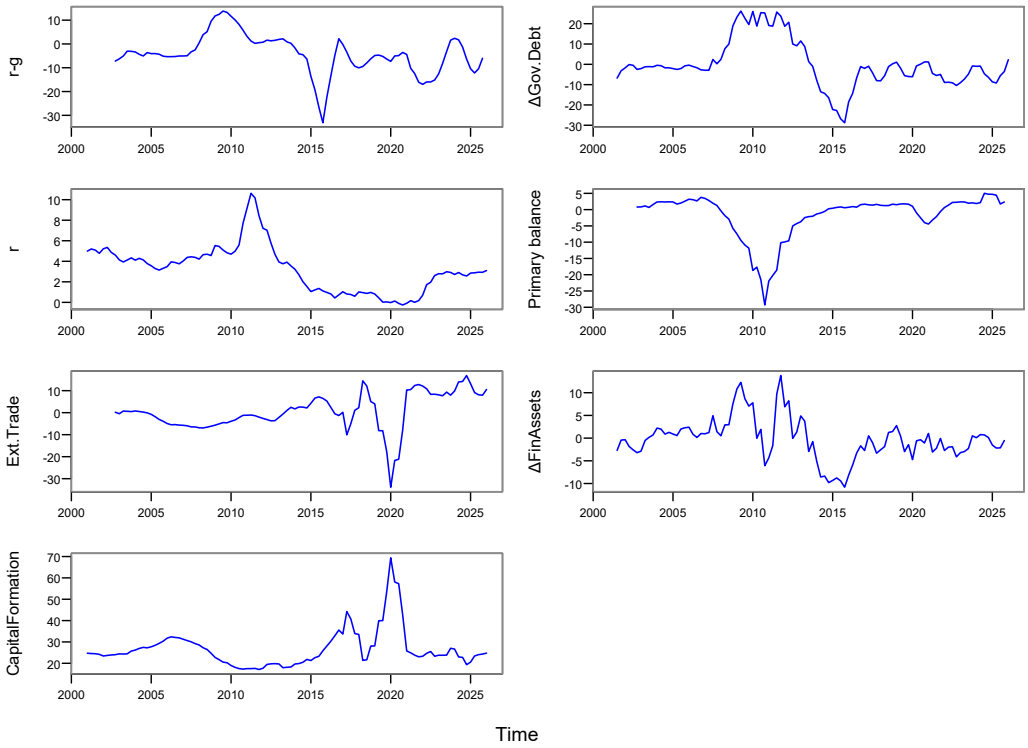


Table D14: Ireland: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.458	0.153	-0.221	0.218	0.286	-0.034	2013Q4	-0.175	-0.292	0.757	0.142	-0.508	-0.828
2001Q3	0.317	0.124	0.498	0.066	0.169	-0.016	2014Q1	-2.438	-0.637	0.015	-0.656	0.195	-0.297
2001Q4	1.225	0.427	0.680	-0.254	-0.272	0.092	2014Q2	-2.011	-0.965	0.298	-0.784	-0.292	-0.223
2002Q1	0.696	0.043	-0.751	0.149	0.440	-0.008	2014Q3	-0.431	-0.041	1.057	-0.520	-0.493	-0.077
2002Q2	0.485	0.325	-0.806	-0.769	0.309	0.122	2014Q4	-1.180	-0.183	0.594	-0.316	-0.371	0.148
2002Q3	-0.128	0.018	0.763	-0.826	0.186	-0.039	2015Q1	-1.867	-1.314	0.425	0.641	-1.773	-0.539
2002Q4	-0.589	-0.202	0.540	-0.218	0.249	-0.204	2015Q2	-0.716	-0.454	-0.393	1.125	-1.093	0.377
2003Q1	0.054	0.238	0.701	-0.568	-0.220	-0.073	2015Q3	-1.829	-0.769	-0.674	1.142	-1.260	0.005
2003Q2	0.668	0.081	0.712	0.115	-0.093	-0.325	2015Q4	-1.733	-0.068	-0.372	0.358	-1.554	0.470
2003Q3	0.460	0.258	-0.261	-0.037	0.440	-0.433	2016Q1	3.316	2.168	1.032	-1.192	1.569	0.590
2003Q4	-0.146	0.243	-0.017	0.576	0.308	0.015	2016Q2	1.835	1.807	0.870	-0.935	1.485	-0.439
2004Q1	0.151	0.128	0.684	0.271	-0.240	0.024	2016Q3	2.422	2.128	1.095	-1.222	0.914	0.180
2004Q2	-0.276	0.102	-0.370	0.321	-0.042	0.079	2016Q4	1.860	1.513	0.281	-0.172	1.562	0.260
2004Q3	-0.553	-0.072	-0.021	-0.264	-0.129	0.041	2017Q1	-0.387	-0.775	-0.595	0.538	-0.041	0.254
2004Q4	0.146	0.104	0.594	-0.493	0.017	-0.212	2017Q2	-1.294	3.347	-0.052	0.818	-1.661	-0.177
2005Q1	-0.334	0.095	0.167	-0.248	-0.239	0.014	2017Q3	-0.726	-1.881	-0.191	0.169	-0.469	-0.033
2005Q2	-0.251	0.236	0.064	-0.500	-0.330	-0.152	2017Q4	-0.453	-2.653	0.175	-0.236	0.050	-0.064
2005Q3	0.029	0.429	0.376	0.243	-0.300	-0.114	2018Q1	0.228	-0.190	-0.632	0.640	0.078	-0.178
2005Q4	-0.038	0.367	-0.148	0.498	-0.020	0.382	2018Q2	2.353	-3.533	0.661	-0.106	0.248	0.377
2006Q1	-0.184	0.398	-0.129	0.401	0.189	0.143	2018Q3	1.261	1.041	0.456	0.280	-0.280	0.189
2006Q2	-0.360	0.095	-0.941	0.218	0.544	0.083	2018Q4	-0.533	2.329	-0.206	0.040	0.301	0.089
2006Q3	-0.175	-0.130	-0.150	-0.279	-0.005	-0.081	2019Q1	0.412	0.431	0.325	-0.149	-0.032	-0.266
2006Q4	-0.187	-0.062	0.608	0.286	0.179	-0.217	2019Q2	-2.705	3.350	-0.207	-0.719	-0.180	-0.235
2007Q1	0.065	-0.063	-0.515	0.232	0.184	-0.124	2019Q3	-1.385	-0.542	0.124	-0.975	0.095	-0.066
2007Q2	0.213	0.030	-0.698	0.287	0.140	-0.211	2019Q4	-1.714	3.719	-0.482	0.503	-0.764	-0.390
2007Q3	1.819	0.793	0.133	0.292	-0.402	0.073	2020Q1	-2.933	4.743	-1.285	-0.609	-0.523	0.477
2007Q4	-0.694	-0.437	-0.528	-0.962	0.474	0.016	2020Q2	3.817	-2.707	0.214	0.042	-0.179	-0.210
2008Q1	0.798	0.306	-0.270	-1.166	0.167	0.015	2020Q3	0.437	-0.097	-0.123	-0.595	-0.537	-0.155
2008Q2	2.095	0.693	-0.626	0.057	0.314	0.008	2020Q4	2.448	-4.256	0.296	-0.915	0.283	0.052
2008Q3	0.760	0.137	-0.357	-0.376	-0.003	0.144	2021Q1	2.815	-5.349	0.548	0.522	0.141	-0.303
2008Q4	3.331	1.028	0.028	-0.643	-0.627	0.059	2021Q2	-2.443	-1.284	-0.762	0.687	-0.083	0.359
2009Q1	2.110	0.471	-1.383	0.947	0.400	-0.312	2021Q3	-0.310	-0.679	0.550	0.568	-0.367	0.043
2009Q2	1.167	0.221	-0.317	-0.415	-0.659	-0.083	2021Q4	-0.146	-0.279	0.316	1.470	-0.544	0.325
2009Q3	-0.771	-0.668	-0.306	-1.558	0.266	-0.444	2022Q1	-1.352	-0.404	-0.763	0.586	0.990	-0.058
2009Q4	-0.824	-0.386	-0.175	-0.734	-0.253	-0.473	2022Q2	0.172	0.613	-1.159	1.340	1.133	-0.161
2010Q1	1.936	0.462	-2.028	-1.496	-2.453	0.153	2022Q3	-0.346	0.499	-0.178	0.527	0.374	0.030
2010Q2	-2.655	-1.424	-1.064	-0.600	1.338	0.282	2022Q4	-0.202	-0.512	-1.100	0.238	1.078	0.030
2010Q3	1.878	0.522	-1.980	0.342	-1.319	0.490	2023Q1	0.680	0.474	-0.030	0.056	0.604	-0.151
2010Q4	-0.155	-0.329	-6.579	-0.646	0.221	0.229	2023Q2	0.892	0.496	0.228	-0.579	0.903	-0.140
2011Q1	-1.773	-0.720	0.283	4.228	2.518	-0.151	2023Q3	1.363	0.353	-0.068	-0.436	1.067	-0.210
2011Q2	0.158	0.098	-1.305	2.837	0.893	-0.161	2023Q4	1.395	1.523	0.571	-0.199	0.513	-0.179
2011Q3	2.509	1.195	2.354	2.107	-1.725	-0.315	2024Q1	0.134	-0.346	0.222	-0.448	0.064	-0.075
2011Q4	-1.075	-0.401	5.626	0.830	0.310	0.351	2024Q2	0.584	-1.199	-0.009	0.500	0.134	-0.005
2012Q1	-2.298	-0.767	0.965	-2.569	0.061	0.324	2024Q3	-1.451	-0.708	1.044	0.891	0.012	0.029
2012Q2	0.391	0.652	0.498	-0.019	-0.147	0.121	2024Q4	-0.634	-1.423	-0.171	0.417	-1.066	0.210
2012Q3	-4.080	-1.596	2.222	-1.683	1.396	0.207	2025Q1	-1.511	0.246	-0.957	0.604	-0.478	0.206
2012Q4	-0.239	-0.089	1.909	-0.805	-0.785	-0.313	2025Q2	-0.930	0.915	-0.494	0.126	-0.450	0.099
2013Q1	0.896	0.113	1.666	-0.117	-0.856	-0.090	2025Q3	1.064	0.800	-0.932	-0.835	-0.375	0.068
2013Q2	-0.593	-0.861	0.585	-0.104	0.529	-0.150	2025Q4	1.075	0.582	0.592	-0.167	0.894	-0.230
2013Q3	-2.440	-1.301	-0.972	-0.746	1.000	-0.141	2026Q1	2.330	0.676	-0.130	-0.155	0.115	-0.030

D.8 Italy

Italy is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D29 shows the loading of each series on each factor and Figure D30 the same loadings as a heat map; Table D15 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.519); Factor 2, y^{10} (0.517); Factor 3, Primary balance (0.634); Factor 4, y^{10} (0.794); Factor 5, ΔFinAssets (0.683); Factor 6, CapitalFormation (0.486). The sum of squared loadings is largest for y^{10} (0.990) and smallest for $r - g$ (0.338); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D29: Italy: factor loadings by series

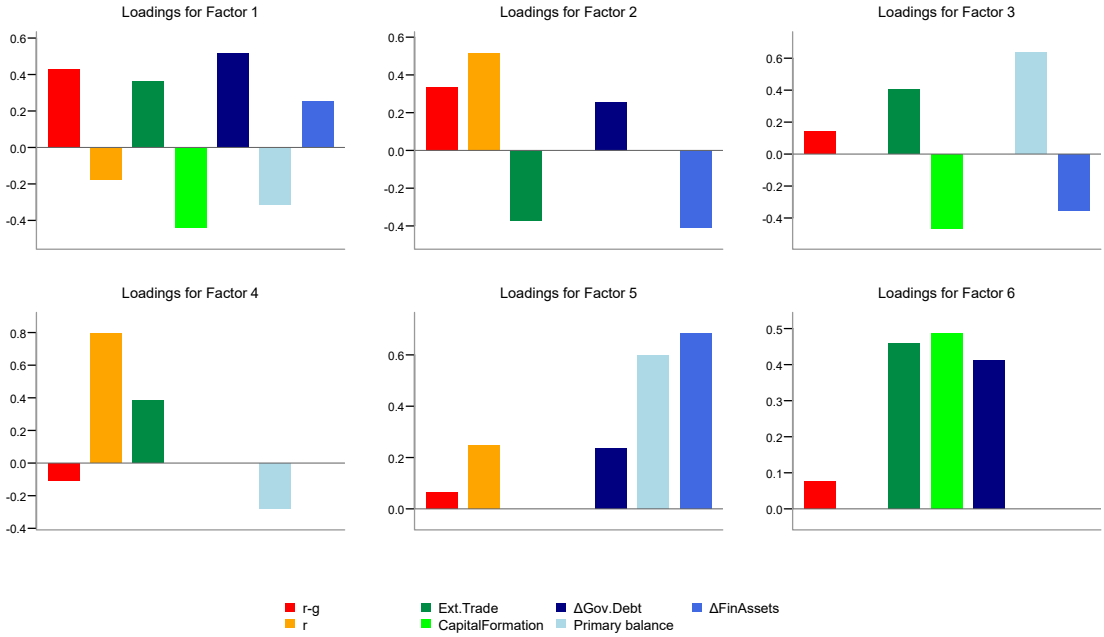


Figure D30: Italy: loading heat map

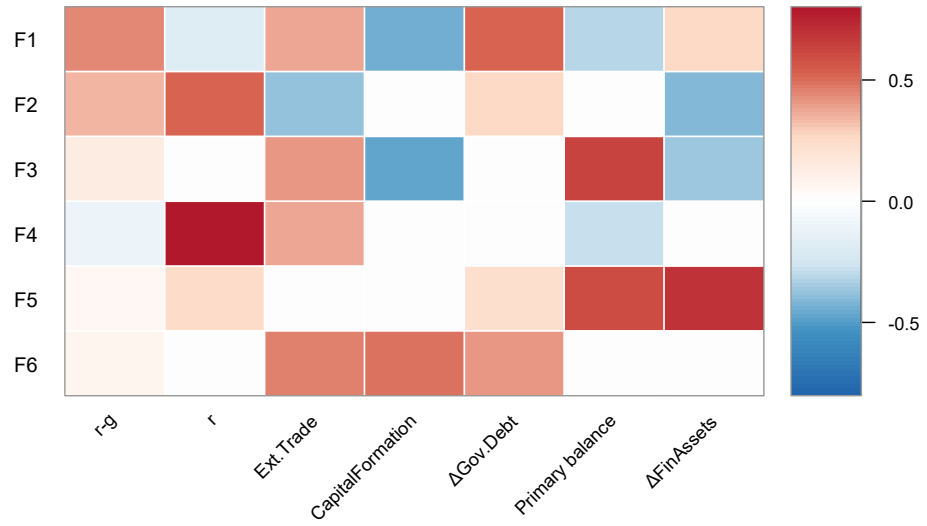


Table D15: Italy: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.430	0.334	0.141	-0.106	0.063	0.077	0.338
y^{10}	-0.177	0.517	0.000	0.794	0.247	0.000	0.990
Ext. Trade	0.366	-0.375	0.403	0.381	0.000	0.459	0.794
CapitalFormation	-0.443	0.000	-0.464	0.000	0.000	0.486	0.648
ΔGov.Debt	0.519	0.256	0.000	0.000	0.235	0.412	0.559
Primary balance	-0.313	0.000	0.634	-0.281	0.600	0.000	0.938
ΔFinAssets	0.254	-0.412	-0.351	0.000	0.683	0.000	0.825
$\sum \lambda^2$	0.979	0.755	0.923	0.866	0.946	0.623	5.091
Prop. of common var.	0.192	0.148	0.181	0.170	0.186	0.122	1.000

Figure D31 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D32 shows the seven underlying series. Table D16 lists the factor estimates quarter by quarter.

Figure D31: Italy: standardised factor estimates and standardised data

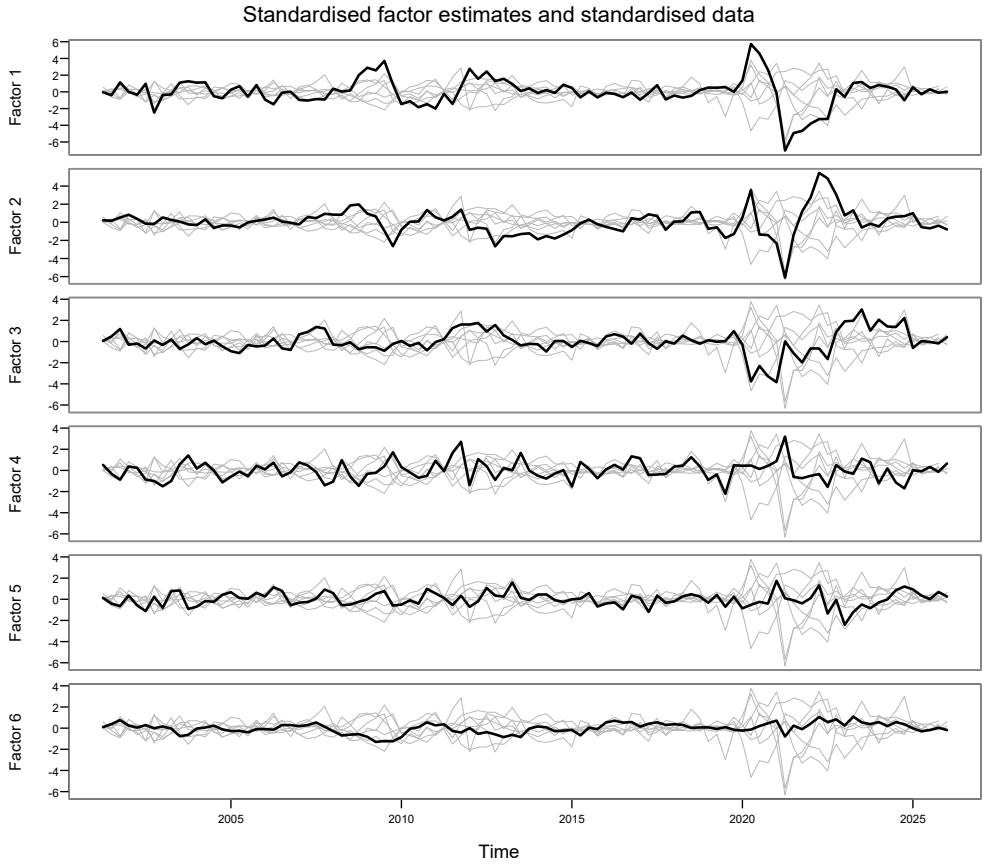


Figure D32: Italy: input series over the estimation window

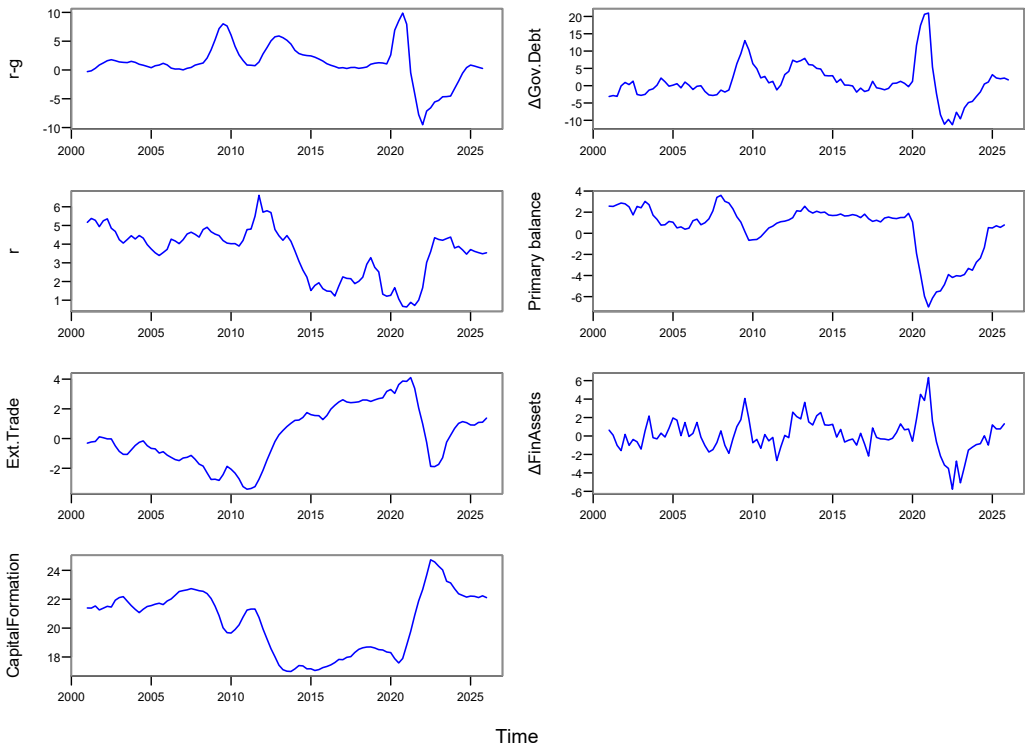


Table D16: Italy: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-0.052	0.224	0.061	0.515	0.134	0.099	2013Q4	0.419	-1.216	-0.223	-0.013	-0.062	-0.031
2001Q3	-0.400	0.175	0.494	-0.330	-0.417	0.385	2014Q1	-0.141	-1.887	-0.259	-0.509	0.445	0.153
2001Q4	1.139	0.523	1.195	-0.884	-0.648	0.766	2014Q2	0.223	-1.517	-0.950	-0.775	0.455	0.077
2002Q1	0.004	0.839	-0.306	0.375	0.362	0.240	2014Q3	-0.136	-1.793	0.050	-0.301	-0.085	-0.293
2002Q2	-0.344	0.384	-0.185	0.238	-0.586	0.047	2014Q4	0.862	-1.367	0.053	0.031	-0.247	-0.238
2002Q3	0.970	-0.150	-0.661	-0.864	-1.113	0.291	2015Q1	0.466	-0.876	-0.525	-1.525	-0.003	-0.171
2002Q4	-2.487	-0.180	0.105	-1.007	0.251	-0.010	2015Q2	-0.661	-0.114	0.076	0.801	0.065	-0.683
2003Q1	-0.373	0.537	-0.334	-1.503	-0.813	0.159	2015Q3	0.053	0.295	-0.141	0.146	0.590	0.040
2003Q2	-0.309	0.259	0.193	-1.005	0.778	-0.042	2015Q4	-0.662	-0.208	-0.432	-0.748	-0.685	-0.090
2003Q3	1.113	0.078	-0.699	0.580	0.832	-0.762	2016Q1	-0.109	-0.503	0.476	0.092	-0.403	0.535
2003Q4	1.266	-0.235	-0.270	1.416	-0.922	-0.632	2016Q2	-0.248	-0.750	0.696	0.533	-0.296	0.703
2004Q1	1.105	-0.312	0.326	0.169	-0.695	-0.035	2016Q3	-0.642	-1.000	0.467	0.069	-0.958	0.521
2004Q2	1.144	0.323	-0.259	0.728	-0.161	0.058	2016Q4	-0.089	0.443	-0.204	1.329	0.323	0.580
2004Q3	-0.529	-0.634	0.089	-0.018	-0.244	0.234	2017Q1	-0.946	0.301	0.764	1.137	0.131	0.138
2004Q4	-0.768	-0.343	-0.459	-1.133	0.420	-0.116	2017Q2	-0.140	0.885	-0.153	-0.429	-1.196	0.413
2005Q1	0.264	-0.355	-0.911	-0.598	0.673	-0.279	2017Q3	0.793	0.696	-0.690	-0.391	0.353	0.554
2005Q2	0.674	-0.565	-1.095	-0.107	0.118	-0.253	2017Q4	-0.907	-0.833	0.026	-0.330	-0.335	0.308
2005Q3	-0.569	-0.019	-0.322	-0.542	0.053	-0.398	2018Q1	-0.436	0.078	-0.200	0.360	-0.204	0.367
2005Q4	0.817	0.167	-0.468	0.437	0.611	-0.090	2018Q2	-0.685	0.102	0.555	0.447	0.274	0.311
2006Q1	-0.919	0.304	-0.383	0.101	0.267	-0.089	2018Q3	-0.486	1.086	0.137	1.249	0.476	0.015
2006Q2	-1.493	0.511	0.279	0.739	1.136	-0.145	2018Q4	0.198	1.146	-0.204	0.390	0.256	0.068
2006Q3	-0.099	0.084	-0.644	-0.566	0.791	0.293	2019Q1	0.517	-0.723	0.125	-0.897	-0.328	0.071
2006Q4	0.024	-0.053	-0.793	-0.177	-0.564	0.293	2019Q2	0.491	-0.549	-0.025	-0.376	0.413	-0.082
2007Q1	-0.962	-0.236	0.680	0.763	-0.322	0.181	2019Q3	0.578	-1.735	0.041	-2.218	-0.713	0.102
2007Q2	-1.042	0.578	0.905	0.477	-0.253	0.288	2019Q4	0.008	-1.294	0.987	0.486	0.244	-0.156
2007Q3	-0.860	0.461	1.378	-0.156	0.063	0.533	2020Q1	1.353	0.517	-0.307	0.441	-0.862	-0.233
2007Q4	-0.919	0.936	1.227	-1.410	0.923	0.126	2020Q2	5.724	3.555	-3.772	0.462	-0.521	-0.133
2008Q1	0.377	0.846	-0.297	-1.060	0.582	-0.277	2020Q3	4.584	-1.350	-2.306	0.134	-0.245	0.197
2008Q2	0.037	0.835	-0.354	0.965	-0.563	-0.691	2020Q4	2.645	-1.415	-3.289	0.428	-0.415	0.456
2008Q3	0.164	1.859	-0.061	-0.576	-0.499	-0.617	2021Q1	-0.156	-2.326	-3.842	0.871	1.742	0.713
2008Q4	2.005	1.974	-0.729	-1.465	-0.237	-0.570	2021Q2	-7.021	-6.106	0.012	3.196	0.088	-0.794
2009Q1	2.893	0.957	-0.531	-0.320	-0.010	-0.819	2021Q3	-4.928	-1.405	-1.100	-0.618	-0.096	0.235
2009Q2	2.572	0.645	-0.552	-0.226	0.519	-1.306	2021Q4	-4.676	1.192	-1.974	-0.736	-0.389	-0.102
2009Q3	3.706	-0.933	-0.882	0.393	0.772	-1.216	2022Q1	-3.807	2.736	-0.650	-0.512	0.103	0.421
2009Q4	0.902	-2.636	-0.223	1.714	-0.602	-1.241	2022Q2	-3.260	5.455	-0.657	-0.346	1.319	1.056
2010Q1	-1.471	-0.827	0.051	0.307	-0.503	-0.852	2022Q3	-3.246	4.842	-1.661	-1.566	-1.351	0.571
2010Q2	-1.110	0.063	-0.415	-0.187	-0.100	-0.065	2022Q4	0.323	3.066	0.908	0.509	-0.049	0.830
2010Q3	-1.848	0.088	-0.124	-0.696	-0.418	0.091	2023Q1	-0.599	0.757	1.910	-0.110	-2.418	0.245
2010Q4	-1.464	1.351	-0.833	-0.517	0.994	0.548	2023Q2	1.066	1.322	1.965	-0.322	-1.216	1.092
2011Q1	-2.007	0.555	-0.053	0.912	0.547	0.263	2023Q3	1.179	-0.576	3.035	1.117	-0.488	0.537
2011Q2	-0.232	0.195	0.224	-0.076	0.117	0.363	2023Q4	0.471	-0.154	1.025	0.769	-0.850	0.380
2011Q3	-1.447	0.616	1.270	1.657	-0.541	-0.269	2024Q1	0.810	-0.454	2.062	-1.213	-0.302	0.575
2011Q4	0.649	1.403	1.614	2.683	0.332	-0.416	2024Q2	0.616	0.425	1.425	0.187	-0.017	0.188
2012Q1	2.768	-0.841	1.613	-1.398	-0.717	0.007	2024Q3	0.293	0.620	1.367	-1.140	0.850	0.569
2012Q2	1.559	-0.598	1.762	1.071	-0.192	-0.542	2024Q4	-1.011	0.675	2.226	-1.705	1.208	0.367
2012Q3	2.439	-0.707	0.945	0.398	1.069	-0.378	2025Q1	0.550	1.005	-0.607	0.010	0.934	-0.022
2012Q4	1.294	-2.655	1.565	-0.904	0.343	-0.592	2025Q2	-0.282	-0.535	0.040	-0.085	0.348	-0.302
2013Q1	1.565	-1.518	0.574	0.238	0.254	-0.852	2025Q3	0.303	-0.669	-0.033	0.330	0.010	-0.183
2013Q2	0.942	-1.542	0.150	0.001	1.597	-0.629	2025Q4	-0.107	-0.373	-0.180	-0.139	0.697	0.039
2013Q3	0.089	-1.299	-0.369	1.658	0.133	-0.849	2026Q1	0.013	-0.787	0.412	0.653	0.254	-0.174

D.9 Latvia

Latvia is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D33 shows the loading of each series on each factor and Figure D34 the same loadings as a heat map; Table D17 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, $r-g$ (0.507); Factor 2, ΔFinAssets (0.662); Factor 3, Primary balance (0.651); Factor 4, y^{10} (0.734); Factor 5, Ext. Trade (0.772); Factor 6, $r-g$ (0.737). The sum of squared loadings is largest for Capital-Formation (1.000) and smallest for ΔGov.Debt (0.397); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D33: Latvia: factor loadings by series

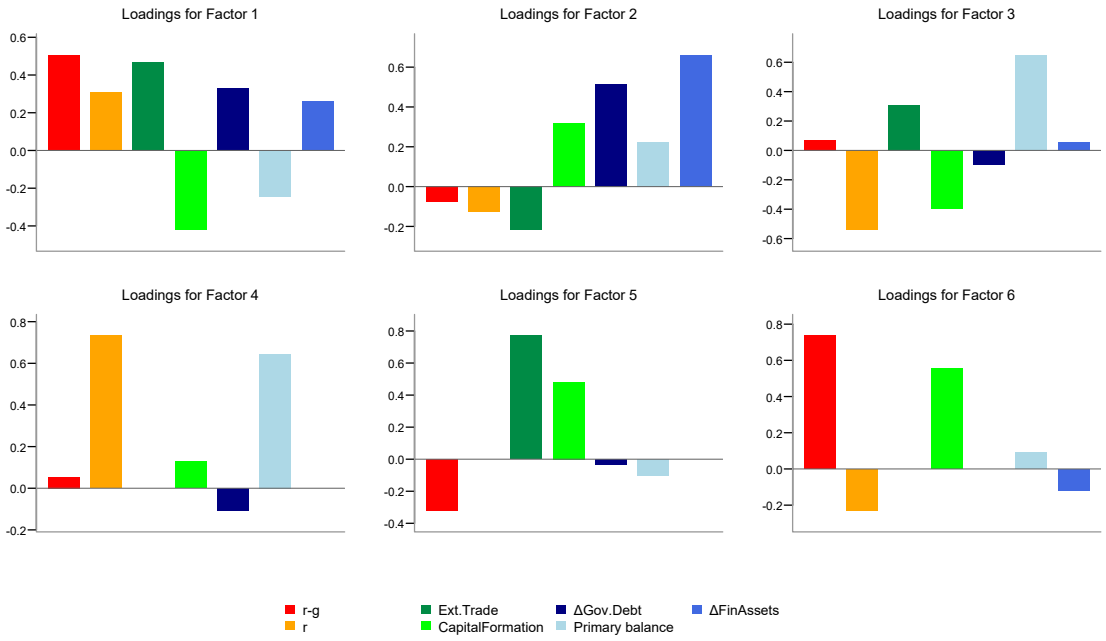


Figure D34: Latvia: loading heat map

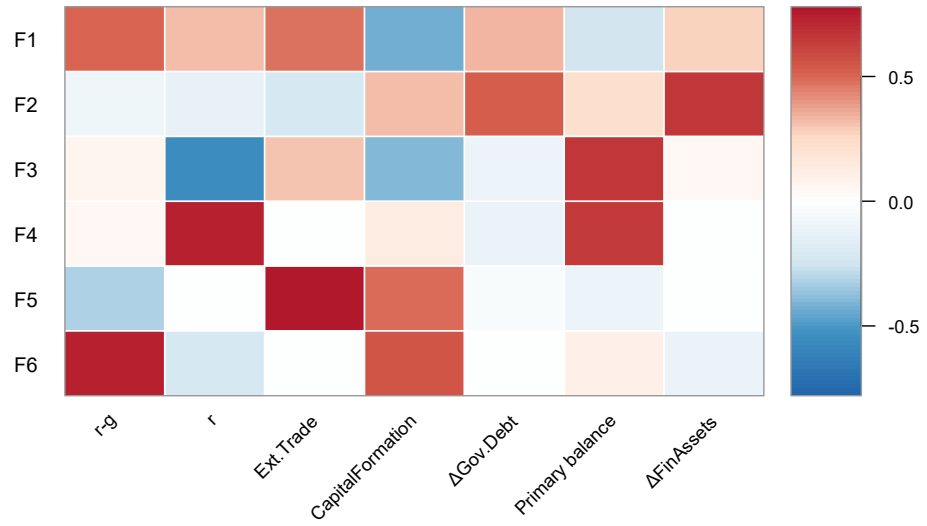


Table D17: Latvia: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.507	-0.078	0.072	0.055	-0.322	0.737	0.919
y^{10}	0.309	-0.128	-0.543	0.734	0.000	-0.232	0.998
Ext. Trade	0.467	-0.219	0.307	0.000	0.772	0.000	0.956
CapitalFormation	-0.423	0.320	-0.401	0.129	0.483	0.556	1.000
ΔGov.Debt	0.331	0.514	-0.098	-0.109	-0.037	0.000	0.397
Primary balance	-0.247	0.225	0.651	0.646	-0.101	0.094	0.971
ΔFinAssets	0.264	0.662	0.056	0.000	0.000	-0.119	0.525
$\sum \lambda^2$	0.989	0.926	0.991	0.987	0.945	0.929	5.766
Prop. of common var.	0.172	0.161	0.172	0.171	0.164	0.161	1.000

Figure D35 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D36 shows the seven underlying series. Table D18 lists the factor estimates quarter by quarter.

Figure D35: Latvia: standardised factor estimates and standardised data

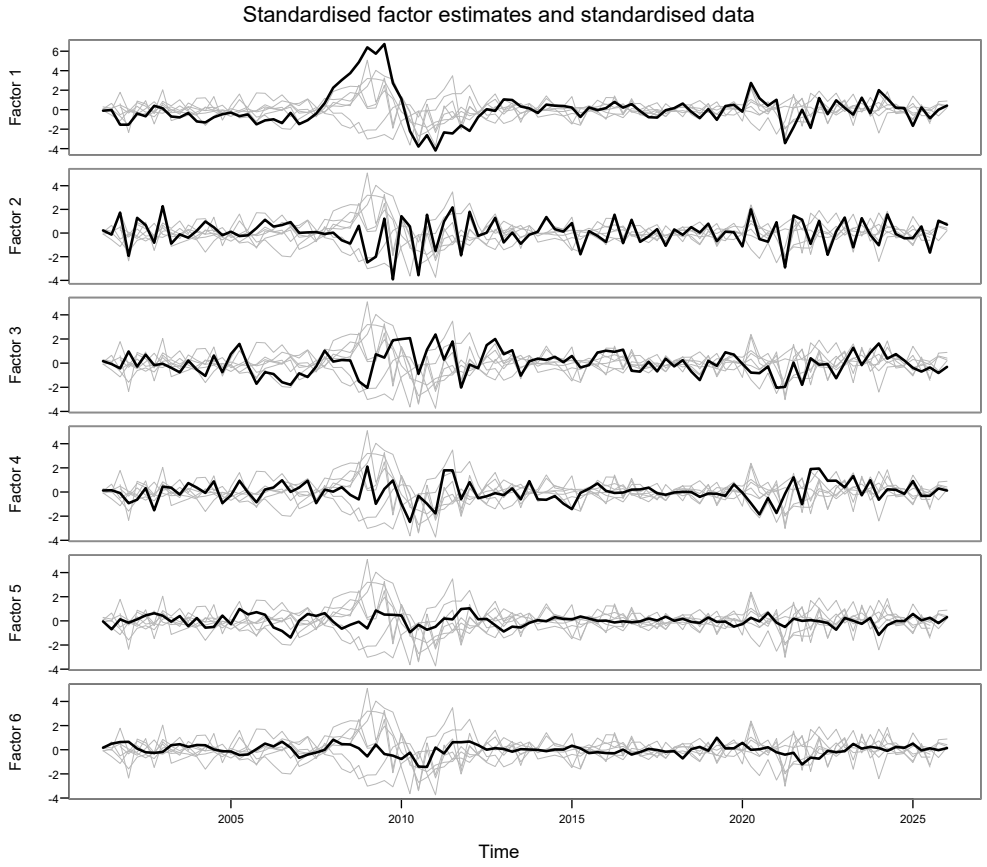


Figure D36: Latvia: input series over the estimation window

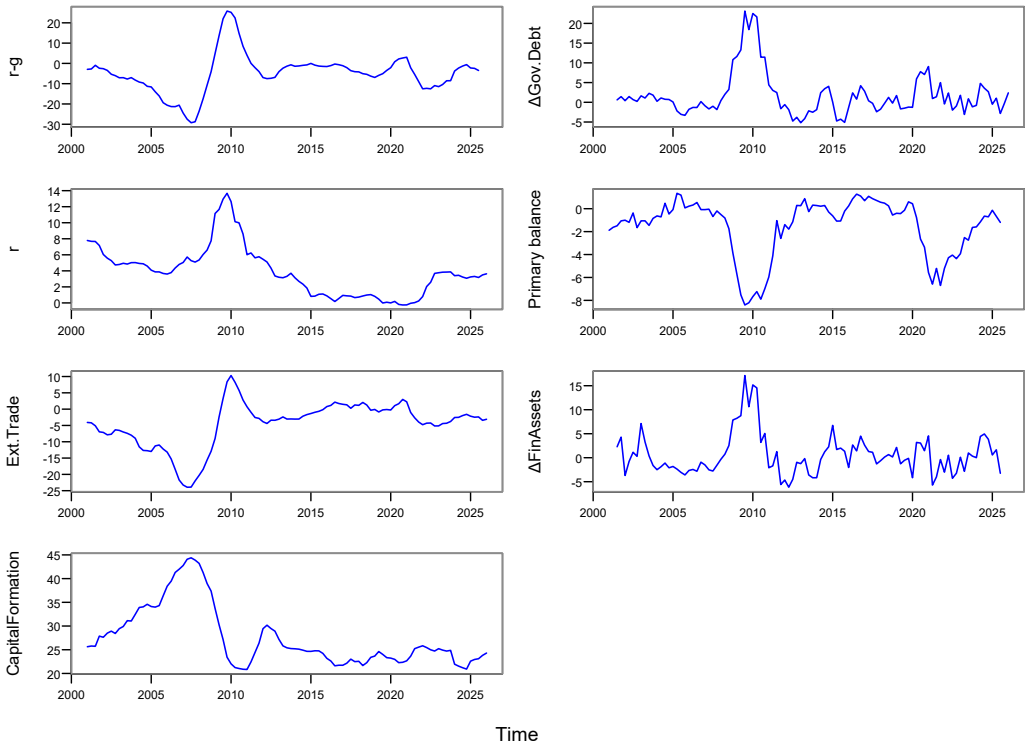


Table D18: Latvia: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-0.105	0.221	0.166	0.127	-0.045	0.166	2013Q4	0.133	-0.145	0.155	0.898	-0.169	0.018
2001Q3	-0.037	-0.133	-0.075	0.144	-0.697	0.499	2014Q1	-0.344	0.110	0.374	-0.611	0.059	-0.014
2001Q4	-1.544	1.729	-0.432	-0.087	0.128	0.636	2014Q2	0.521	1.354	0.266	-0.644	-0.028	-0.117
2002Q1	-1.540	-1.933	0.968	-0.927	-0.154	0.657	2014Q3	0.408	0.324	0.525	-0.334	0.309	0.015
2002Q2	-0.392	1.284	-0.289	-0.635	0.127	0.095	2014Q4	0.378	0.120	0.077	-0.964	0.162	0.011
2002Q3	-0.678	0.674	0.706	0.321	0.454	-0.210	2015Q1	0.216	0.845	0.593	-1.416	0.124	0.335
2002Q4	0.396	-0.820	-0.167	-1.515	0.629	-0.260	2015Q2	-0.752	-1.798	-0.361	-0.077	0.363	0.113
2003Q1	0.138	2.273	-0.051	0.444	0.432	-0.185	2015Q3	0.231	0.163	-0.167	0.271	0.194	-0.279
2003Q2	-0.704	-0.894	-0.388	0.357	-0.065	0.376	2015Q4	-0.045	-0.237	0.876	0.711	0.002	-0.205
2003Q3	-0.789	-0.107	-0.762	-0.215	0.400	0.459	2016Q1	0.161	-0.751	1.032	0.093	0.022	-0.276
2003Q4	-0.341	-0.398	0.213	0.738	-0.446	0.239	2016Q2	0.799	1.552	0.936	-0.079	-0.133	-0.293
2004Q1	-1.209	0.198	-0.547	0.367	0.236	0.394	2016Q3	0.203	-0.856	1.116	-0.044	-0.037	0.010
2004Q2	-1.311	0.978	-1.060	-0.075	-0.565	0.372	2016Q4	0.577	1.121	-0.630	0.197	-0.116	-0.403
2004Q3	-0.776	0.483	0.632	0.883	-0.504	0.025	2017Q1	-0.144	-0.747	-0.707	0.214	-0.043	-0.190
2004Q4	-0.464	-0.182	-0.754	-0.918	0.435	-0.124	2017Q2	-0.773	-0.282	0.122	0.360	0.199	0.074
2005Q1	-0.306	0.116	0.755	-0.277	-0.266	-0.144	2017Q3	-0.813	0.334	-0.688	-0.099	0.029	-0.023
2005Q2	-0.665	-0.253	1.594	0.923	0.985	-0.445	2017Q4	-0.054	-1.078	0.375	-0.230	0.349	-0.162
2005Q3	-0.486	-0.192	-0.225	-0.036	0.540	-0.392	2018Q1	0.158	0.307	-0.269	-0.041	0.009	-0.124
2005Q4	-1.505	0.412	-1.710	-0.854	0.721	0.052	2018Q2	0.628	-0.150	0.247	0.022	0.168	-0.721
2006Q1	-1.102	1.122	-0.751	0.202	0.508	0.510	2018Q3	-0.212	0.500	-0.697	0.000	-0.080	0.048
2006Q2	-0.987	0.546	-0.900	0.376	-0.544	0.275	2018Q4	-0.862	0.041	-1.403	-0.403	-0.167	0.235
2006Q3	-1.378	0.680	-1.579	0.974	-0.838	0.666	2019Q1	0.059	0.794	0.190	-0.124	0.284	-0.082
2006Q4	-0.326	0.928	-1.791	0.008	-1.385	0.172	2019Q2	-1.040	-0.693	-0.228	-0.148	-0.087	0.998
2007Q1	-1.485	0.017	-0.845	0.363	0.010	-0.671	2019Q3	0.371	0.113	0.899	-0.323	-0.061	0.110
2007Q2	-1.080	0.056	-1.151	0.933	0.571	-0.405	2019Q4	0.499	0.068	0.703	0.636	-0.474	0.117
2007Q3	-0.407	0.079	-0.288	-0.926	0.410	-0.231	2020Q1	-0.207	-1.126	-0.061	-0.057	-0.267	0.569
2007Q4	0.670	-0.078	1.031	0.193	0.641	-0.011	2020Q2	2.743	2.003	-0.786	-0.934	0.252	-0.010
2008Q1	2.252	0.034	0.128	0.027	-0.071	0.835	2020Q3	1.173	-0.510	-0.830	-1.856	-0.045	0.042
2008Q2	3.037	-0.610	0.258	0.399	-0.648	0.465	2020Q4	0.410	-0.733	-0.277	-0.497	0.665	0.207
2008Q3	3.749	-0.889	0.224	-0.237	-0.326	0.436	2021Q1	1.017	0.916	-2.039	-1.741	-0.151	-0.210
2008Q4	4.860	0.614	-1.501	-0.615	-0.069	0.122	2021Q2	-3.436	-2.898	-1.965	-0.147	-0.506	-0.408
2009Q1	6.396	-2.485	-2.043	2.110	-0.627	-0.556	2021Q3	-1.801	1.476	0.044	1.214	0.183	-0.250
2009Q2	5.743	-2.001	0.735	-0.982	0.858	0.428	2021Q4	-0.005	1.125	-1.799	-1.020	0.000	-1.218
2009Q3	6.733	1.219	0.457	0.252	0.523	-0.355	2022Q1	-1.872	-0.918	0.404	1.902	0.060	-0.653
2009Q4	2.729	-3.917	1.893	0.955	0.501	-0.490	2022Q2	1.193	1.030	-0.130	1.944	-0.035	-0.741
2010Q1	1.102	1.425	1.992	-0.970	0.451	-0.772	2022Q3	-0.464	-1.840	-0.085	0.930	-0.176	-0.092
2010Q2	-2.196	0.552	2.073	-2.480	-0.947	-0.239	2022Q4	0.960	0.121	-1.253	0.937	-0.733	-0.200
2010Q3	-3.787	-3.564	-0.897	-0.319	-0.324	-1.394	2023Q1	0.186	1.310	0.033	0.381	0.240	-0.119
2010Q4	-2.616	1.548	1.117	-0.970	-0.737	-1.424	2023Q2	-0.512	-1.212	1.278	1.326	0.011	0.480
2011Q1	-4.193	-1.519	2.374	-1.776	-0.488	0.183	2023Q3	1.224	1.255	-0.157	-0.273	-0.249	0.095
2011Q2	-2.333	0.975	0.289	1.793	0.197	-0.310	2023Q4	-0.369	-0.159	0.869	0.977	0.242	0.241
2011Q3	-2.443	2.172	1.797	1.794	0.124	0.625	2024Q1	2.006	-1.034	1.626	-0.645	-1.168	0.137
2011Q4	-1.612	-1.875	-2.025	-0.591	0.980	0.624	2024Q2	1.140	1.574	0.371	0.221	-0.353	-0.098
2012Q1	-2.180	1.799	-0.114	0.816	1.043	0.678	2024Q3	0.187	-0.081	0.749	0.181	-0.023	0.228
2012Q2	-0.762	-0.238	-0.426	-0.515	0.132	0.379	2024Q4	0.156	-0.442	0.216	-0.156	-0.013	0.163
2012Q3	0.045	0.027	1.490	-0.352	0.163	0.003	2025Q1	-1.663	-0.423	-0.388	0.910	0.570	0.502
2012Q4	-0.131	1.291	2.001	-0.096	-0.362	0.129	2025Q2	0.235	0.557	-0.701	-0.329	0.043	-0.059
2013Q1	1.054	-0.794	0.736	-0.257	-0.881	0.062	2025Q3	-0.891	-1.654	-0.356	-0.315	0.258	0.119
2013Q2	0.993	0.046	1.074	0.296	-0.479	-0.162	2025Q4	-0.006	1.045	-0.817	0.294	-0.186	-0.035
2013Q3	0.327	-0.904	-1.040	-0.612	-0.552	0.049	2026Q1	0.417	0.711	-0.310	0.119	0.312	0.136

D.10 Luxembourg

Luxembourg is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D37 shows the loading of each series on each factor and Figure D38 the same loadings as a heat map; Table D19 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, CapitalFormation (0.502); Factor 2, ΔGov.Debt (0.577); Factor 3, Primary balance (0.359); Factor 4, ΔFinAssets (0.445); Factor 5, CapitalFormation (0.537); Factor 6, $r - g$ (0.616). The sum of squared loadings is largest for $r - g$ (0.977) and smallest for y^{10} (0.051); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D37: Luxembourg: factor loadings by series

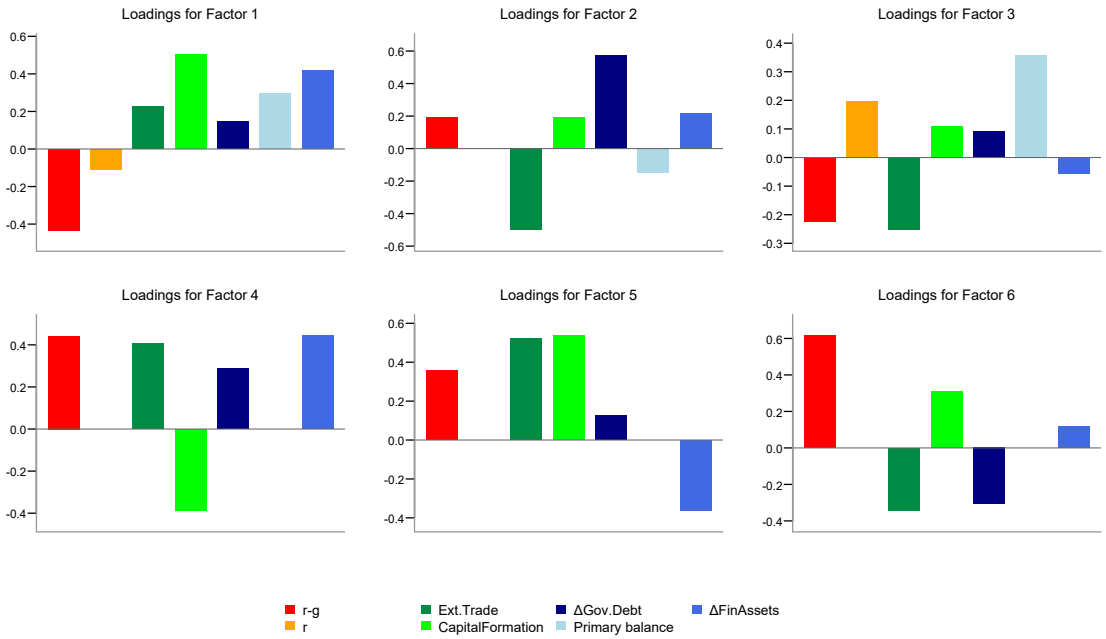


Figure D38: Luxembourg: loading heat map

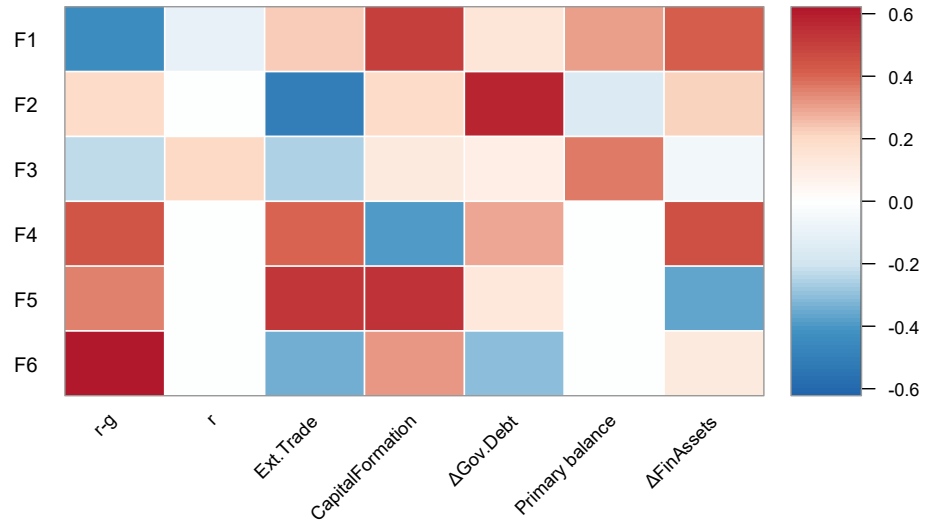


Table D19: Luxembourg: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	-0.432	0.194	-0.225	0.442	0.356	0.616	0.977
y^{10}	-0.109	0.000	0.199	0.000	0.000	0.000	0.051
Ext. Trade	0.226	-0.502	-0.254	0.405	0.520	-0.345	0.921
CapitalFormation	0.502	0.191	0.111	-0.389	0.537	0.309	0.836
ΔGov.Debt	0.146	0.577	0.091	0.289	0.125	-0.308	0.556
Primary balance	0.297	-0.149	0.359	0.000	0.000	0.000	0.239
ΔFinAssets	0.416	0.221	-0.057	0.445	-0.364	0.118	0.569
$\sum \lambda^2$	0.784	0.729	0.308	0.792	0.834	0.703	4.150
Prop. of common var.	0.189	0.176	0.074	0.191	0.201	0.169	1.000

Figure D39 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D40 shows the seven underlying series. Table D20 lists the factor estimates quarter by quarter.

Figure D39: Luxembourg: standardised factor estimates and standardised data

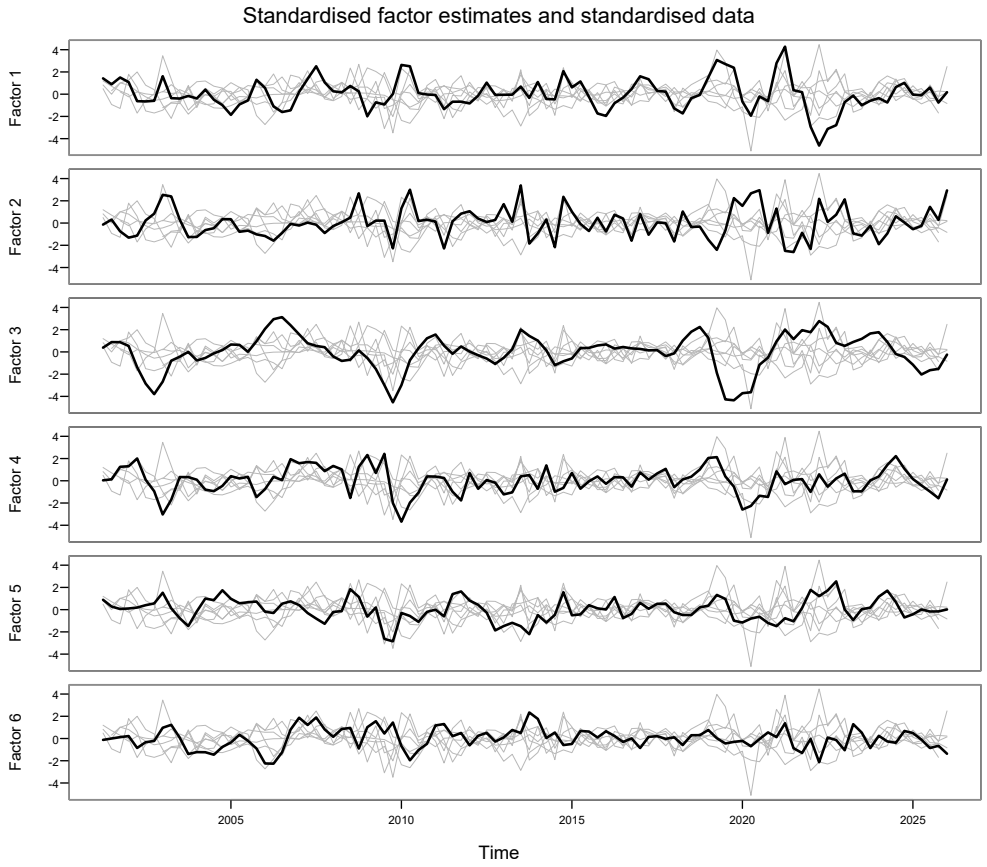


Figure D40: Luxembourg: input series over the estimation window

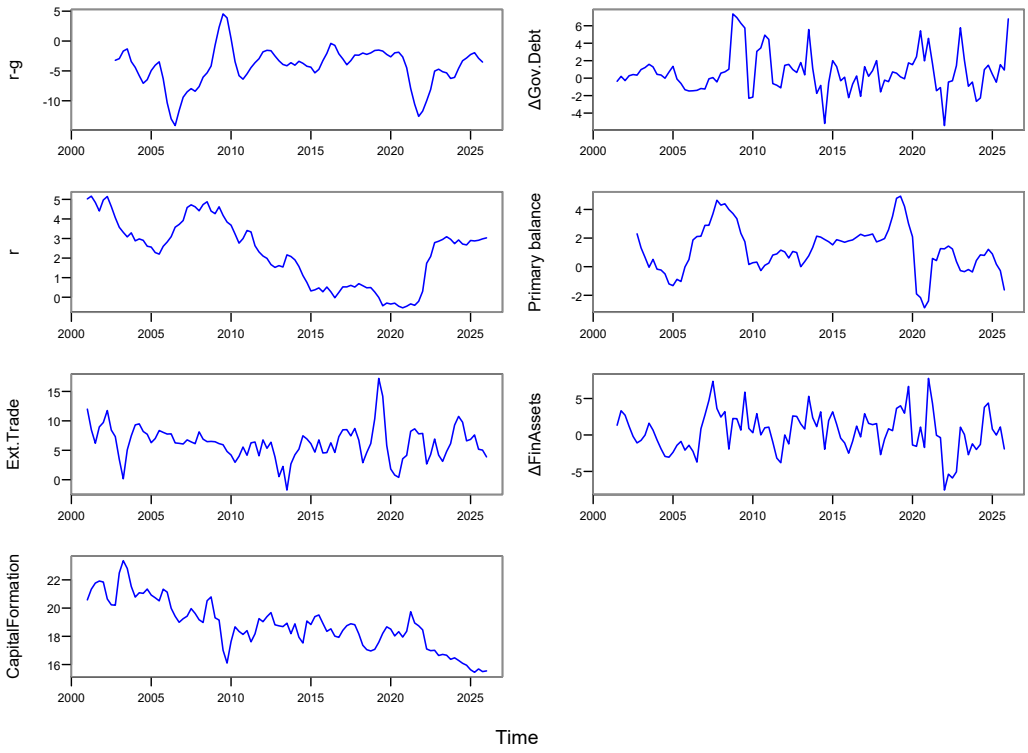


Table D20: Luxembourg: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	1.422	-0.139	0.382	0.040	0.885	-0.125	2013Q4	-0.337	-1.838	1.438	0.512	-2.206	2.343
2001Q3	0.888	0.311	0.882	0.131	0.279	0.002	2014Q1	1.094	-0.933	1.017	-0.721	-0.482	1.768
2001Q4	1.500	-0.740	0.875	1.257	0.058	0.120	2014Q2	-0.434	0.312	0.145	1.384	-1.162	0.046
2002Q1	1.089	-1.314	0.541	1.307	0.105	0.223	2014Q3	-0.466	-2.173	-1.196	-0.986	-0.467	0.540
2002Q2	-0.629	-1.132	-1.376	2.007	0.187	-0.850	2014Q4	2.074	2.361	-0.844	-0.638	1.566	-0.589
2002Q3	-0.652	0.236	-2.855	0.092	0.401	-0.331	2015Q1	0.608	1.023	-0.590	0.703	-0.507	-0.490
2002Q4	-0.594	0.844	-3.800	-0.941	0.560	-0.221	2015Q2	1.146	-0.076	0.342	-0.716	-0.425	0.679
2003Q1	1.615	2.536	-2.676	-3.030	1.522	0.968	2015Q3	-0.235	-0.717	0.363	-0.045	0.403	0.627
2003Q2	-0.359	2.398	-0.790	-1.703	0.119	1.231	2015Q4	-1.736	0.478	0.577	0.392	0.118	0.090
2003Q3	-0.393	0.306	-0.430	0.333	-0.774	0.066	2016Q1	-1.955	-0.759	0.675	-0.269	0.016	0.655
2003Q4	-0.160	-1.260	0.001	0.329	-1.478	-1.394	2016Q2	-0.807	0.752	0.297	0.334	1.137	0.225
2004Q1	-0.385	-1.247	-0.766	0.094	-0.154	-1.223	2016Q3	-0.308	0.406	0.443	0.305	-0.761	-0.312
2004Q2	0.415	-0.631	-0.543	-0.797	1.000	-1.235	2016Q4	0.466	-1.587	0.318	-0.535	-0.384	0.013
2004Q3	-0.499	-0.466	-0.128	-0.936	0.851	-1.458	2017Q1	1.609	0.807	0.266	0.744	0.614	-0.855
2004Q4	-0.983	0.351	0.165	-0.462	1.745	-0.757	2017Q2	1.344	-1.051	0.141	0.103	0.077	0.148
2005Q1	-1.861	0.346	0.666	0.414	0.974	-0.359	2017Q3	0.293	0.058	0.158	0.658	0.527	0.213
2005Q2	-0.921	-0.786	0.635	0.208	0.573	0.340	2017Q4	0.230	-0.005	-0.378	1.073	0.539	-0.034
2005Q3	-0.564	-0.681	0.012	0.353	0.662	-0.204	2018Q1	-1.280	-1.657	-0.130	-0.578	-0.235	0.112
2005Q4	1.291	-1.017	0.955	-1.470	0.731	-0.897	2018Q2	-1.732	1.039	1.011	0.117	-0.481	-0.589
2006Q1	0.560	-1.166	2.071	-0.754	-0.184	-2.244	2018Q3	-0.361	-0.358	1.813	0.365	-0.482	0.295
2006Q2	-1.103	-1.586	2.943	0.360	-0.302	-2.264	2018Q4	-0.069	-0.317	2.245	1.012	0.204	0.302
2006Q3	-1.607	-0.895	3.129	0.057	0.534	-1.244	2019Q1	1.514	-1.495	1.273	2.051	0.357	0.767
2006Q4	-1.452	-0.075	2.396	1.945	0.748	0.804	2019Q2	3.075	-2.410	-1.885	2.127	1.323	0.028
2007Q1	0.208	-0.222	1.583	1.578	0.409	1.870	2019Q3	2.708	-0.679	-4.271	0.390	0.964	-0.447
2007Q2	1.371	0.040	0.781	1.687	-0.304	1.218	2019Q4	2.382	2.243	-4.351	-0.526	-0.994	-0.299
2007Q3	2.512	-0.149	0.539	1.610	-0.802	1.894	2020Q1	-0.666	1.544	-3.714	-2.591	-1.157	-0.213
2007Q4	1.060	-0.898	0.422	0.868	-1.274	0.817	2020Q2	-1.937	2.672	-3.629	-2.259	-0.802	-0.696
2008Q1	0.258	-0.271	-0.412	1.336	-0.209	0.151	2020Q3	-0.201	2.954	-1.158	-1.346	-0.635	-0.054
2008Q2	0.161	0.071	-0.811	1.024	-0.132	0.853	2020Q4	-0.623	-0.884	-0.513	-1.453	-1.198	0.550
2008Q3	0.740	0.485	-0.701	-1.549	1.841	0.954	2021Q1	2.793	1.306	0.945	0.846	-1.487	0.124
2008Q4	0.250	2.674	0.136	1.253	1.148	-0.902	2021Q2	4.275	-2.501	2.009	-0.304	-0.770	1.375
2009Q1	-1.980	-0.275	-0.551	2.318	-0.629	1.019	2021Q3	0.356	-2.608	1.153	0.082	-1.049	-0.874
2009Q2	-0.731	0.217	-1.534	0.702	0.197	1.556	2021Q4	0.181	-0.877	1.958	0.151	0.150	-1.303
2009Q3	-0.930	0.217	-2.980	2.424	-2.632	0.446	2022Q1	-2.927	-2.338	1.772	-1.002	1.778	-0.032
2009Q4	0.017	-2.280	-4.536	-2.003	-2.849	1.438	2022Q2	-4.622	2.180	2.773	0.579	1.214	-2.132
2010Q1	2.630	1.320	-2.991	-3.677	-0.294	-0.674	2022Q3	-3.124	0.103	2.253	-0.532	1.724	0.085
2010Q2	2.509	2.998	-0.749	-1.966	-0.570	-1.958	2022Q4	-2.790	0.752	0.811	0.207	2.542	-0.134
2010Q3	0.096	0.178	0.329	-1.070	-1.096	-1.039	2023Q1	-0.712	2.130	0.540	0.643	0.015	-1.054
2010Q4	-0.029	0.304	1.223	0.390	-0.202	-0.478	2023Q2	-0.128	-0.938	0.894	-0.943	-0.934	1.289
2011Q1	-0.070	0.132	1.564	0.380	0.000	1.176	2023Q3	-0.990	-1.131	1.175	-0.953	0.040	0.527
2011Q2	-1.349	-2.285	0.567	0.253	-0.607	1.302	2023Q4	-0.590	-0.252	1.657	0.028	0.178	-0.863
2011Q3	-0.680	0.157	-0.173	-0.983	1.398	0.212	2024Q1	-0.356	-1.903	1.776	0.388	1.170	0.243
2011Q4	-0.673	0.845	0.491	-1.768	1.636	0.500	2024Q2	-0.748	-0.939	0.894	1.426	1.711	-0.264
2012Q1	-0.832	1.066	0.033	0.693	0.822	-0.615	2024Q3	0.639	0.610	-0.191	2.221	0.679	-0.383
2012Q2	-0.133	0.401	-0.299	-0.722	0.435	0.250	2024Q4	1.024	0.049	-0.453	1.088	-0.704	0.679
2012Q3	1.040	0.093	-0.598	0.067	-0.262	0.509	2025Q1	-0.046	-0.557	-1.149	0.159	-0.420	0.493
2012Q4	-0.063	0.274	-1.078	-0.170	-1.869	-0.276	2025Q2	-0.092	-0.252	-2.032	-0.415	0.018	-0.104
2013Q1	-0.052	1.697	-0.530	-1.225	-1.476	0.094	2025Q3	0.602	1.456	-1.645	-0.969	-0.168	-0.858
2013Q2	-0.044	0.115	0.242	-1.041	-1.183	0.769	2025Q4	-0.754	0.281	-1.526	-1.569	-0.158	-0.650
2013Q3	0.688	3.386	2.017	0.401	-1.492	0.494	2026Q1	0.176	2.934	-0.242	0.118	0.017	-1.368

D.11 Netherlands

Netherlands is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D41 shows the loading of each series on each factor and Figure D42 the same loadings as a heat map; Table D21 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, $r - g$ (0.529); Factor 2, Ext. Trade (-0.661); Factor 3, y^{10} (0.863); Factor 4, Primary balance (0.630); Factor 5, ΔFinAssets (0.769); Factor 6, $r - g$ (0.672). The sum of squared loadings is largest for y^{10} (0.993) and smallest for CapitalFormation (0.389); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D41: Netherlands: factor loadings by series

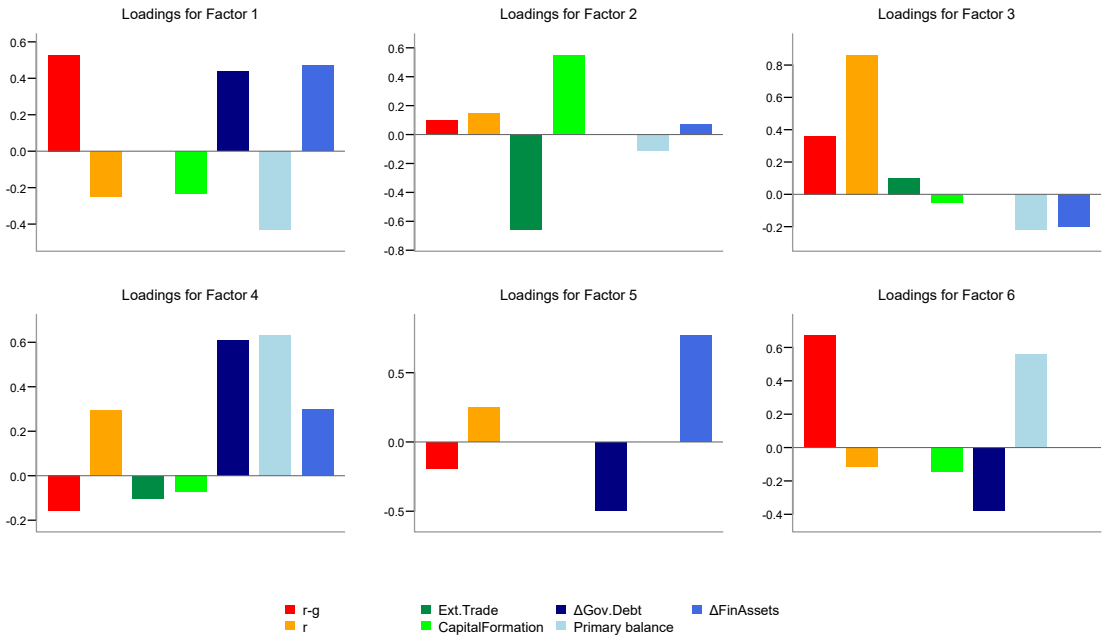


Figure D42: Netherlands: loading heat map

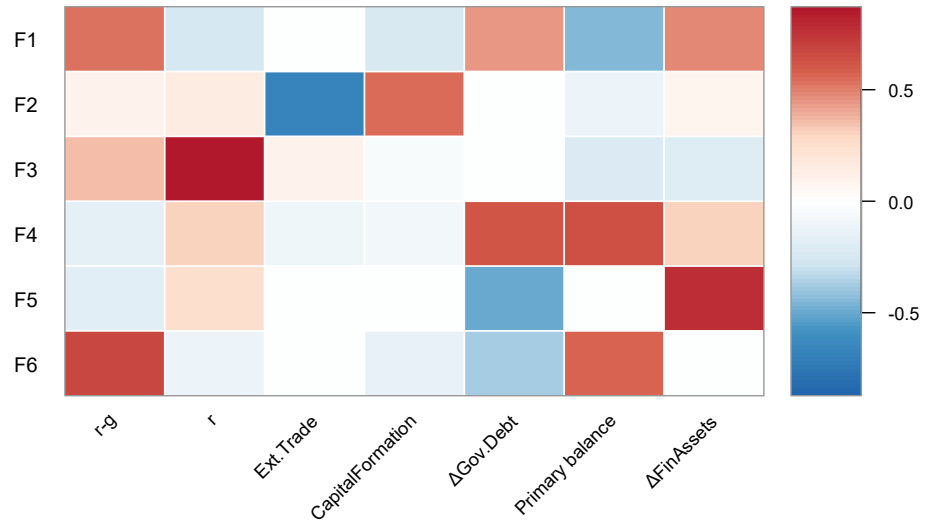


Table D21: Netherlands: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.529	0.101	0.360	-0.158	-0.188	0.672	0.931
y^{10}	-0.251	0.148	0.863	0.295	0.248	-0.118	0.993
Ext. Trade	0.000	-0.661	0.102	-0.103	0.000	0.000	0.458
CapitalFormation	-0.235	0.552	-0.052	-0.073	0.000	-0.147	0.389
ΔGov.Debt	0.438	0.000	0.000	0.609	-0.497	-0.379	0.954
Primary balance	-0.433	-0.116	-0.222	0.630	0.000	0.559	0.961
ΔFinAssets	0.472	0.074	-0.201	0.298	0.769	0.000	0.948
$\sum \lambda^2$	1.000	0.792	0.978	0.985	0.935	0.943	5.634
Prop. of common var.	0.178	0.141	0.174	0.175	0.166	0.167	1.000

Figure D43 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D44 shows the seven underlying series. Table D22 lists the factor estimates quarter by quarter.

Figure D43: Netherlands: standardised factor estimates and standardised data

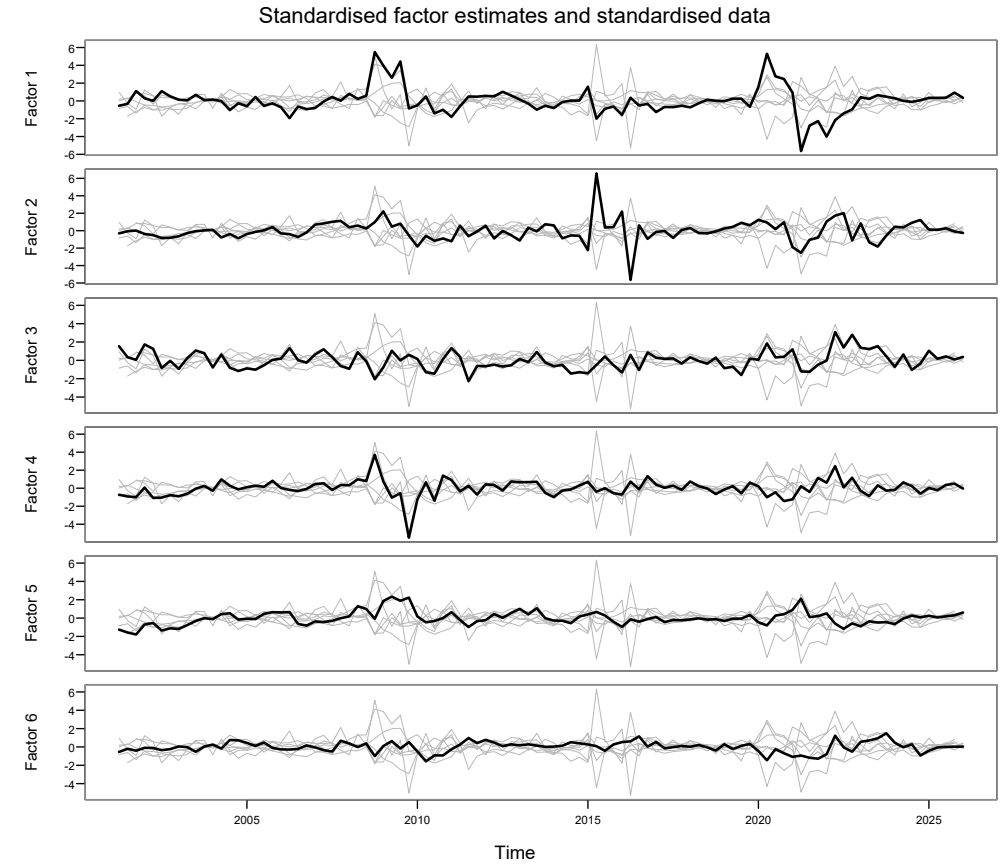


Figure D44: Netherlands: input series over the estimation window

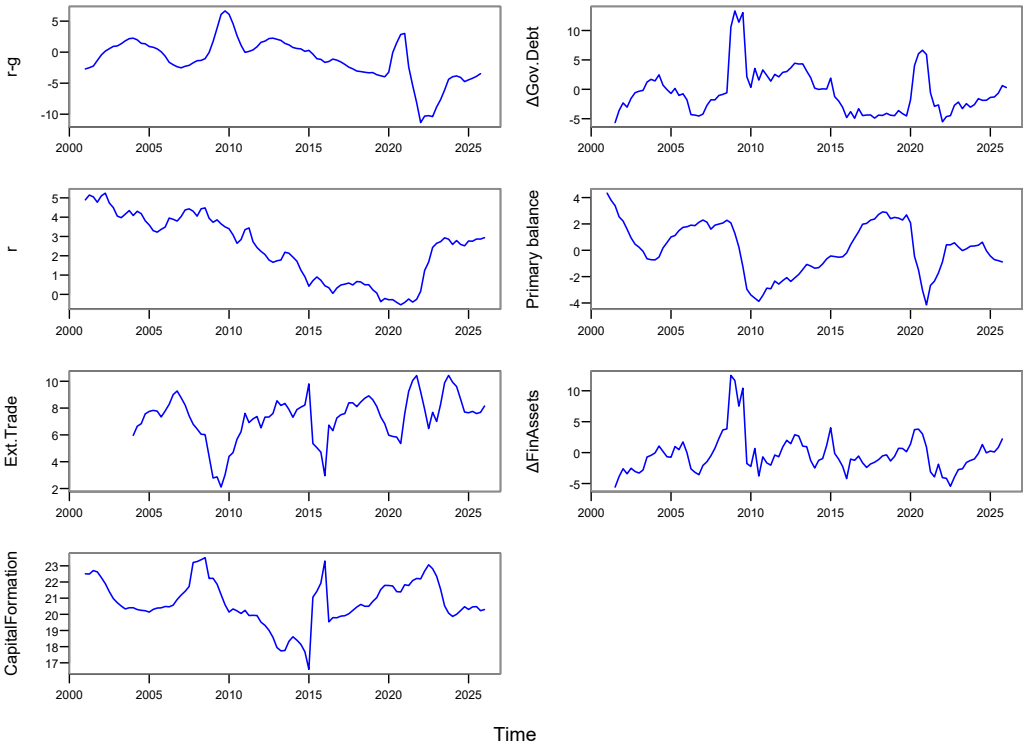


Table D22: Netherlands: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-0.545	-0.310	1.536	-0.730	-1.262	-0.544	2013Q4	-0.538	0.736	-0.234	-0.522	-0.015	0.013
2001Q3	-0.324	-0.066	0.343	-0.914	-1.574	-0.199	2014Q1	-0.805	0.608	-0.641	-0.999	-0.271	0.036
2001Q4	1.108	0.014	0.060	-1.001	-1.789	-0.408	2014Q2	-0.118	-0.876	-0.494	-0.266	-0.292	0.129
2002Q1	0.300	-0.373	1.738	0.075	-0.704	-0.095	2014Q3	0.031	-0.545	-1.439	-0.137	-0.560	0.519
2002Q2	-0.003	-0.512	1.257	-1.085	-0.530	-0.110	2014Q4	0.037	-0.589	-1.298	0.303	0.175	0.397
2002Q3	1.101	-0.844	-0.835	-1.042	-1.363	-0.350	2015Q1	1.599	-2.239	-1.409	0.709	0.389	0.272
2002Q4	0.519	-0.816	-0.051	-0.768	-1.117	-0.237	2015Q2	-1.997	6.561	-0.495	-0.390	0.672	0.084
2003Q1	0.141	-0.644	-0.925	-0.894	-1.183	0.052	2015Q3	-0.881	0.354	0.420	-0.002	0.284	-0.423
2003Q2	0.066	-0.279	0.235	-0.620	-0.768	-0.019	2015Q4	-0.632	0.413	-0.528	-0.555	-0.400	0.273
2003Q3	0.689	-0.057	1.074	-0.048	-0.314	-0.531	2016Q1	-1.573	2.196	-1.327	-0.698	-0.965	0.530
2003Q4	0.088	0.032	0.787	0.247	-0.016	0.038	2016Q2	0.362	-5.648	0.592	0.736	-0.134	0.611
2004Q1	0.157	0.095	-0.776	-0.282	-0.113	0.262	2016Q3	-0.516	0.610	-1.061	-0.127	-0.392	1.154
2004Q2	-0.011	-0.780	0.631	0.978	0.429	-0.177	2016Q4	-0.329	-0.937	0.852	1.339	-0.115	0.036
2004Q3	-1.033	-0.383	-0.813	0.280	0.532	0.760	2017Q1	-1.253	-0.140	0.293	0.401	0.116	0.551
2004Q4	-0.258	-0.872	-1.159	-0.136	-0.170	0.715	2017Q2	-0.676	-0.037	0.168	0.044	-0.440	-0.158
2005Q1	-0.601	-0.388	-0.877	0.118	-0.060	0.439	2017Q3	-0.698	-0.835	0.212	0.286	-0.202	-0.020
2005Q2	0.439	-0.141	-1.021	0.275	-0.107	0.113	2017Q4	-0.540	0.078	-0.352	-0.189	-0.245	0.106
2005Q3	-0.570	0.040	-0.537	0.150	0.466	0.489	2018Q1	-0.720	0.308	0.339	0.746	-0.144	0.016
2005Q4	-0.267	0.413	0.049	0.828	0.644	-0.109	2018Q2	-0.267	-0.279	-0.084	0.218	-0.030	0.215
2006Q1	-0.711	-0.310	0.186	0.018	0.619	-0.266	2018Q3	0.128	-0.307	-0.361	-0.033	-0.168	-0.048
2006Q2	-1.935	-0.398	1.338	-0.193	0.656	-0.286	2018Q4	0.018	-0.086	0.306	-0.652	-0.119	-0.398
2006Q3	-0.628	-0.724	0.022	-0.317	-0.638	-0.215	2019Q1	-0.027	0.254	-0.856	-0.150	-0.317	0.298
2006Q4	-0.920	-0.205	-0.246	-0.079	-0.821	0.166	2019Q2	0.256	0.440	-0.703	0.230	-0.063	-0.233
2007Q1	-0.795	0.692	0.636	0.463	-0.363	-0.044	2019Q3	0.253	0.909	-1.606	-0.577	-0.055	0.125
2007Q2	-0.069	0.830	1.204	0.543	-0.431	-0.349	2019Q4	-0.669	0.617	0.186	0.616	0.338	0.325
2007Q3	0.447	1.019	0.352	-0.202	-0.304	-0.504	2020Q1	1.577	1.241	0.064	0.221	-0.457	-0.443
2007Q4	0.023	1.125	-0.618	0.349	-0.016	0.679	2020Q2	5.300	0.940	1.848	-1.007	-0.799	-1.453
2008Q1	0.803	0.393	-0.922	0.334	0.179	0.385	2020Q3	2.757	0.175	0.324	-0.457	0.270	-0.245
2008Q2	0.230	0.586	0.890	0.988	1.294	-0.020	2020Q4	2.462	0.961	0.388	-1.434	0.428	-0.688
2008Q3	0.581	0.250	-0.104	0.791	0.993	0.377	2021Q1	0.945	-1.886	1.201	-1.236	0.889	-1.079
2008Q4	5.489	0.978	-2.069	3.709	-0.081	-1.007	2021Q2	-5.632	-2.537	-1.204	0.219	2.116	-0.940
2009Q1	3.975	2.216	-0.768	0.751	1.862	0.096	2021Q3	-2.771	-1.046	-1.254	-0.419	0.107	-1.180
2009Q2	2.598	0.466	1.046	-1.051	2.342	0.643	2021Q4	-2.263	-0.800	-0.474	1.141	0.217	-1.283
2009Q3	4.432	0.808	-0.002	-0.554	1.881	-0.174	2022Q1	-4.014	1.053	-0.031	0.611	0.511	-0.793
2009Q4	-0.847	-0.544	0.605	-5.491	2.237	0.521	2022Q2	-2.169	1.720	3.089	2.448	-0.603	1.232
2010Q1	-0.487	-1.821	0.150	-1.238	0.173	-0.492	2022Q3	-1.403	2.015	1.398	0.087	-1.174	-0.046
2010Q2	0.504	-0.585	-1.286	0.656	-0.464	-1.575	2022Q4	-0.951	-1.120	2.797	1.170	-0.575	-0.515
2010Q3	-1.401	-1.169	-1.463	-1.393	-0.319	-0.919	2023Q1	0.419	0.829	1.366	-0.304	-0.901	0.559
2010Q4	-0.986	-0.891	0.133	1.401	0.003	-0.926	2023Q2	0.226	-1.334	1.231	-0.881	-0.353	0.711
2011Q1	-1.797	-1.223	1.343	0.899	0.641	-0.253	2023Q3	0.646	-1.825	1.553	0.316	-0.487	0.944
2011Q2	-0.592	0.583	0.390	-0.349	-0.205	0.264	2023Q4	0.478	-0.567	0.448	-0.274	-0.443	1.489
2011Q3	0.501	-0.636	-2.262	0.263	-0.980	0.969	2024Q1	0.297	0.454	-0.708	-0.180	-0.656	0.467
2011Q4	0.479	-0.091	-0.602	-0.714	-0.333	0.420	2024Q2	0.002	0.376	0.630	0.642	-0.055	-0.042
2012Q1	0.565	0.567	-0.646	0.425	-0.231	0.776	2024Q3	-0.105	0.906	-1.044	0.231	0.263	0.290
2012Q2	0.515	-0.896	-0.469	0.356	0.445	0.480	2024Q4	0.071	1.227	-0.351	-0.605	0.086	-0.931
2012Q3	1.032	-0.031	-0.676	-0.253	0.023	0.085	2025Q1	0.351	0.115	1.047	0.027	0.256	-0.411
2012Q4	0.612	-0.520	-0.537	0.752	0.564	0.284	2025Q2	0.345	0.092	0.172	-0.223	0.066	-0.064
2013Q1	0.165	-1.131	0.154	0.667	1.013	0.174	2025Q3	0.360	0.267	0.429	0.357	0.193	0.016
2013Q2	-0.297	0.330	-0.198	0.677	0.411	0.283	2025Q4	0.933	-0.094	0.091	0.511	0.299	0.020
2013Q3	-0.985	-0.082	0.916	0.715	1.076	0.148	2026Q1	0.352	-0.262	0.373	-0.031	0.607	0.038

D.12 Portugal

Portugal is estimated over 2001Q3–2026Q1, that is 99 quarterly observations once the transformations of Appendix C are applied, which costs 2 quarters. Figure D45 shows the loading of each series on each factor and Figure D46 the same loadings as a heat map; Table D23 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.595); Factor 2, Ext. Trade (-0.623); Factor 3, y^{10} (0.705); Factor 4, Ext. Trade (0.656); Factor 5, Primary balance (0.619); Factor 6, $r - g$ (0.713). The sum of squared loadings is largest for y^{10} (0.985) and smallest for ΔGov.Debt (0.375); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D45: Portugal: factor loadings by series

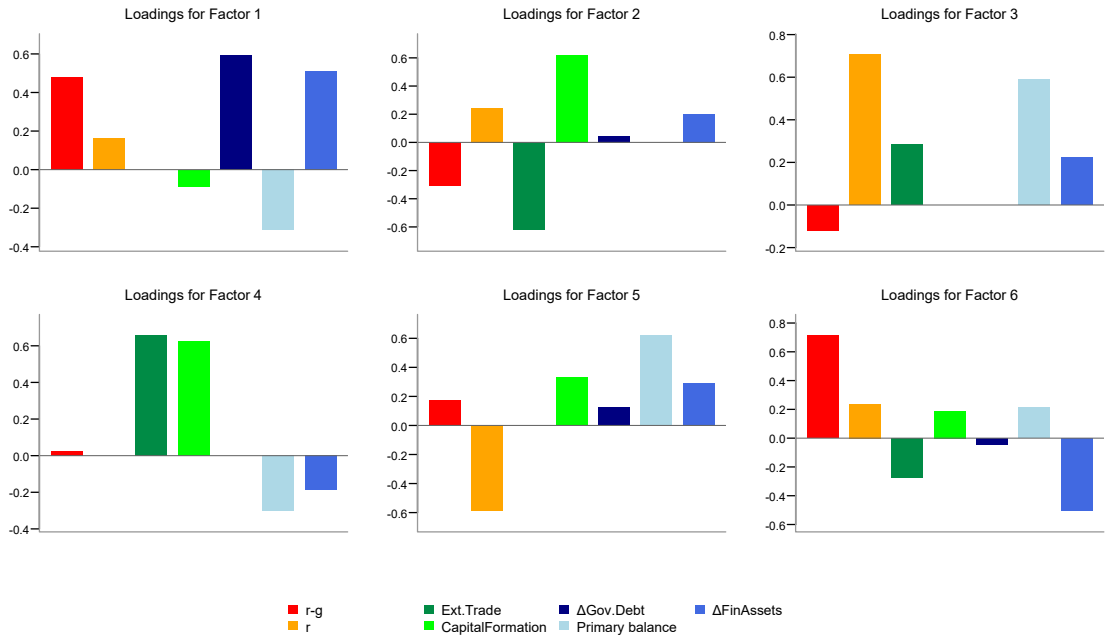


Figure D46: Portugal: loading heat map

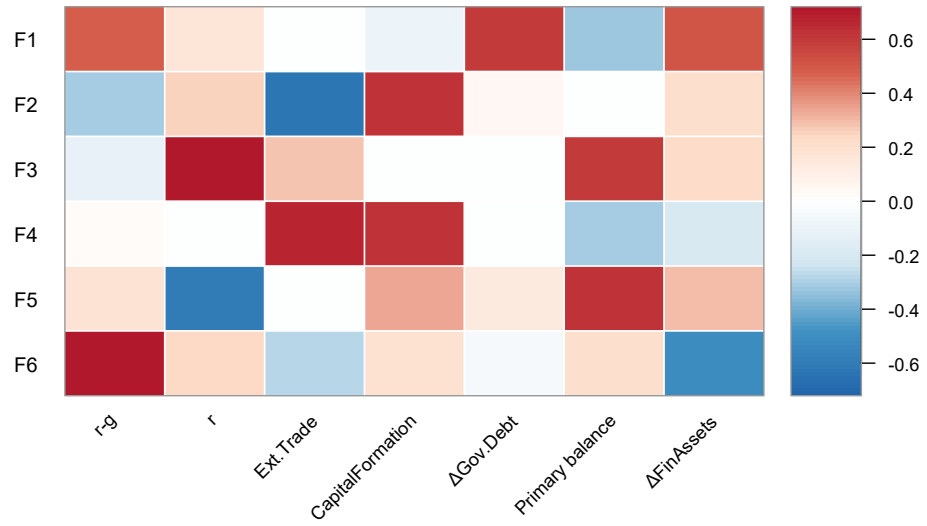


Table D23: Portugal: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.480	-0.305	-0.118	0.025	0.173	0.713	0.877
y^{10}	0.161	0.245	0.705	0.000	-0.588	0.237	0.985
Ext. Trade	0.000	-0.623	0.283	0.656	0.000	-0.273	0.974
CapitalFormation	-0.087	0.622	0.000	0.624	0.336	0.188	0.931
ΔGov.Debt	0.595	0.048	0.000	0.000	0.130	-0.043	0.375
Primary balance	-0.314	0.000	0.587	-0.301	0.619	0.212	0.963
ΔFinAssets	0.510	0.204	0.221	-0.189	0.292	-0.505	0.727
$\sum \lambda^2$	0.977	0.972	0.985	0.947	0.975	0.976	5.832
Prop. of common var.	0.168	0.167	0.169	0.162	0.167	0.167	1.000

Figure D47 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D48 shows the seven underlying series. Table D24 lists the factor estimates quarter by quarter.

Figure D47: Portugal: standardised factor estimates and standardised data

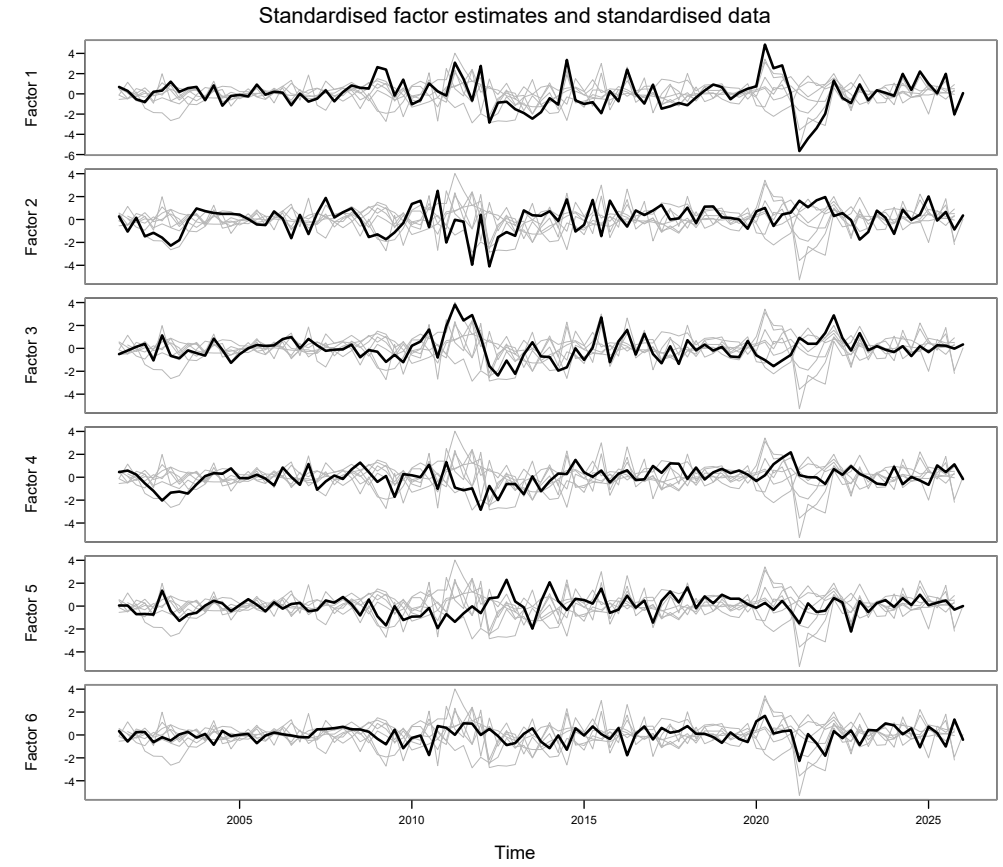


Figure D48: Portugal: input series over the estimation window

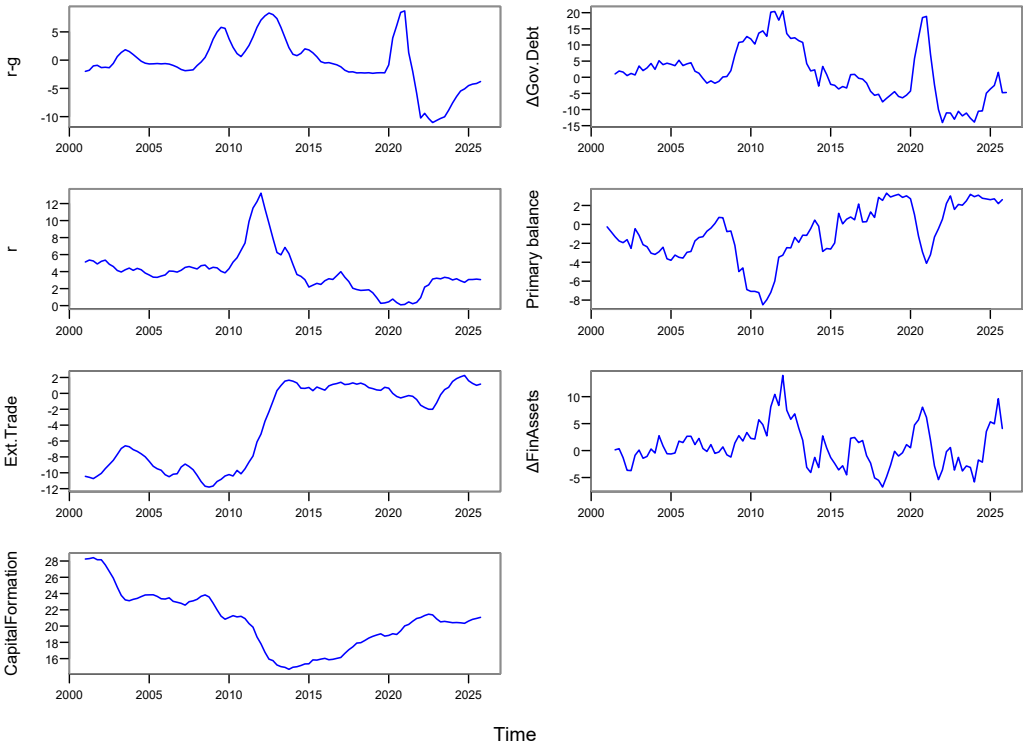


Table D24: Portugal: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q3	0.683	0.270	-0.504	0.451	0.051	0.331	2014Q1	-0.431	0.717	-0.759	-0.333	2.079	-1.157
2001Q4	0.288	-1.055	-0.190	0.580	0.049	-0.574	2014Q2	-1.066	-0.134	-1.946	0.318	0.439	-0.035
2002Q1	-0.536	0.155	0.134	0.234	-0.707	0.252	2014Q3	3.354	1.751	-1.657	0.282	-0.347	-1.291
2002Q2	-0.808	-1.474	0.396	-0.535	-0.689	0.256	2014Q4	-0.642	-1.052	0.006	1.513	0.635	0.580
2002Q3	0.205	-1.160	-1.055	-1.198	-0.744	-0.634	2015Q1	-1.000	-0.467	-1.013	0.427	0.529	-0.043
2002Q4	0.322	-1.584	1.134	-2.035	1.354	-0.195	2015Q2	-0.824	1.706	0.067	0.031	0.217	0.746
2003Q1	1.206	-2.284	-0.651	-1.346	-0.413	-0.489	2015Q3	-1.912	-1.454	2.703	0.585	1.525	0.071
2003Q2	0.196	-1.801	-0.865	-1.255	-1.295	0.047	2015Q4	0.265	1.646	-1.214	-0.458	-0.590	-0.355
2003Q3	0.568	-0.108	-0.175	-1.430	-0.735	0.273	2016Q1	-0.736	0.318	0.611	0.303	-0.308	0.598
2003Q4	0.687	0.963	-0.416	-0.622	-0.581	-0.237	2016Q2	2.401	-0.621	1.602	0.601	0.913	-1.776
2004Q1	-0.626	0.733	-0.633	0.099	0.047	0.092	2016Q3	0.028	0.788	-0.545	-0.250	-0.141	0.096
2004Q2	0.838	0.571	0.851	0.355	0.466	-0.860	2016Q4	-0.962	0.388	1.276	-0.185	0.484	0.747
2004Q3	-1.169	0.493	-0.136	0.301	0.241	0.357	2017Q1	0.907	0.780	-0.492	0.984	-1.448	-0.391
2004Q4	-0.209	0.490	-1.266	0.773	-0.445	-0.090	2017Q2	-1.464	1.273	-1.303	0.382	0.459	0.615
2005Q1	-0.099	0.427	-0.517	-0.080	0.084	0.023	2017Q3	-1.229	-0.003	0.180	1.211	1.274	0.196
2005Q2	-0.253	0.041	0.004	-0.093	0.602	0.088	2017Q4	-0.899	0.101	-1.357	1.180	0.324	0.334
2005Q3	0.918	-0.435	0.290	0.225	0.124	-0.720	2018Q1	-1.127	1.039	0.713	-0.149	1.635	0.773
2005Q4	-0.085	-0.484	0.210	-0.124	-0.458	-0.057	2018Q2	-0.367	-0.321	-0.170	0.850	-0.178	0.108
2006Q1	0.214	0.708	0.259	-0.716	0.330	0.212	2018Q3	0.325	1.122	0.331	-0.199	0.847	0.099
2006Q2	0.118	0.078	0.809	0.848	-0.226	0.059	2018Q4	0.918	1.140	-0.201	0.427	0.247	-0.182
2006Q3	-1.133	-1.634	0.994	0.024	0.183	-0.047	2019Q1	0.668	0.186	0.130	0.729	0.993	-0.690
2006Q4	0.030	0.402	-0.008	-0.663	0.278	-0.189	2019Q2	-0.533	0.138	-0.709	0.359	0.643	0.221
2007Q1	-0.767	-1.273	0.825	1.157	-0.458	-0.209	2019Q3	0.121	0.001	-0.768	0.587	0.647	-0.350
2007Q2	-0.476	0.535	0.262	-1.092	-0.343	0.489	2019Q4	0.508	-0.810	0.652	0.227	0.162	-0.619
2007Q3	0.346	1.883	-0.217	-0.367	0.489	0.516	2020Q1	0.742	0.712	-0.610	-0.331	-0.150	1.193
2007Q4	-0.744	0.183	-0.114	0.150	0.298	0.594	2020Q2	4.870	1.011	-1.025	0.168	0.276	1.668
2008Q1	0.176	0.630	-0.082	-0.153	0.795	0.714	2020Q3	2.532	-0.567	-1.555	1.135	-0.324	0.124
2008Q2	0.832	0.974	0.326	0.682	0.163	0.493	2020Q4	2.814	0.437	-1.055	1.694	0.455	0.310
2008Q3	0.602	0.035	-0.781	1.277	-0.818	0.479	2021Q1	0.042	0.599	-0.572	2.177	-0.447	0.401
2008Q4	0.521	-1.535	-0.150	0.451	0.572	0.280	2021Q2	-5.655	1.639	0.922	0.176	-1.507	-2.272
2009Q1	2.641	-1.297	-0.294	-0.409	-0.884	-0.333	2021Q3	-4.419	1.068	0.399	0.005	0.237	0.077
2009Q2	2.404	-1.729	-1.181	0.109	-1.691	-0.817	2021Q4	-3.393	1.674	0.404	-0.015	-0.520	-0.731
2009Q3	-0.176	-1.128	-0.563	-1.715	-0.012	0.461	2022Q1	-1.972	1.959	1.341	-0.586	-0.402	-1.817
2009Q4	1.406	-0.317	-1.213	0.271	-1.221	-1.163	2022Q2	1.316	0.302	2.881	0.706	0.697	0.333
2010Q1	-1.037	1.356	0.228	0.168	-0.922	-0.261	2022Q3	-0.409	0.558	0.906	0.214	0.324	-0.283
2010Q2	-0.619	1.636	0.596	0.001	-0.903	-0.044	2022Q4	-0.919	-0.054	-0.192	0.962	-2.226	0.394
2010Q3	1.032	-0.665	1.648	1.091	-0.160	-1.757	2023Q1	0.937	-1.758	1.329	0.296	0.427	-0.878
2010Q4	0.249	2.504	-0.805	-1.016	-1.933	0.779	2023Q2	-0.633	-1.091	-0.173	-0.057	-0.525	0.437
2011Q1	-0.176	-2.010	1.940	1.318	-0.704	0.627	2023Q3	0.352	0.775	0.192	-0.558	0.262	0.387
2011Q2	3.084	-0.025	3.818	-0.910	-1.374	0.013	2023Q4	0.102	0.158	-0.122	-0.657	0.517	1.011
2011Q3	1.478	-0.188	2.435	-1.136	-0.639	1.013	2024Q1	-0.192	-1.257	-0.323	0.909	-0.070	0.839
2011Q4	-0.669	-3.950	2.908	-0.967	-0.023	0.992	2024Q2	1.976	0.853	0.187	-0.613	0.701	0.059
2012Q1	2.764	0.412	0.947	-2.844	-0.617	-0.002	2024Q3	0.390	-0.031	-0.675	0.025	0.095	0.575
2012Q2	-2.840	-4.099	-1.548	-0.764	0.680	0.526	2024Q4	2.210	0.425	0.191	-0.289	0.996	-1.083
2012Q3	-0.870	-1.551	-2.375	-1.996	0.763	-0.092	2025Q1	1.003	2.013	-0.331	-0.664	0.069	0.728
2012Q4	-0.776	-1.097	-1.066	-0.591	2.297	-0.885	2025Q2	0.047	-0.090	0.268	1.036	0.297	0.176
2013Q1	-1.515	-1.429	-2.226	-0.596	0.402	-0.709	2025Q3	1.979	0.663	0.234	0.448	0.492	-1.008
2013Q2	-1.892	0.796	-0.534	-1.484	-0.108	0.099	2025Q4	-2.032	-0.873	-0.025	1.125	-0.309	1.348
2013Q3	-2.453	0.369	0.535	0.081	-1.968	0.580	2026Q1	0.072	0.352	0.342	-0.161	0.004	-0.417
2013Q4	-1.811	0.322	-0.705	-1.226	0.418	-0.585							

D.13 Slovakia

Slovakia is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D49 shows the loading of each series on each factor and Figure D50 the same loadings as a heat map; Table D25 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, CapitalFormation (-0.540); Factor 2, y^{10} (0.663); Factor 3, Primary balance (0.737); Factor 4, Ext. Trade (-0.634); Factor 5, y^{10} (-0.600); Factor 6, CapitalFormation (0.470). The sum of squared loadings is largest for ΔFinAssets (0.983) and smallest for ΔGov.Debt (0.197); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D49: Slovakia: factor loadings by series

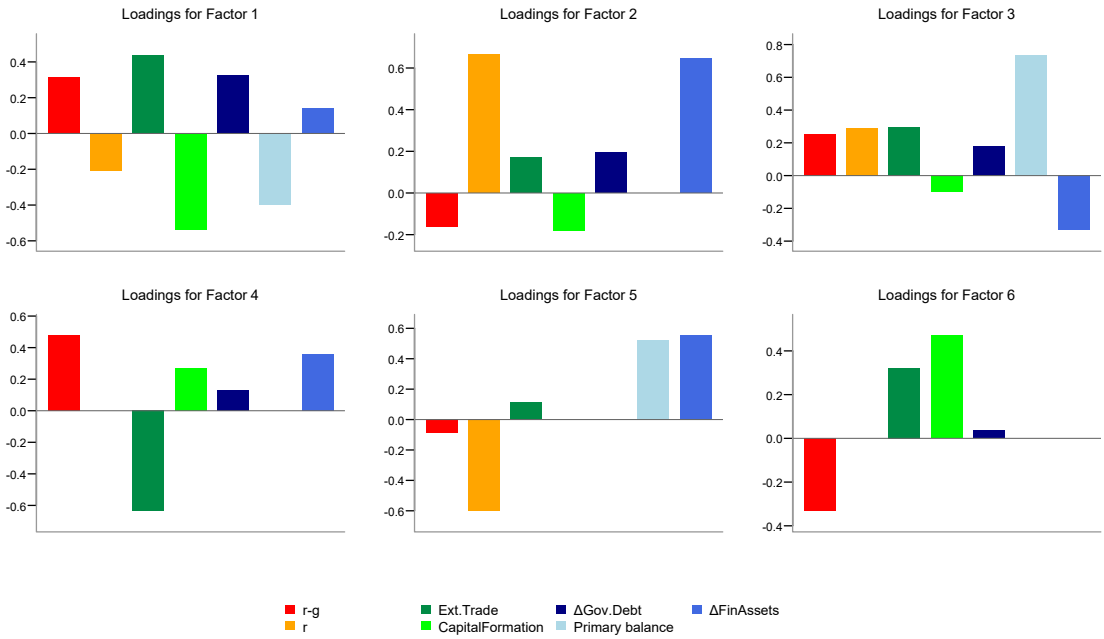


Figure D50: Slovakia: loading heat map

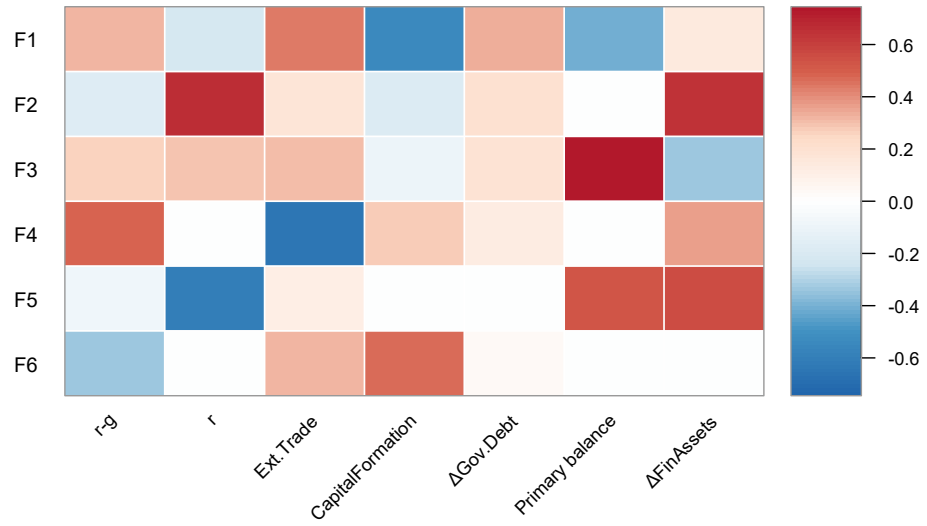


Table D25: Slovakia: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.318	-0.163	0.256	0.477	-0.088	-0.332	0.538
y^{10}	-0.211	0.663	0.292	0.000	-0.600	0.000	0.930
Ext. Trade	0.440	0.170	0.299	-0.634	0.115	0.320	0.830
CapitalFormation	-0.540	-0.179	-0.101	0.270	0.000	0.470	0.629
ΔGov.Debt	0.328	0.196	0.178	0.132	0.000	0.036	0.197
Primary balance	-0.400	0.000	0.737	0.000	0.520	0.000	0.974
ΔFinAssets	0.143	0.646	-0.333	0.357	0.553	0.000	0.983
$\sum \lambda^2$	0.919	0.984	0.937	0.848	0.958	0.435	5.080
Prop. of common var.	0.181	0.194	0.184	0.167	0.188	0.086	1.000

Figure D51 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D52 shows the seven underlying series. Table D26 lists the factor estimates quarter by quarter.

Figure D51: Slovakia: standardised factor estimates and standardised data

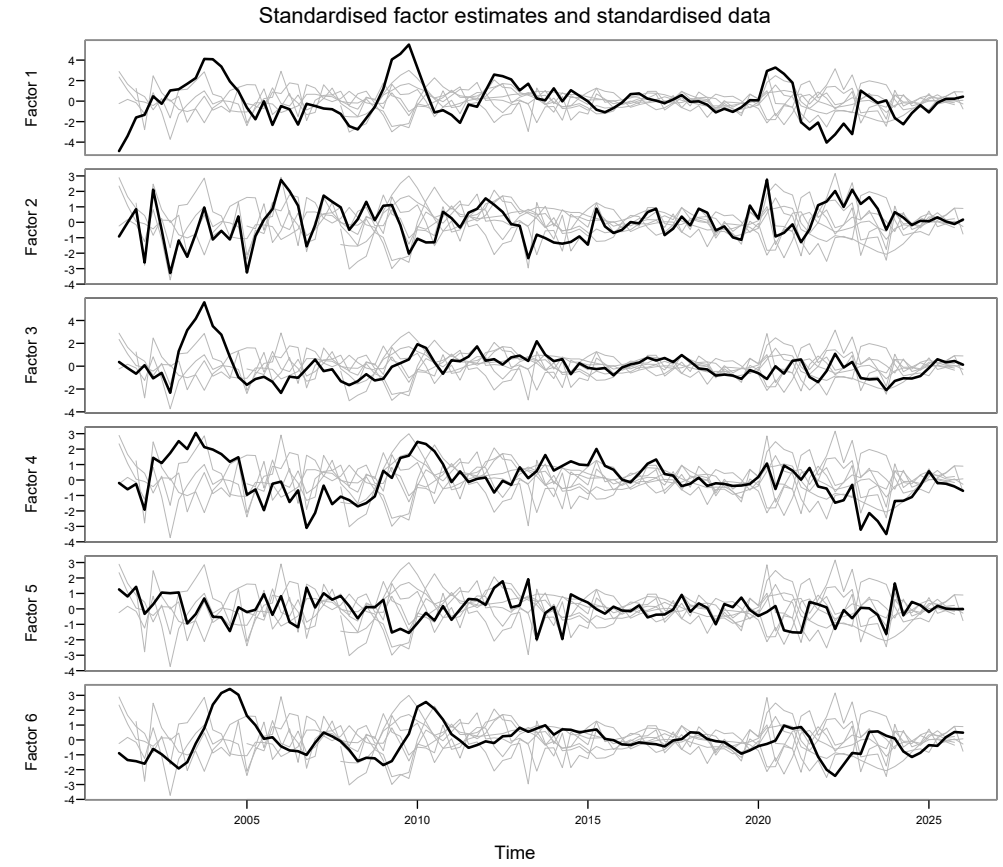


Figure D52: Slovakia: input series over the estimation window

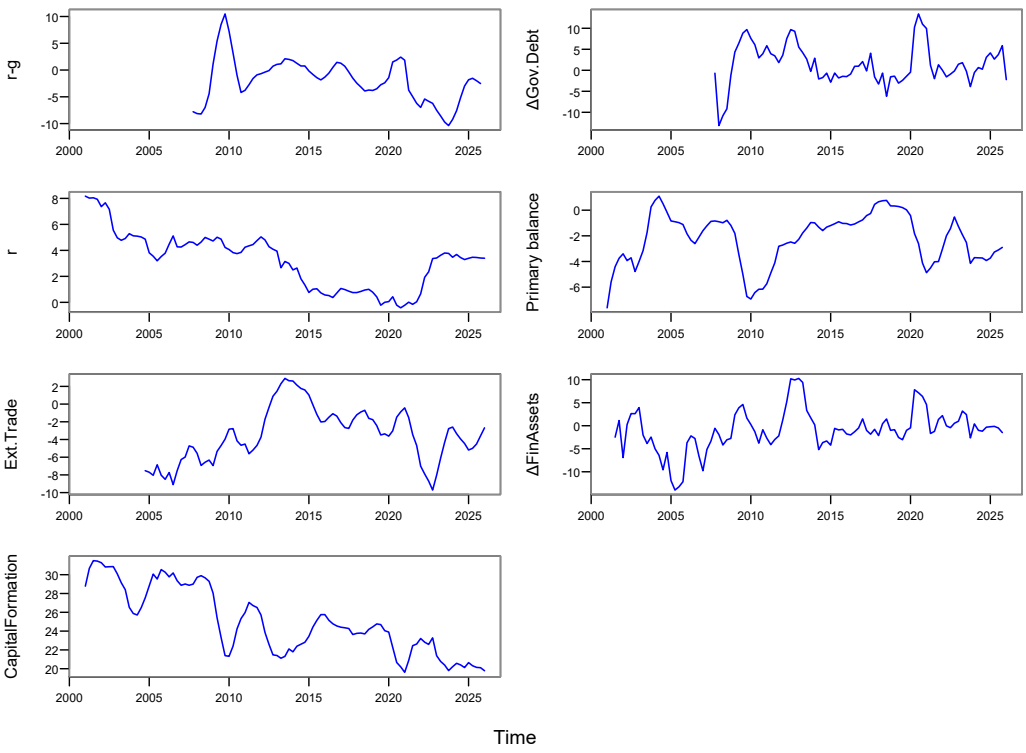


Table D26: Slovakia: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	-4.871	-0.918	0.368	-0.192	1.261	-0.890	2013Q4	0.077	-1.026	0.999	1.618	-0.263	0.984
2001Q3	-3.405	-0.020	-0.165	-0.602	0.803	-1.350	2014Q1	1.247	-1.308	0.436	0.612	0.124	0.355
2001Q4	-1.595	0.849	-0.660	-0.257	1.429	-1.430	2014Q2	-0.023	-1.384	0.620	0.915	-1.958	0.712
2002Q1	-1.336	-2.604	0.085	-1.926	-0.323	-1.600	2014Q3	1.065	-1.271	-0.703	1.211	0.942	0.682
2002Q2	0.475	2.110	-1.066	1.440	0.293	-0.600	2014Q4	0.519	-0.914	0.254	1.000	0.651	0.503
2002Q3	-0.262	-0.494	-0.585	1.097	1.057	-0.965	2015Q1	-0.023	-1.451	-0.146	0.957	0.410	0.614
2002Q4	1.038	-3.288	-2.330	1.744	1.016	-1.449	2015Q2	-0.837	0.883	-0.254	2.014	-0.017	0.707
2003Q1	1.158	-1.177	1.312	2.518	1.064	-1.920	2015Q3	-1.089	-0.268	-0.162	0.915	-0.309	0.070
2003Q2	1.687	-2.235	3.151	2.002	-0.937	-1.498	2015Q4	-0.680	-0.696	-0.804	0.667	0.144	-0.018
2003Q3	2.249	-0.742	4.160	3.049	-0.335	-0.236	2016Q1	-0.145	-0.493	-0.119	0.025	-0.107	-0.297
2003Q4	4.113	0.957	5.589	2.129	0.676	0.789	2016Q2	0.679	0.016	0.157	-0.144	-0.143	-0.341
2004Q1	4.082	-1.116	3.505	1.970	-0.507	2.372	2016Q3	0.748	-0.074	0.309	0.421	0.228	-0.167
2004Q2	3.371	-0.557	2.758	1.679	-0.541	3.154	2016Q4	0.238	0.656	0.749	1.056	-0.551	-0.241
2004Q3	1.924	-1.112	0.812	1.181	-1.440	3.431	2017Q1	0.044	0.877	0.495	1.330	-0.376	-0.290
2004Q4	0.970	0.383	-0.958	1.475	0.107	3.040	2017Q2	-0.198	-0.832	0.714	0.387	-0.363	-0.437
2005Q1	-0.636	-3.251	-1.630	-0.955	-0.208	1.626	2017Q3	0.158	-0.416	0.358	0.290	-0.043	-0.035
2005Q2	-1.771	-0.897	-1.125	-0.613	-0.058	0.993	2017Q4	0.588	0.371	0.973	-0.398	0.904	0.055
2005Q3	-0.018	0.166	-0.927	-1.954	0.951	0.073	2018Q1	-0.065	-0.202	0.420	-0.242	-0.191	0.514
2005Q4	-2.322	0.856	-1.371	-0.242	-0.388	0.185	2018Q2	-0.022	0.882	-0.198	0.151	0.355	0.479
2006Q1	-0.484	2.739	-2.343	-0.105	0.825	-0.429	2018Q3	-0.354	0.622	-0.288	-0.383	0.065	0.064
2006Q2	-0.792	2.020	-0.916	-1.425	-0.857	-0.705	2018Q4	-1.101	-0.528	-0.823	-0.211	-0.999	-0.082
2006Q3	-2.291	1.074	-1.017	-0.672	-1.192	-0.761	2019Q1	-0.746	-0.266	-0.735	-0.254	0.313	-0.152
2006Q4	-0.261	-1.559	-0.228	-3.100	1.379	-1.010	2019Q2	-1.053	-0.978	-0.823	-0.381	0.108	-0.520
2007Q1	-0.474	-0.198	0.581	-2.129	0.089	-0.188	2019Q3	-0.603	-1.138	-1.114	-0.357	0.734	-0.927
2007Q2	-0.721	1.728	-0.434	-0.362	1.012	0.496	2019Q4	0.093	1.083	-0.338	-0.257	-0.068	-0.705
2007Q3	-0.794	1.332	-0.273	-1.564	0.595	0.241	2020Q1	0.071	0.232	-0.613	0.204	-0.458	-0.416
2007Q4	-1.248	0.969	-1.306	-1.074	0.837	-0.092	2020Q2	2.937	2.753	-1.125	1.069	-0.199	-0.268
2008Q1	-2.441	-0.487	-1.661	-1.303	0.182	-0.665	2020Q3	3.274	-0.920	-0.007	-0.585	0.187	-0.045
2008Q2	-2.758	0.194	-1.313	-1.706	-0.617	-1.426	2020Q4	2.661	-0.681	-0.652	0.938	-1.389	0.964
2008Q3	-1.663	1.326	-0.703	-1.492	0.120	-1.201	2021Q1	1.794	-0.133	0.465	0.607	-1.515	0.765
2008Q4	-0.566	0.136	-1.250	-1.061	0.102	-1.237	2021Q2	-2.051	-1.289	0.583	0.017	-1.539	0.870
2009Q1	1.197	1.070	-1.104	0.594	0.585	-1.693	2021Q3	-2.764	-0.457	-0.951	0.782	0.451	0.183
2009Q2	4.057	1.118	-0.054	0.133	-1.530	-1.423	2021Q4	-2.088	1.104	-1.397	-0.421	0.290	-1.126
2009Q3	4.606	-0.188	0.264	1.429	-1.291	-0.481	2022Q1	-4.044	1.344	-0.423	-0.553	0.096	-2.003
2009Q4	5.512	-2.031	0.591	1.575	-1.559	0.409	2022Q2	-3.250	2.026	1.085	-1.471	-1.299	-2.418
2010Q1	3.248	-1.063	1.920	2.471	-0.924	2.229	2022Q3	-2.194	0.992	-0.116	-1.309	-0.077	-1.650
2010Q2	0.979	-1.304	1.601	2.330	-0.255	2.552	2022Q4	-3.215	2.110	0.375	-0.310	-0.606	-0.875
2010Q3	-1.111	-1.295	0.354	1.864	-0.753	2.087	2023Q1	1.023	1.186	-1.036	-3.219	0.072	-0.943
2010Q4	-0.888	0.678	-0.668	1.030	0.181	1.368	2023Q2	0.428	1.625	-1.162	-2.136	0.038	0.539
2011Q1	-1.345	0.267	0.513	-0.136	-0.710	0.383	2023Q3	-0.175	0.871	-1.116	-2.665	-0.393	0.577
2011Q2	-2.121	-0.343	0.432	0.561	-0.034	-0.040	2023Q4	0.065	-0.479	-2.105	-3.499	-1.626	0.282
2011Q3	-0.341	0.624	0.843	-0.130	0.641	-0.533	2024Q1	-1.647	0.657	-1.295	-1.360	1.651	0.100
2011Q4	-0.552	0.869	1.725	0.075	0.601	-0.343	2024Q2	-2.258	0.308	-1.060	-1.343	-0.414	-0.771
2012Q1	1.016	1.548	0.477	0.160	0.258	-0.093	2024Q3	-1.174	-0.182	-1.080	-1.101	0.451	-1.151
2012Q2	2.587	1.136	0.614	-0.812	1.366	-0.210	2024Q4	-0.394	0.097	-0.865	-0.356	0.236	-0.865
2012Q3	2.433	0.667	0.140	-0.050	1.789	0.246	2025Q1	-1.099	0.064	-0.163	0.569	-0.204	-0.351
2012Q4	2.116	-0.120	0.771	-0.317	0.094	0.284	2025Q2	-0.159	0.321	0.605	-0.190	0.198	-0.397
2013Q1	1.046	-0.224	0.926	0.824	0.223	0.824	2025Q3	0.221	0.052	0.339	-0.242	0.016	0.186
2013Q2	1.699	-2.327	0.455	0.116	1.922	0.548	2025Q4	0.234	-0.112	0.452	-0.419	-0.017	0.533
2013Q3	0.250	-0.800	2.182	0.582	-1.976	0.774	2026Q1	0.435	0.175	0.131	-0.698	-0.017	0.490

D.14 Slovenia

Slovenia is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D53 shows the loading of each series on each factor and Figure D54 the same loadings as a heat map; Table D27 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, $r - g$ (0.472); Factor 2, CapitalFormation (0.548); Factor 3, Primary balance (0.817); Factor 4, y^{10} (-0.827); Factor 5, $r - g$ (-0.691); Factor 6, CapitalFormation (0.621). The sum of squared loadings is largest for y^{10} (0.994) and smallest for ΔGov.Debt (0.342); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D53: Slovenia: factor loadings by series

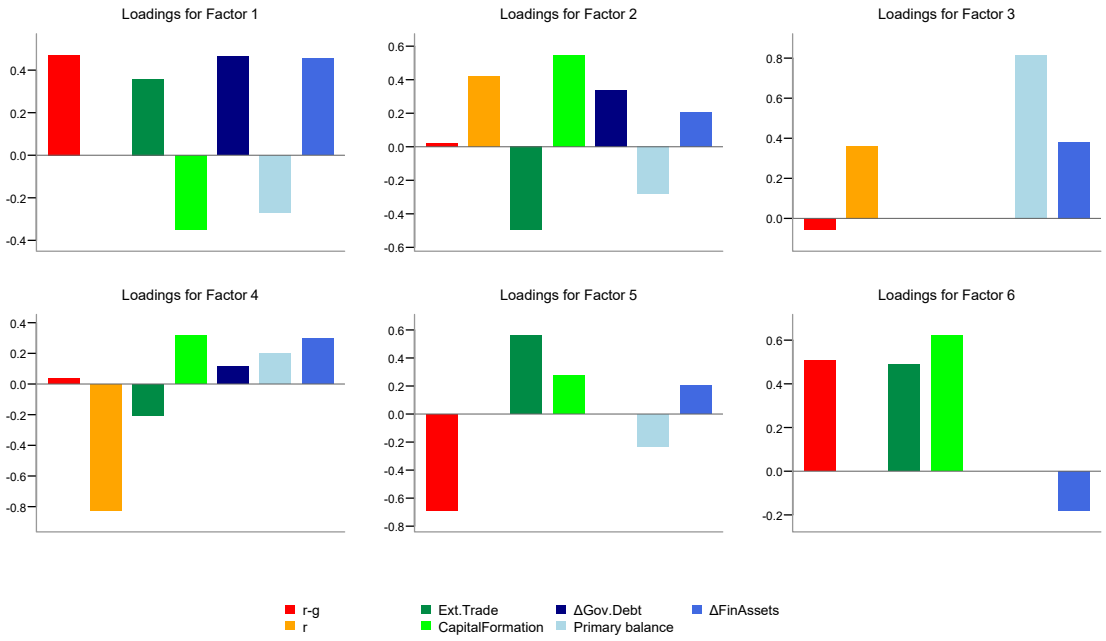


Figure D54: Slovenia: loading heat map

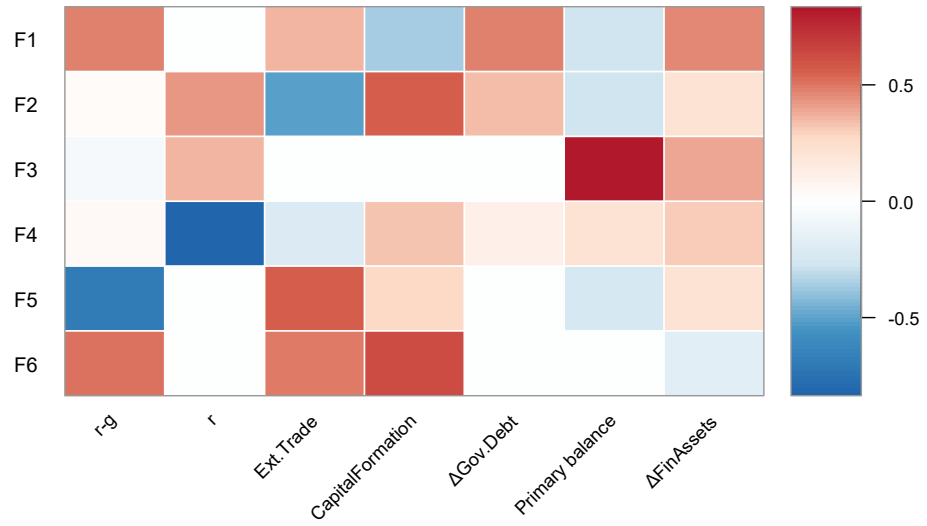


Table D27: Slovenia: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.472	0.023	-0.059	0.036	-0.691	0.507	0.962
y^{10}	0.000	0.422	0.362	-0.827	0.000	0.000	0.994
Ext. Trade	0.359	-0.498	0.000	-0.203	0.560	0.490	0.972
CapitalFormation	-0.352	0.548	0.000	0.317	0.276	0.621	0.987
ΔGov.Debt	0.465	0.335	0.000	0.114	0.000	0.000	0.342
Primary balance	-0.269	-0.281	0.817	0.199	-0.233	0.000	0.912
ΔFinAssets	0.454	0.204	0.382	0.300	0.205	-0.181	0.558
$\sum \lambda^2$	0.971	0.960	0.947	0.970	0.964	0.915	5.727
Prop. of common var.	0.169	0.168	0.165	0.169	0.168	0.160	1.000

Figure D55 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D56 shows the seven underlying series. Table D28 lists the factor estimates quarter by quarter.

Figure D55: Slovenia: standardised factor estimates and standardised data

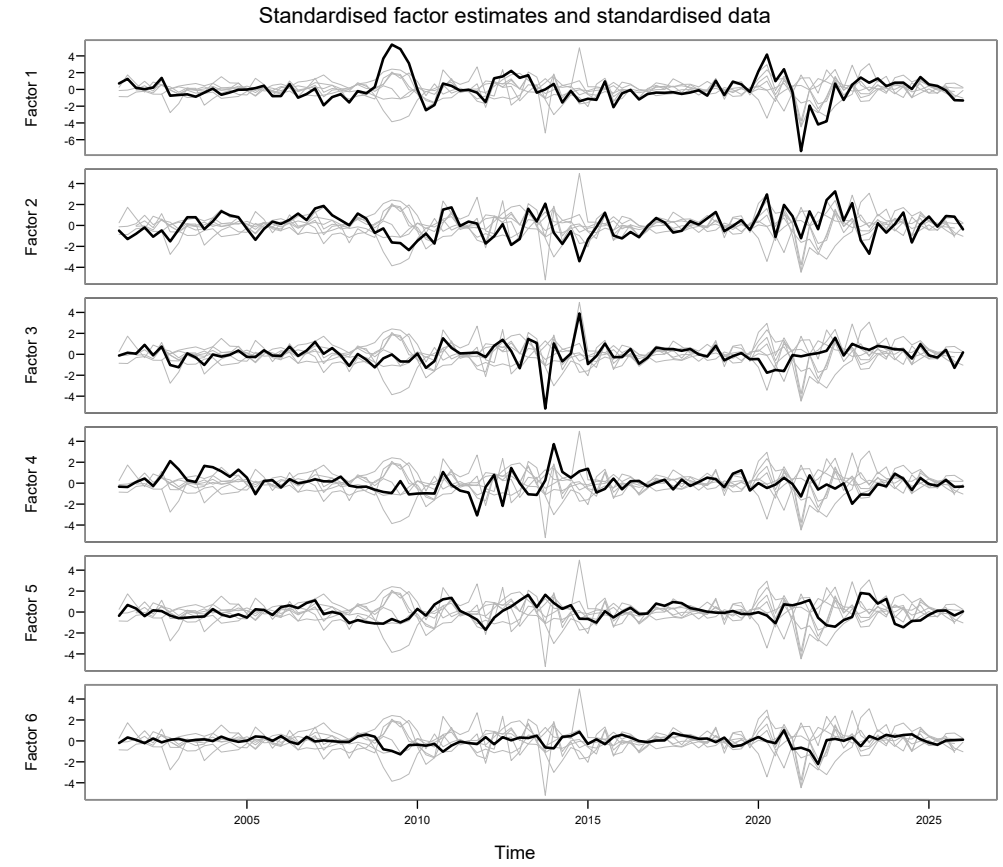


Figure D56: Slovenia: input series over the estimation window

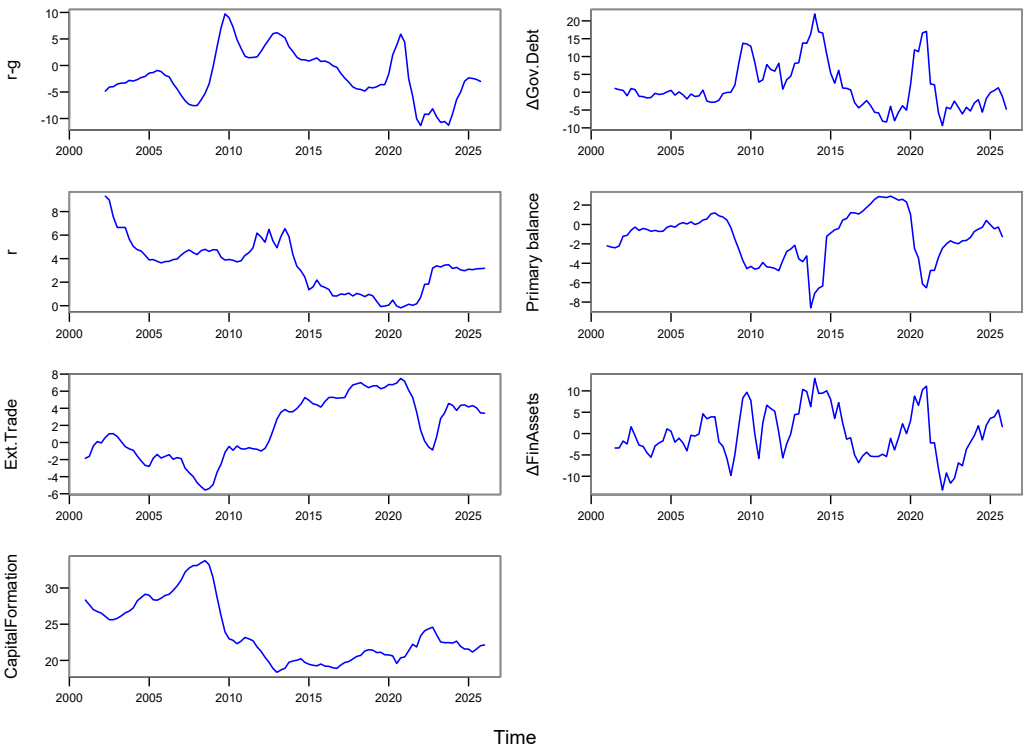


Table D28: Slovenia: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.709	-0.496	-0.122	-0.335	-0.360	-0.204	2013Q4	0.000	2.080	-5.182	0.271	1.648	-0.628
2001Q3	1.257	-1.302	0.147	-0.365	0.676	0.331	2014Q1	0.654	-0.700	1.040	3.732	0.864	-0.714
2001Q4	0.183	-0.797	0.055	0.107	0.334	0.096	2014Q2	-1.564	-1.767	-0.678	1.073	0.299	0.389
2002Q1	0.028	-0.194	0.900	0.443	-0.393	-0.213	2014Q3	-0.186	-0.549	0.074	0.519	0.655	0.457
2002Q2	0.222	-1.077	-0.094	-0.263	0.163	0.230	2014Q4	-1.421	-3.421	3.898	1.135	-0.645	0.863
2002Q3	1.372	-0.476	0.743	0.742	0.088	-0.141	2015Q1	-1.119	-1.348	-0.937	1.376	-0.657	-0.286
2002Q4	-0.753	-1.525	-1.025	2.114	-0.337	0.113	2015Q2	-1.248	-0.152	-0.163	-0.895	-1.014	0.151
2003Q1	-0.667	-0.383	-1.241	1.311	-0.590	0.185	2015Q3	0.953	1.205	1.025	-0.543	0.074	-0.315
2003Q2	-0.586	0.782	0.083	0.277	-0.530	-0.013	2015Q4	-2.125	-0.964	-0.274	0.440	-0.519	0.372
2003Q3	-0.871	0.783	-0.305	0.108	-0.465	0.102	2016Q1	-0.478	-1.243	-0.266	-0.589	0.000	0.589
2003Q4	-0.386	-0.355	-1.019	1.651	-0.429	0.147	2016Q2	-0.070	-0.625	0.508	0.187	0.404	0.315
2004Q1	0.100	0.387	-0.026	1.525	0.277	-0.014	2016Q3	-1.202	-1.130	-0.897	0.213	-0.197	-0.013
2004Q2	-0.644	1.353	-0.222	1.130	-0.235	0.408	2016Q4	-0.555	-0.069	-0.321	-0.309	-0.140	-0.115
2004Q3	-0.365	0.961	-0.052	0.604	-0.463	0.097	2017Q1	-0.380	0.706	0.639	0.056	0.808	0.034
2004Q4	-0.042	0.791	0.358	1.292	-0.218	-0.090	2017Q2	-0.405	0.280	0.509	0.329	0.593	0.049
2005Q1	-0.021	-0.319	-0.251	0.525	-0.533	0.021	2017Q3	-0.335	-0.684	0.498	-0.600	0.960	0.752
2005Q2	0.153	-1.379	-0.255	-1.054	0.262	0.424	2017Q4	-0.539	-0.512	0.337	0.341	0.850	0.540
2005Q3	0.444	-0.339	0.384	0.213	0.195	0.368	2018Q1	-0.372	0.407	0.507	-0.268	0.371	0.400
2005Q4	-0.806	0.365	-0.121	0.286	-0.286	0.000	2018Q2	-0.101	0.080	-0.006	0.117	0.223	0.202
2006Q1	-0.795	0.144	-0.156	-0.415	0.474	0.457	2018Q3	-0.747	0.676	-0.213	0.526	0.030	0.225
2006Q2	0.645	0.542	0.732	0.381	0.633	-0.079	2018Q4	1.037	1.283	0.823	0.382	-0.059	-0.121
2006Q3	-0.990	1.132	-0.152	-0.007	0.378	-0.306	2019Q1	-0.686	-0.547	-0.580	-0.388	-0.080	0.315
2006Q4	-0.556	0.508	0.355	0.133	0.864	0.390	2019Q2	0.932	-0.058	-0.173	0.906	0.100	-0.547
2007Q1	0.104	1.622	1.183	0.367	1.112	-0.056	2019Q3	0.603	0.507	0.116	1.230	-0.184	-0.414
2007Q2	-1.899	1.864	0.050	0.178	-0.191	0.046	2019Q4	-0.296	-0.432	-0.470	-0.707	-0.195	0.002
2007Q3	-0.917	0.978	0.597	0.159	0.003	-0.027	2020Q1	2.160	1.073	-0.449	0.004	-0.028	0.372
2007Q4	-0.569	0.503	-0.100	0.625	-0.178	-0.099	2020Q2	4.159	2.963	-1.762	-0.464	-0.339	-0.031
2008Q1	-1.551	0.012	-1.116	-0.235	-1.054	-0.090	2020Q3	0.984	-1.111	-1.498	-0.135	-1.074	-0.223
2008Q2	-0.205	1.136	0.028	-0.380	-0.770	0.425	2020Q4	2.410	1.961	-1.604	0.506	0.735	1.014
2008Q3	-0.464	0.651	-0.510	-0.332	-0.978	0.612	2021Q1	-0.345	0.889	-0.076	-0.081	0.639	-0.793
2008Q4	0.282	-0.712	-1.251	-0.622	-1.087	0.399	2021Q2	-7.358	-1.226	-0.195	-1.277	0.851	-0.650
2009Q1	3.684	-0.284	-0.389	-0.841	-1.107	-0.799	2021Q3	-1.929	1.341	-0.018	0.745	1.143	-0.935
2009Q2	5.348	-1.627	-0.018	-0.947	-0.668	-0.958	2021Q4	-4.183	-0.355	0.109	-0.610	-0.543	-2.211
2009Q3	4.817	-1.687	-0.685	0.201	-0.990	-1.286	2022Q1	-3.804	2.426	0.333	-0.125	-1.265	0.074
2009Q4	3.124	-2.359	-0.692	-1.075	-0.628	-0.412	2022Q2	0.701	3.252	1.583	-0.522	-1.404	0.194
2010Q1	0.000	-1.476	0.071	-0.995	0.302	-0.358	2022Q3	-1.264	0.465	-0.118	-0.017	-0.771	0.012
2010Q2	-2.485	-0.774	-1.295	-0.964	-0.341	-0.455	2022Q4	0.551	2.133	0.996	-1.969	-0.467	0.315
2010Q3	-1.861	-1.735	-0.651	-0.993	0.716	-0.288	2023Q1	1.417	-1.430	0.651	-1.078	1.819	-0.507
2010Q4	0.699	1.509	1.529	1.068	1.212	-1.033	2023Q2	0.790	-2.707	0.423	-1.108	1.716	0.465
2011Q1	0.423	1.725	0.611	-0.165	1.354	-0.446	2023Q3	1.310	0.214	0.796	-0.093	0.803	0.148
2011Q2	-0.138	-0.048	0.082	-0.712	0.112	-0.065	2023Q4	0.390	-0.691	0.681	-0.302	1.255	0.576
2011Q3	-0.039	0.367	0.124	-0.898	-0.272	-0.205	2024Q1	0.802	0.117	0.473	0.910	-1.132	0.404
2011Q4	-0.389	0.181	0.170	-3.081	-0.740	-0.303	2024Q2	0.812	1.236	0.448	0.431	-1.460	0.561
2012Q1	-1.503	-1.734	-0.263	-0.365	-1.701	0.359	2024Q3	0.042	-1.630	-0.412	-0.655	-0.855	0.626
2012Q2	1.328	-1.036	0.813	0.787	-0.538	-0.319	2024Q4	1.479	0.094	0.965	0.492	-0.808	0.163
2012Q3	1.545	0.123	1.392	-2.175	0.110	0.356	2025Q1	0.597	0.851	-0.108	-0.090	-0.263	-0.159
2012Q4	2.213	-1.864	0.312	1.454	0.530	0.054	2025Q2	0.412	-0.122	-0.310	-0.250	0.124	-0.371
2013Q1	1.375	-1.298	-1.333	0.003	1.105	0.320	2025Q3	-0.132	0.901	0.431	0.310	0.144	0.035
2013Q2	1.700	1.589	1.461	-1.063	1.642	0.274	2025Q4	-1.284	0.837	-1.300	-0.357	-0.330	0.070
2013Q3	-0.392	0.374	1.082	-1.121	0.464	0.498	2026Q1	-1.321	-0.377	0.182	-0.320	0.058	0.113

D.15 Spain

Spain is estimated over 2001Q3–2026Q1, that is 99 quarterly observations once the transformations of Appendix C are applied, which costs 2 quarters. Figure D57 shows the loading of each series on each factor and Figure D58 the same loadings as a heat map; Table D29 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.547); Factor 2, Ext. Trade (0.690); Factor 3, y^{10} (0.939); Factor 4, ΔFinAssets (0.735); Factor 5, CapitalFormation (0.604); Factor 6, Primary balance (0.636). The sum of squared loadings is largest for CapitalFormation (0.974) and smallest for ΔGov.Debt (0.311); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D57: Spain: factor loadings by series

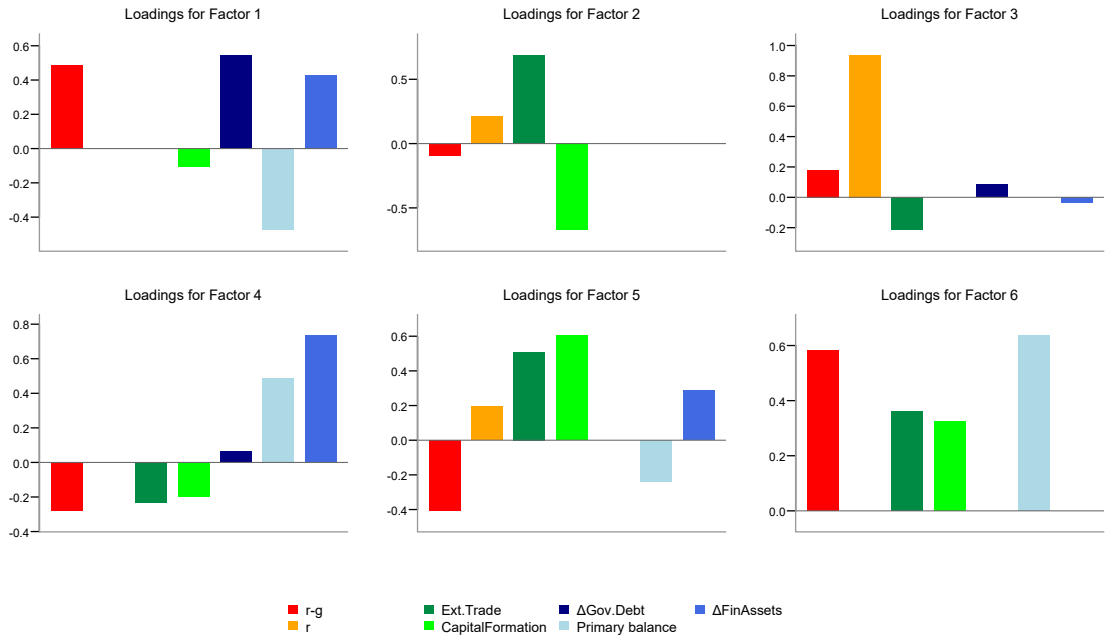


Figure D58: Spain: loading heat map

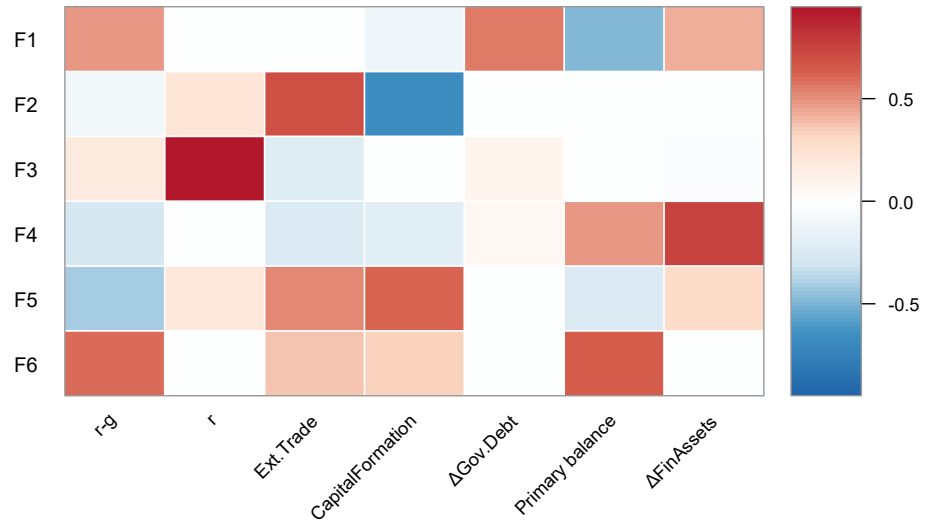


Table D29: Spain: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.487	-0.092	0.179	-0.280	-0.408	0.583	0.862
y^{10}	0.000	0.211	0.939	0.000	0.196	0.000	0.964
Ext. Trade	0.000	0.690	-0.216	-0.236	0.508	0.361	0.968
CapitalFormation	-0.106	-0.673	0.000	-0.197	0.604	0.324	0.974
ΔGov.Debt	0.547	0.000	0.085	0.066	0.000	0.000	0.311
Primary balance	-0.476	0.000	0.000	0.486	-0.240	0.636	0.926
ΔFinAssets	0.430	0.000	-0.034	0.735	0.285	0.000	0.807
$\sum \lambda^2$	0.959	0.983	0.968	0.953	0.967	0.981	5.812
Prop. of common var.	0.165	0.169	0.167	0.164	0.166	0.169	1.000

Figure D59 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D60 shows the seven underlying series. Table D30 lists the factor estimates quarter by quarter.

Figure D59: Spain: standardised factor estimates and standardised data

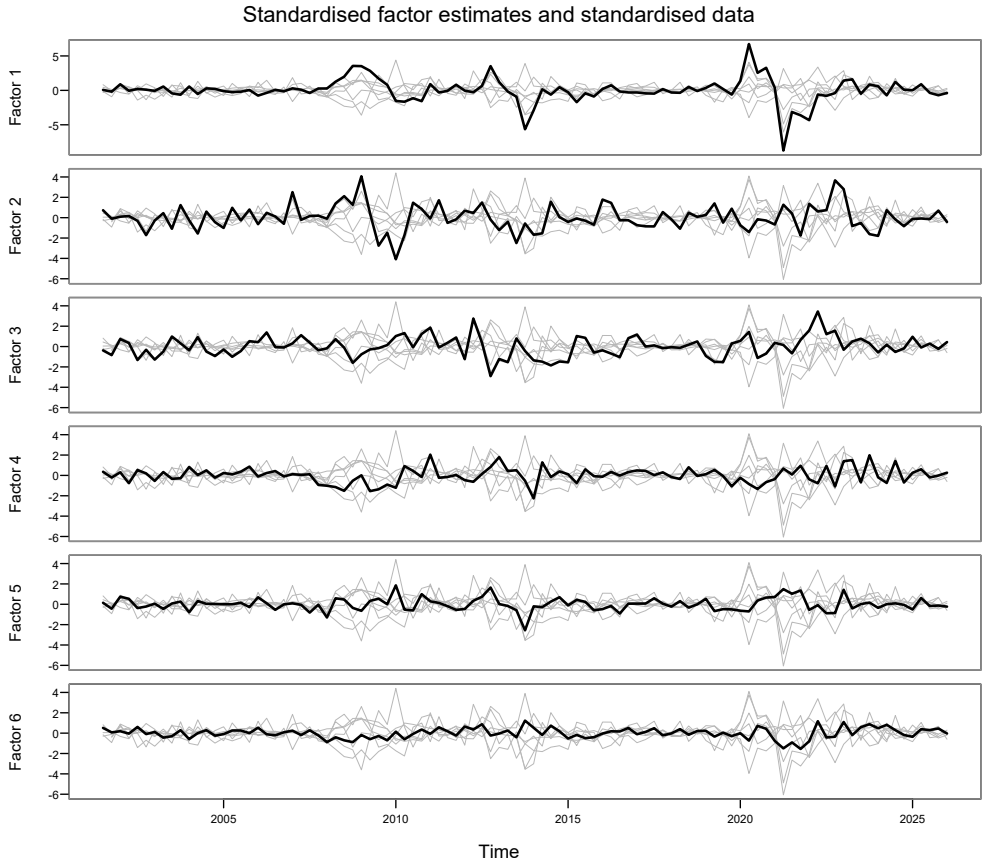


Figure D60: Spain: input series over the estimation window

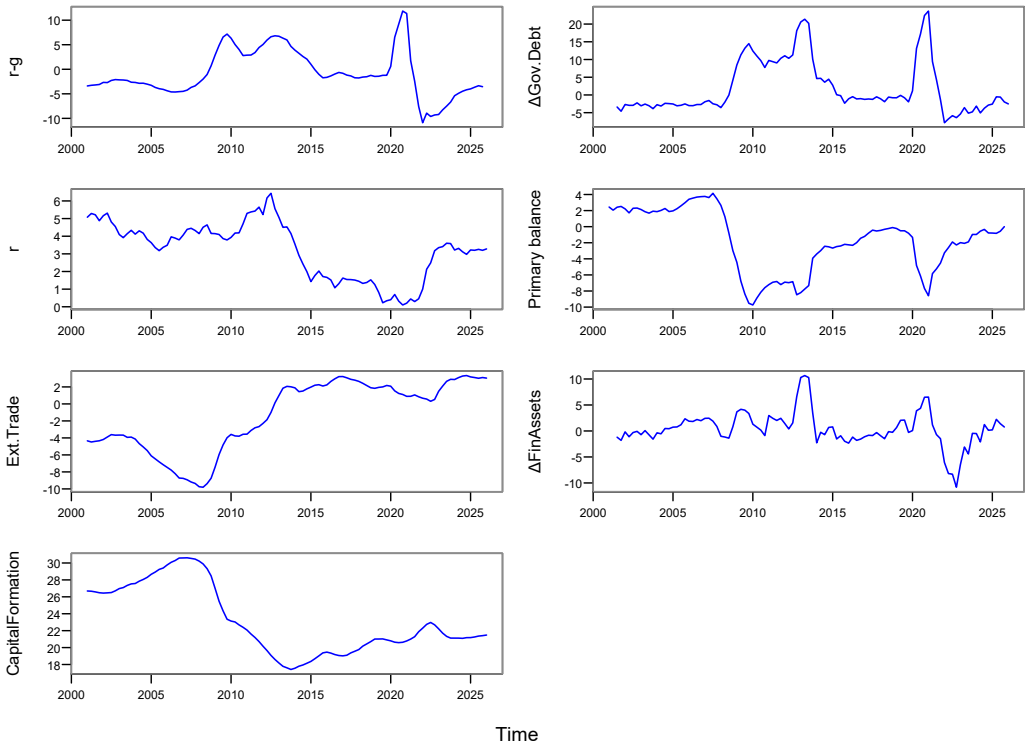


Table D30: Spain: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q3	0.072	0.726	-0.380	0.351	0.148	0.516	2014Q1	-2.900	-1.678	-1.360	-2.259	-0.204	0.526
2001Q4	-0.193	-0.106	-0.837	-0.209	-0.436	0.074	2014Q2	0.163	-1.538	-1.485	1.282	-0.280	-0.179
2002Q1	0.887	0.094	0.738	0.304	0.745	0.190	2014Q3	-0.619	1.569	-1.855	-0.162	0.262	0.738
2002Q2	-0.064	0.162	0.349	-0.766	0.539	-0.049	2014Q4	0.459	0.053	-1.468	0.400	0.695	0.179
2002Q3	0.208	-0.306	-1.320	0.527	-0.376	0.610	2015Q1	-0.276	-0.417	-1.550	0.112	-0.107	-0.528
2002Q4	0.097	-1.716	-0.315	0.199	-0.221	-0.094	2015Q2	-1.713	-0.079	1.032	-0.754	0.436	-0.173
2003Q1	-0.069	-0.280	-1.284	-0.542	0.059	0.121	2015Q3	-0.419	-0.300	0.856	0.601	0.239	-0.494
2003Q2	0.558	0.447	-0.495	0.329	-0.458	-0.443	2015Q4	-0.893	-0.693	-0.595	-0.076	-0.587	-0.396
2003Q3	-0.433	-1.088	0.986	-0.335	0.076	-0.301	2016Q1	0.199	1.778	-0.370	-0.127	-0.476	-0.041
2003Q4	-0.638	1.236	0.317	-0.278	0.265	0.290	2016Q2	0.748	1.455	-0.693	0.342	-0.159	0.182
2004Q1	0.557	-0.257	-0.369	0.832	-0.792	-0.583	2016Q3	-0.159	-0.245	-1.037	-0.022	-0.910	0.164
2004Q2	-0.535	-1.557	0.924	0.038	0.323	0.004	2016Q4	-0.303	-0.252	0.823	0.332	0.059	0.511
2004Q3	0.289	0.597	-0.497	0.494	0.041	0.294	2017Q1	-0.306	-0.734	1.181	0.491	0.064	-0.106
2004Q4	0.222	-0.438	-0.934	-0.226	0.037	-0.251	2017Q2	-0.436	-0.851	-0.006	0.441	0.072	0.121
2005Q1	-0.118	-1.009	-0.284	0.241	0.006	-0.097	2017Q3	-0.475	-0.859	0.116	0.032	0.591	0.486
2005Q2	-0.246	0.971	-1.007	0.100	0.007	0.257	2017Q4	0.177	0.553	-0.161	0.302	0.058	-0.219
2005Q3	-0.196	-0.273	-0.444	0.367	0.149	0.261	2018Q1	-0.311	-0.166	-0.062	-0.160	-0.233	-0.025
2005Q4	0.054	0.801	0.523	0.863	-0.278	-0.002	2018Q2	-0.352	-1.085	-0.125	-0.353	0.313	0.396
2006Q1	-0.762	-0.638	0.428	-0.110	0.688	0.538	2018Q3	0.502	0.482	0.183	0.802	-0.334	-0.162
2006Q2	-0.358	0.480	1.385	0.234	0.099	-0.113	2018Q4	-0.098	0.084	0.501	-0.055	-0.005	0.213
2006Q3	0.073	0.116	-0.053	0.429	-0.546	-0.212	2019Q1	0.355	0.265	-0.924	0.118	0.537	0.230
2006Q4	-0.138	-0.602	-0.092	-0.100	-0.008	0.086	2019Q2	1.003	1.402	-1.503	0.538	-0.674	-0.343
2007Q1	0.295	2.512	0.310	0.117	0.093	0.243	2019Q3	0.197	-0.488	-1.523	-0.063	-0.478	0.046
2007Q2	0.099	-0.216	1.117	0.055	-0.054	-0.194	2019Q4	-0.603	0.884	0.302	-1.071	-0.501	-0.291
2007Q3	-0.373	0.161	0.389	0.113	-0.815	0.280	2020Q1	1.378	-0.741	0.554	-0.227	-0.632	-0.004
2007Q4	0.271	0.196	-0.367	-0.934	-0.033	-0.220	2020Q2	6.726	-1.421	1.439	-0.846	-0.703	-0.735
2008Q1	0.268	-0.101	-0.172	-1.013	-1.299	-0.886	2020Q3	2.545	-0.156	-1.125	-1.337	0.368	0.720
2008Q2	1.260	1.395	0.725	-1.145	0.587	-0.404	2020Q4	3.279	-0.281	-0.694	-0.660	0.658	0.428
2008Q3	1.992	2.112	-0.108	-1.514	0.469	-0.705	2021Q1	0.381	-0.673	0.367	-0.367	0.716	-0.763
2008Q4	3.554	1.261	-1.603	-0.520	-0.356	-0.886	2021Q2	-8.736	1.272	0.155	0.674	1.490	-1.496
2009Q1	3.519	4.066	-0.765	0.013	-0.644	-0.182	2021Q3	-3.139	0.410	-0.673	0.085	1.031	-0.911
2009Q2	2.841	0.431	-0.284	-1.546	0.332	-0.596	2021Q4	-3.620	-1.759	0.661	0.952	1.356	-1.553
2009Q3	1.717	-2.751	-0.157	-1.363	0.548	-0.275	2022Q1	-4.327	1.352	1.583	-0.385	-0.547	-0.791
2009Q4	0.801	-1.481	0.143	-0.898	0.003	-0.708	2022Q2	-0.602	0.601	3.443	-0.775	-0.089	1.171
2010Q1	-1.535	-4.085	1.029	-1.216	1.858	0.146	2022Q3	-0.796	0.741	1.219	0.936	-0.881	-0.426
2010Q2	-1.651	-1.720	1.324	0.914	-0.525	-0.611	2022Q4	-0.392	3.666	1.562	-1.083	-0.861	-0.352
2010Q3	-1.134	1.471	-0.083	0.445	-0.605	-0.076	2023Q1	1.423	2.804	-0.318	1.395	1.425	1.095
2010Q4	-1.569	0.850	1.239	-0.156	0.995	0.356	2023Q2	1.605	-0.822	0.495	1.510	-0.409	-0.194
2011Q1	0.911	-0.089	1.856	2.042	0.286	-0.067	2023Q3	-0.496	-0.521	0.771	-0.689	0.017	0.585
2011Q2	-0.317	1.710	-0.093	-0.236	0.137	0.576	2023Q4	0.846	-1.617	0.317	1.979	0.168	0.876
2011Q3	-0.068	-0.512	0.368	-0.150	-0.184	0.184	2024Q1	0.594	-1.788	-0.580	-0.192	-0.352	0.481
2011Q4	0.822	-0.169	0.891	0.039	-0.551	-0.236	2024Q2	-0.717	0.732	0.176	-0.736	0.010	0.819
2012Q1	-0.041	0.684	-1.241	-0.465	-0.466	0.653	2024Q3	1.226	-0.171	-0.532	1.416	0.091	0.296
2012Q2	-0.279	0.464	2.748	-0.631	0.345	0.352	2024Q4	0.090	-0.838	-0.180	-0.699	-0.055	-0.180
2012Q3	0.655	1.478	0.382	0.141	0.739	0.876	2025Q1	0.019	-0.128	0.964	0.260	-0.513	-0.361
2012Q4	3.520	-0.192	-2.907	0.830	1.644	-0.238	2025Q2	0.883	-0.082	-0.096	0.603	0.616	0.391
2013Q1	1.200	-1.226	-1.218	1.804	0.022	-0.038	2025Q3	-0.363	-0.143	0.293	-0.207	-0.175	0.300
2013Q2	-0.171	-0.403	-1.536	0.428	-0.162	0.255	2025Q4	-0.710	0.696	-0.233	-0.007	-0.124	0.459
2013Q3	-0.913	-2.493	0.795	0.510	-0.584	-0.411	2026Q1	-0.382	-0.416	0.441	0.267	-0.229	-0.035
2013Q4	-5.646	-0.570	-0.430	-0.525	-2.557	1.231							

D.16 United Kingdom

United Kingdom is estimated over 2001Q2–2026Q1, that is 100 quarterly observations once the transformations of Appendix C are applied, which costs one quarter. Figure D61 shows the loading of each series on each factor and Figure D62 the same loadings as a heat map; Table D31 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, ΔGov.Debt (0.520); Factor 2, Ext. Trade (-0.705); Factor 3, y^{10} (0.963); Factor 4, ΔFinAssets (0.782); Factor 5, Primary balance (0.710); Factor 6, CapitalFormation (0.528). The sum of squared loadings is largest for CapitalFormation (0.994) and smallest for ΔGov.Debt (0.283); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D61: United Kingdom: factor loadings by series

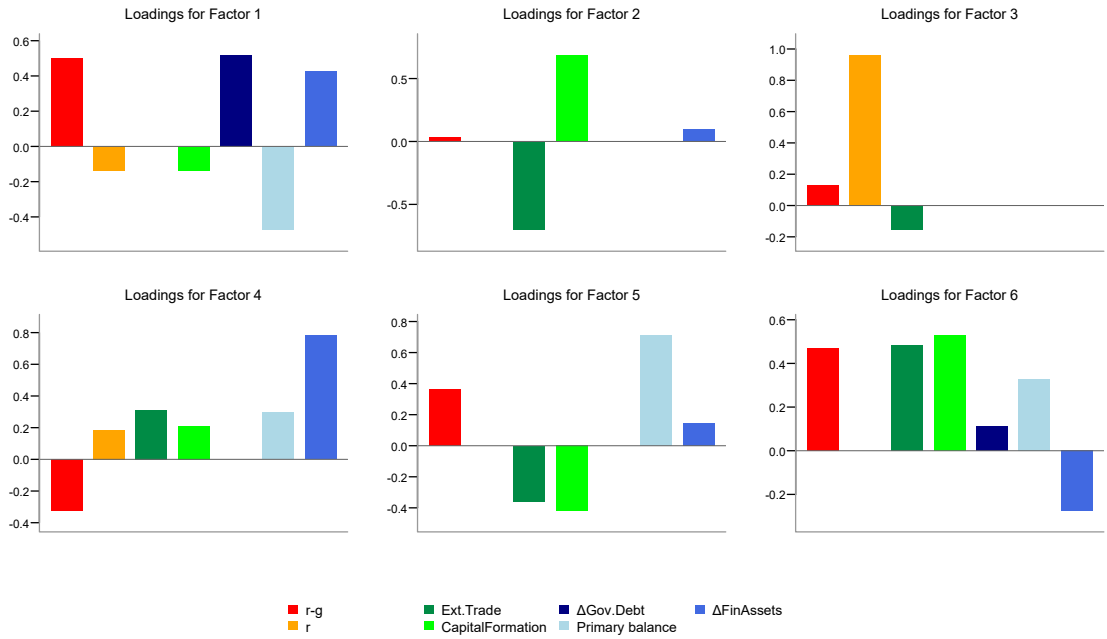


Figure D62: United Kingdom: loading heat map

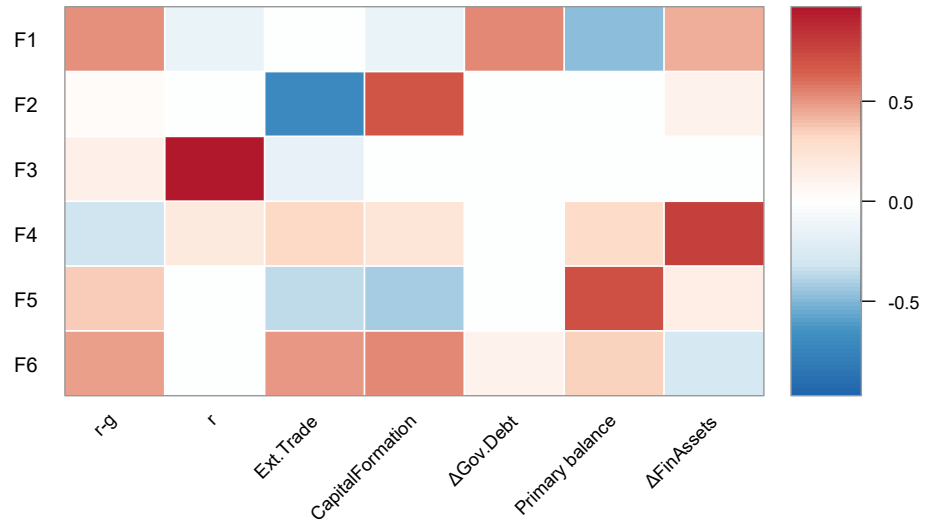


Table D31: United Kingdom: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.504	0.038	0.132	-0.325	0.365	0.471	0.734
y^{10}	-0.139	0.000	0.963	0.187	0.000	0.000	0.981
Ext. Trade	0.000	-0.705	-0.157	0.313	-0.358	0.482	0.980
CapitalFormation	-0.141	0.689	0.000	0.212	-0.419	0.528	0.994
ΔGov.Debt	0.520	0.000	0.000	0.000	0.000	0.112	0.283
Primary balance	-0.475	0.000	0.000	0.295	0.710	0.328	0.925
ΔFinAssets	0.425	0.101	0.000	0.782	0.146	-0.275	0.899
$\sum \lambda^2$	0.970	0.983	0.969	0.982	0.963	0.929	5.795
Prop. of common var.	0.167	0.170	0.167	0.169	0.166	0.160	1.000

Figure D63 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D64 shows the seven underlying series. Table D32 lists the factor estimates quarter by quarter.

Figure D63: United Kingdom: standardised factor estimates and standardised data

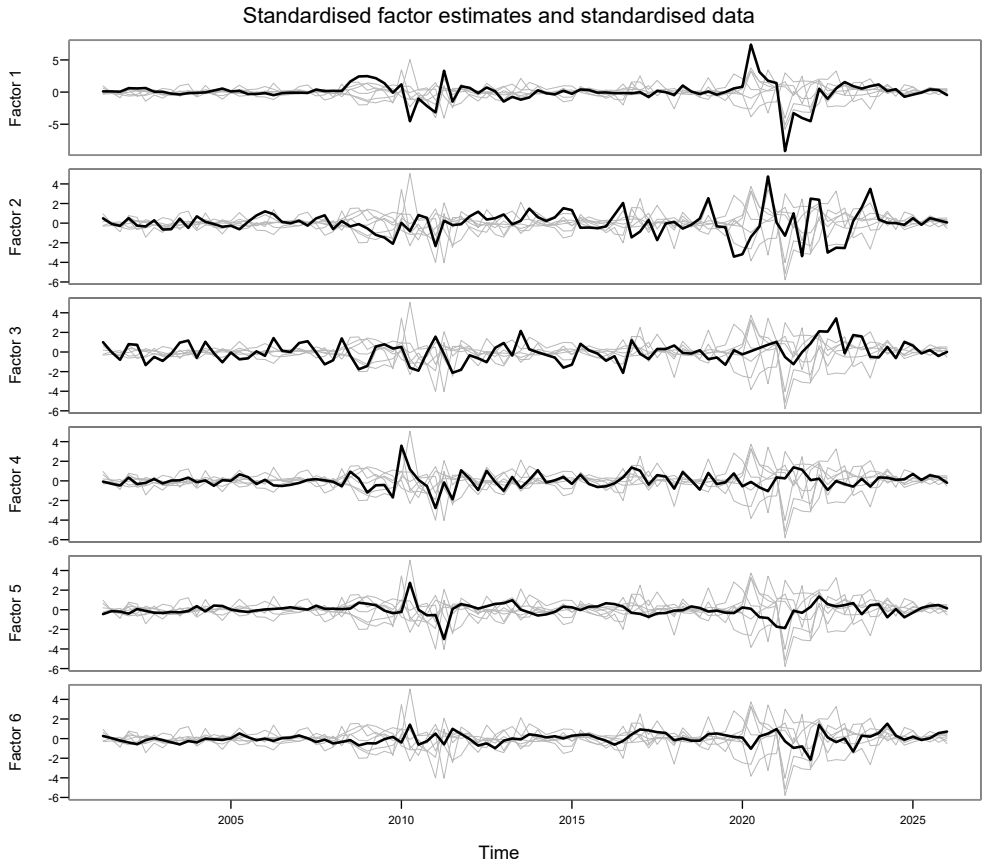


Figure D64: United Kingdom: input series over the estimation window

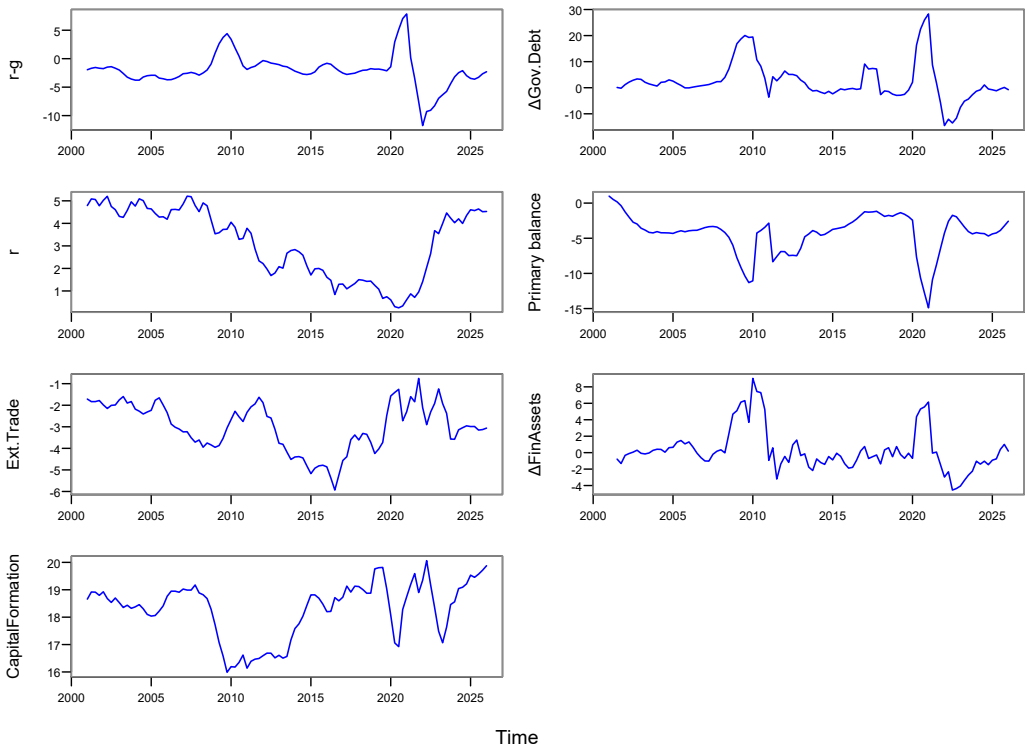


Table D32: United Kingdom: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q2	0.104	0.486	0.999	-0.075	-0.445	0.267	2013Q4	-0.836	1.478	0.272	0.107	-0.279	0.437
2001Q3	0.109	-0.074	-0.009	-0.294	-0.143	0.045	2014Q1	0.286	0.606	-0.033	1.094	-0.590	0.318
2001Q4	0.061	-0.279	-0.795	-0.482	-0.190	-0.199	2014Q2	-0.184	0.219	-0.304	-0.167	-0.475	0.124
2002Q1	0.601	0.509	0.803	0.336	-0.402	-0.395	2014Q3	-0.360	0.582	-0.570	0.063	-0.219	0.243
2002Q2	0.569	-0.255	0.734	-0.335	0.072	-0.568	2014Q4	0.214	1.528	-1.598	0.402	0.320	0.017
2002Q3	0.626	-0.337	-1.331	-0.199	-0.122	-0.150	2015Q1	-0.268	1.326	-1.295	-0.309	0.245	0.297
2002Q4	0.059	0.284	-0.501	0.189	-0.307	0.044	2015Q2	0.402	-0.471	0.842	0.640	-0.034	0.379
2003Q1	0.058	-0.652	-0.911	-0.259	-0.328	-0.157	2015Q3	0.321	-0.451	0.171	-0.323	0.317	0.411
2003Q2	-0.246	-0.622	-0.177	0.037	-0.232	-0.380	2015Q4	-0.090	-0.513	-0.152	-0.629	0.331	0.057
2003Q3	-0.369	0.464	0.954	0.053	-0.268	-0.595	2016Q1	-0.059	-0.334	-0.889	-0.554	0.664	-0.219
2003Q4	-0.164	-0.490	1.165	0.334	-0.135	-0.253	2016Q2	-0.190	0.859	-0.427	-0.252	0.578	-0.618
2004Q1	-0.128	0.695	-0.590	-0.142	0.370	-0.385	2016Q3	-0.142	2.057	-2.132	0.356	0.320	-0.237
2004Q2	-0.051	0.134	1.042	0.045	-0.169	-0.005	2016Q4	-0.186	-1.453	1.200	1.367	-0.312	0.425
2004Q3	0.239	-0.090	-0.123	-0.503	0.427	-0.073	2017Q1	-0.020	-0.855	-0.180	1.041	-0.424	0.934
2004Q4	0.549	-0.369	-1.050	0.069	0.385	-0.140	2017Q2	-0.769	0.331	-0.728	-0.383	-0.722	0.837
2005Q1	0.100	-0.266	-0.052	0.011	0.016	0.027	2017Q3	0.183	-1.730	0.307	0.589	-0.378	0.656
2005Q2	0.197	-0.624	-0.750	0.672	-0.137	0.530	2017Q4	-0.017	-0.036	0.301	0.441	-0.332	0.580
2005Q3	-0.299	0.123	-0.647	0.388	-0.219	0.143	2018Q1	-0.446	0.128	0.665	-0.794	-0.093	-0.166
2005Q4	-0.298	0.786	0.069	-0.328	-0.073	-0.161	2018Q2	1.017	-0.559	-0.092	0.900	-0.053	0.012
2006Q1	-0.160	1.204	-0.391	0.111	0.025	-0.011	2018Q3	0.099	-0.172	-0.160	0.033	0.340	-0.210
2006Q2	-0.481	0.917	1.424	-0.461	0.083	-0.232	2018Q4	-0.305	0.465	0.181	-0.897	0.185	-0.217
2006Q3	-0.159	0.128	0.131	-0.521	0.136	0.050	2019Q1	0.094	2.542	-0.740	0.811	-0.186	0.460
2006Q4	-0.091	-0.008	-0.005	-0.401	0.255	0.106	2019Q2	-0.409	-0.330	-0.556	-0.328	-0.095	0.524
2007Q1	-0.097	0.248	0.919	-0.211	0.098	0.315	2019Q3	0.008	-0.407	-1.307	-0.152	-0.301	0.343
2007Q2	-0.135	-0.258	1.101	0.097	0.019	0.036	2019Q4	0.572	-3.418	0.180	0.763	-0.338	0.176
2007Q3	0.385	0.488	-0.049	0.178	0.399	-0.347	2020Q1	0.821	-3.173	-0.238	-0.555	0.236	0.102
2007Q4	0.158	0.802	-1.262	0.053	0.094	-0.107	2020Q2	7.387	-1.403	0.083	-0.103	0.102	-1.043
2008Q1	0.197	-0.609	-0.836	-0.089	0.103	-0.491	2020Q3	3.132	-0.345	0.414	-0.662	-0.757	0.230
2008Q2	0.159	0.210	1.391	-0.543	0.066	-0.342	2020Q4	1.764	4.756	0.741	-1.050	-0.844	0.511
2008Q3	1.643	-0.334	-0.356	0.946	0.117	-0.152	2021Q1	1.402	0.076	1.036	0.356	-1.721	0.971
2008Q4	2.435	-0.097	-1.760	0.236	0.715	-0.690	2021Q2	-9.174	-1.283	-0.506	0.253	-1.871	-0.271
2009Q1	2.478	-0.536	-1.430	-1.189	0.610	-0.475	2021Q3	-3.275	1.002	-1.243	1.387	-0.099	-0.952
2009Q2	2.161	-1.198	0.562	-0.452	0.459	-0.488	2021Q4	-4.028	-3.370	-0.013	1.138	-0.314	-0.792
2009Q3	1.390	-1.465	0.784	-0.417	-0.090	-0.061	2022Q1	-4.533	2.497	0.868	0.071	0.292	-2.149
2009Q4	-0.092	-2.106	0.349	-1.692	-0.352	0.177	2022Q2	0.514	2.384	2.117	0.222	1.358	1.404
2010Q1	1.204	0.033	0.524	3.591	-0.228	-0.403	2022Q3	-1.043	-3.004	2.077	-0.929	0.570	0.119
2010Q2	-4.529	-0.811	-1.607	1.182	2.742	1.420	2022Q4	0.576	-2.508	3.433	-0.014	0.335	-0.359
2010Q3	-1.004	0.827	-1.908	0.109	0.012	-0.619	2023Q1	1.564	-2.538	-0.119	-0.329	0.472	0.010
2010Q4	-2.174	0.531	-0.056	-0.541	-0.568	-0.262	2023Q2	0.924	0.212	1.729	-0.591	0.686	-1.324
2011Q1	-3.157	-2.346	1.568	-2.773	-0.544	0.516	2023Q3	0.545	1.603	1.591	0.217	-0.458	0.297
2011Q2	3.325	0.245	-0.210	-0.155	-2.993	-0.589	2023Q4	0.918	3.501	-0.489	-0.584	0.467	0.200
2011Q3	-1.490	-0.198	-2.117	-1.883	0.079	1.001	2024Q1	1.188	0.370	-0.550	0.354	0.556	0.572
2011Q4	0.903	-0.113	-1.814	1.073	0.576	0.475	2024Q2	0.164	0.034	0.526	0.321	-0.772	1.525
2012Q1	0.668	0.682	-0.336	0.203	0.397	-0.050	2024Q3	0.451	-0.001	-0.624	0.110	0.065	0.302
2012Q2	-0.164	1.170	-0.635	-0.940	0.114	-0.705	2024Q4	-0.734	-0.169	1.027	0.155	-0.771	-0.116
2012Q3	0.721	0.365	-1.039	1.025	0.353	-0.486	2025Q1	-0.396	0.509	0.646	0.698	-0.292	0.192
2012Q4	0.116	0.507	0.424	-0.119	0.598	-0.981	2025Q2	-0.084	-0.166	-0.145	0.095	0.183	-0.139
2013Q1	-1.475	0.884	0.922	-1.014	0.662	-0.214	2025Q3	0.422	0.516	0.196	0.567	0.406	0.037
2013Q2	-0.735	-0.121	-0.365	0.401	0.971	0.009	2025Q4	0.324	0.289	-0.411	0.390	0.479	0.561
2013Q3	-1.183	0.242	2.152	-0.725	0.014	-0.084	2026Q1	-0.461	0.070	0.013	-0.193	0.148	0.718

D.17 United States

United States is estimated over 2001Q3–2026Q1, that is 99 quarterly observations once the transformations of Appendix C are applied, which costs 2 quarters. Figure D65 shows the loading of each series on each factor and Figure D66 the same loadings as a heat map; Table D33 reports them with each series' communality, the sum of its squared loadings on the six common factors. That sum is a share of the series' variance only when the factors are mutually orthogonal with unit variance, a normalisation the estimated scores do not satisfy exactly, so the communalities should be read as indicative magnitudes rather than exact variance shares.

The largest loading on each factor is: Factor 1, $r - g$ (0.515); Factor 2, Capital-Formation (0.674); Factor 3, y^{10} (0.903); Factor 4, Primary balance (0.727); Factor 5, Ext. Trade (0.655); Factor 6, $r - g$ (0.790). The sum of squared loadings is largest for y^{10} (1.000) and smallest for ΔGov.Debt (0.273); a value at or close to one indicates a series almost wholly spanned by the six factors, not an exact unit share of its variance.

Figure D65: United States: factor loadings by series

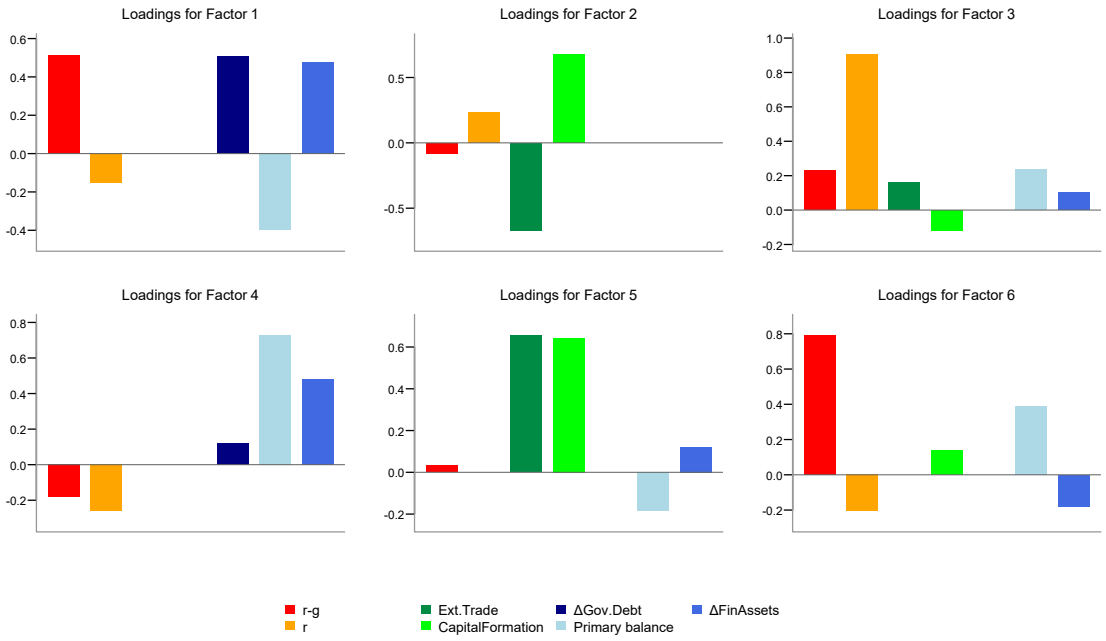


Figure D66: United States: loading heat map

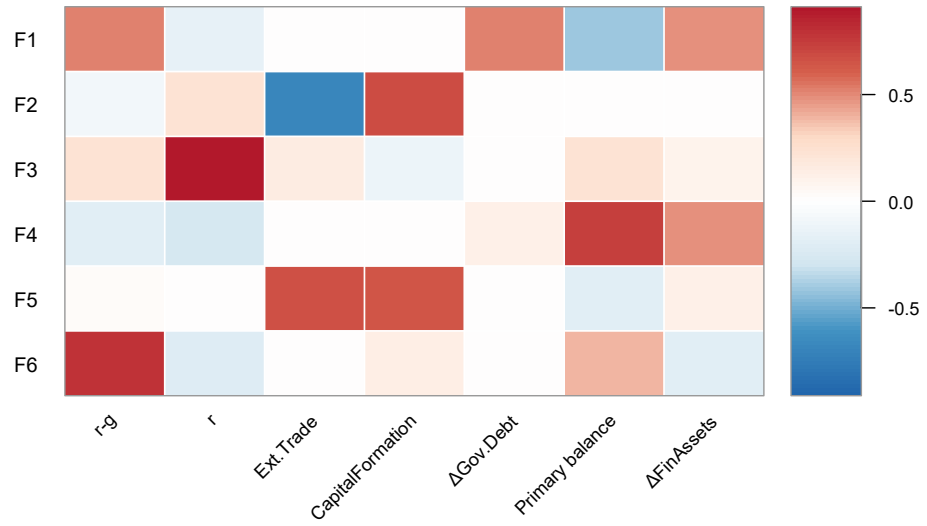


Table D33: United States: factor loadings, communalities and factor variance sums

Variable	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*	*Factor 5*	*Factor 6*	*Communality*
$r-g$	0.515	-0.080	0.232	-0.183	0.033	0.790	0.985
y^{10}	-0.151	0.230	0.903	-0.260	0.000	-0.205	1.000
Ext. Trade	0.000	-0.668	0.158	0.000	0.655	0.000	0.901
CapitalFormation	0.000	0.674	-0.117	0.000	0.643	0.138	0.900
ΔGov.Debt	0.509	0.000	0.000	0.122	0.000	0.000	0.273
Primary balance	-0.399	0.000	0.234	0.727	-0.185	0.390	0.929
ΔFinAssets	0.478	0.000	0.104	0.482	0.122	-0.181	0.519
$\sum \lambda^2$	0.935	0.959	0.973	0.876	0.893	0.871	5.507
Prop. of common var.	0.170	0.174	0.177	0.159	0.162	0.158	1.000

Figure D67 plots the six estimated factors (in black) against the standardised input series (in grey), and Figure D68 shows the seven underlying series. Table D34 lists the factor estimates quarter by quarter.

Figure D67: United States: standardised factor estimates and standardised data

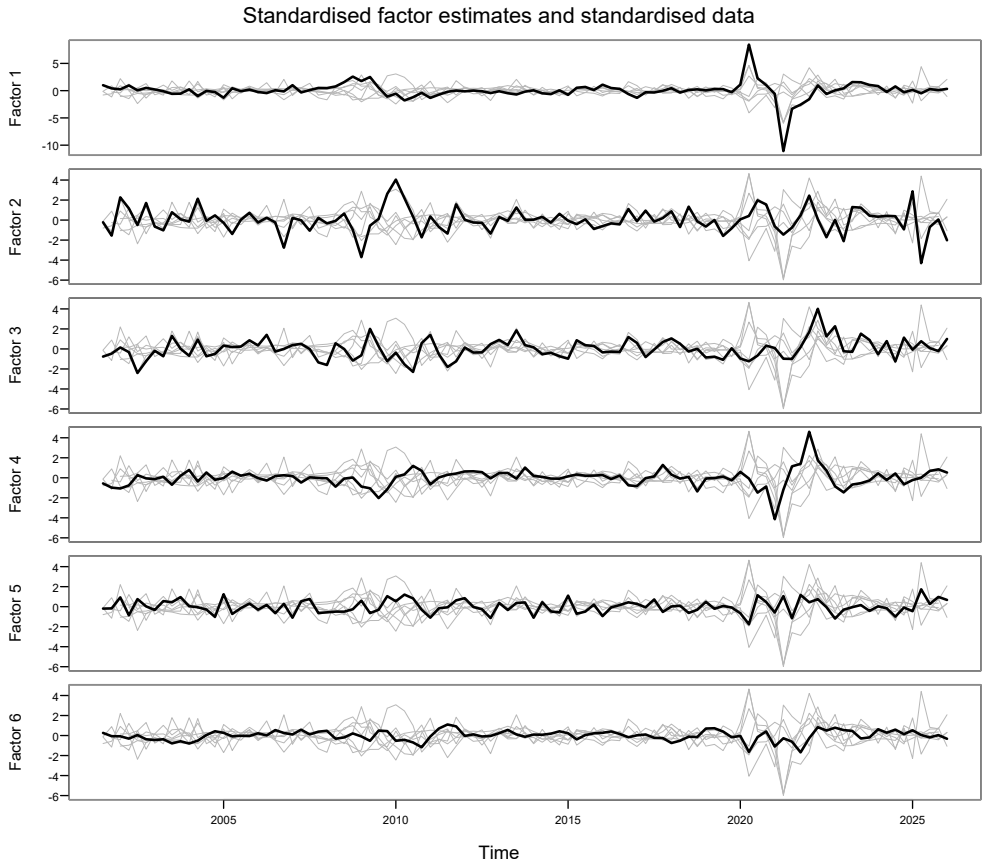


Figure D68: United States: input series over the estimation window

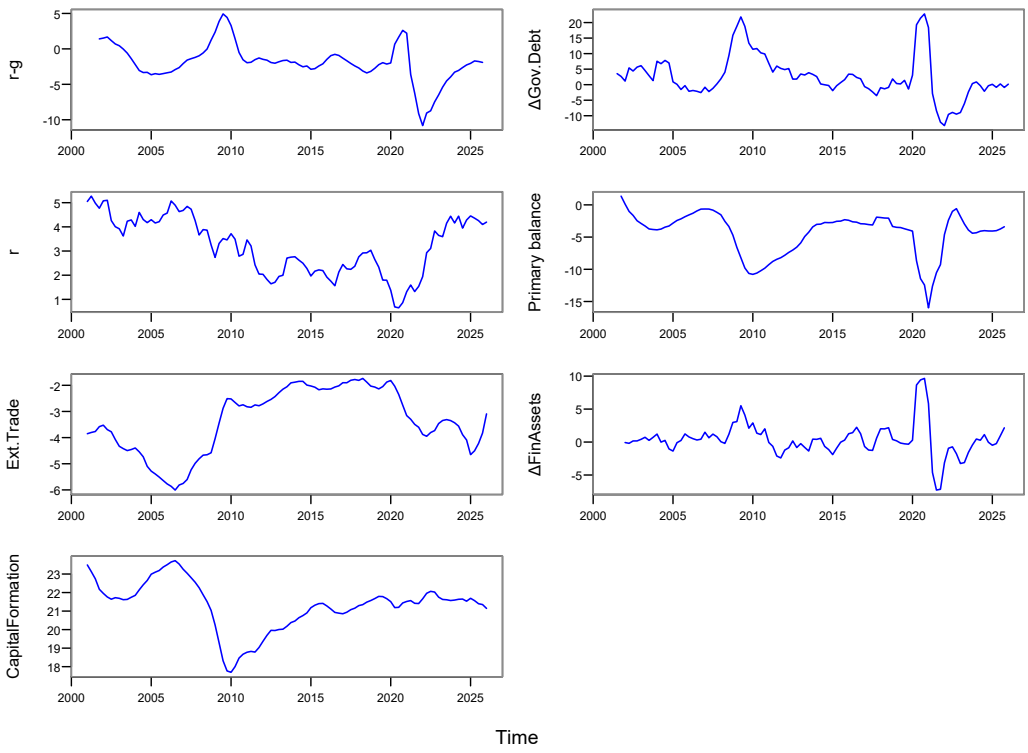


Table D34: United States: estimated factor scores by quarter

Quarter	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*Quarter*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*
2001Q3	1.000	-0.221	-0.764	-0.563	-0.193	0.263	2014Q1	0.066	0.041	0.164	0.189	-1.105	0.098
2001Q4	0.415	-1.551	-0.464	-0.982	-0.173	-0.066	2014Q2	-0.482	0.323	-0.528	0.093	0.448	0.079
2002Q1	0.244	2.268	0.145	-1.068	0.928	-0.074	2014Q3	-0.635	-0.254	-0.389	-0.076	-0.480	0.186
2002Q2	0.962	1.168	-0.318	-0.761	-0.862	-0.311	2014Q4	0.066	0.625	-0.731	-0.090	-0.578	0.424
2002Q3	0.063	-0.495	-2.404	0.270	0.759	0.048	2015Q1	-0.771	-0.065	-0.978	0.148	1.103	0.228
2002Q4	0.511	1.709	-1.221	-0.059	0.030	-0.372	2015Q2	0.532	-0.409	0.869	0.328	-0.708	-0.386
2003Q1	0.223	-0.648	-0.178	-0.132	-0.333	-0.452	2015Q3	0.682	0.071	0.370	0.205	-0.491	0.048
2003Q2	-0.101	-1.026	-0.727	0.105	0.559	-0.374	2015Q4	0.108	-0.893	0.298	0.220	0.204	0.220
2003Q3	-0.583	0.785	1.304	-0.696	0.440	-0.785	2016Q1	1.100	-0.602	-0.343	0.277	-0.947	0.271
2003Q4	-0.551	0.083	0.019	0.204	0.957	-0.572	2016Q2	0.483	-0.329	-0.272	-0.122	-0.072	0.409
2004Q1	0.247	-0.155	-0.685	0.786	0.049	-0.799	2016Q3	0.330	-0.421	-0.307	0.221	0.162	0.152
2004Q2	-1.000	2.135	0.939	-0.374	-0.058	-0.508	2016Q4	-0.620	1.108	1.191	-0.750	0.435	-0.169
2004Q3	-0.045	-0.075	-0.730	0.533	-0.285	0.050	2017Q1	-1.322	-0.121	0.602	-0.834	0.279	0.020
2004Q4	-0.255	0.469	-0.500	-0.199	-1.015	0.430	2017Q2	-0.300	0.946	-0.814	-0.033	-0.064	0.105
2005Q1	-1.293	-0.272	0.332	-0.027	1.241	0.270	2017Q3	-0.333	-0.154	-0.058	0.136	0.723	-0.227
2005Q2	0.447	-1.385	0.186	0.609	-0.729	-0.083	2017Q4	-0.018	0.200	0.695	1.288	-0.527	-0.253
2005Q3	-0.104	0.028	0.235	0.213	-0.150	-0.021	2018Q1	0.450	0.876	1.046	0.320	0.019	-0.749
2005Q4	0.187	0.723	0.862	0.394	0.315	-0.044	2018Q2	-0.345	-0.699	0.522	-0.080	0.102	-0.519
2006Q1	-0.245	-0.246	0.351	-0.033	-0.302	0.224	2018Q3	0.125	1.339	-0.264	0.095	-0.629	-0.115
2006Q2	-0.445	0.239	1.408	-0.276	0.173	0.010	2018Q4	0.240	-0.122	0.018	-1.363	-0.311	-0.151
2006Q3	0.082	-0.237	-0.267	0.174	-0.654	0.547	2019Q1	0.049	-0.649	-0.858	-0.067	0.480	0.698
2006Q4	-0.141	-2.759	0.000	0.255	0.287	0.256	2019Q2	0.299	0.004	-0.774	-0.026	-0.213	0.728
2007Q1	1.007	0.220	0.380	0.176	-1.103	0.107	2019Q3	0.283	-1.588	-1.089	0.124	0.074	0.401
2007Q2	-0.321	-0.023	0.512	-0.475	0.539	0.588	2019Q4	-0.288	-0.822	0.072	-0.258	-0.086	-0.174
2007Q3	0.103	-1.065	-0.009	0.048	0.770	0.108	2020Q1	1.110	0.071	-0.956	0.597	-0.655	-0.034
2007Q4	0.501	0.225	-1.346	-0.012	-0.627	0.386	2020Q2	8.452	0.415	-1.236	-0.076	-1.757	-1.635
2008Q1	0.471	-0.328	-1.601	-0.045	-0.568	0.474	2020Q3	2.277	1.974	-0.666	-1.482	1.143	-0.158
2008Q2	0.777	-0.080	0.566	-0.886	-0.493	-0.318	2020Q4	1.018	1.560	0.328	-0.864	0.467	0.412
2008Q3	1.570	0.649	-0.008	-0.081	-0.498	-0.189	2021Q1	-0.594	-0.627	0.092	-4.139	-0.586	-1.103
2008Q4	2.582	-0.966	-1.164	0.033	-0.259	0.210	2021Q2	-11.081	-1.454	-0.979	-1.191	1.059	-0.272
2009Q1	1.753	-3.706	-0.638	-0.877	0.603	-0.091	2021Q3	-3.324	-0.752	-0.996	1.146	-1.157	-0.612
2009Q2	2.508	-0.558	2.004	-1.049	-0.642	-0.530	2021Q4	-2.560	0.472	0.182	1.394	1.173	-1.680
2009Q3	0.547	0.119	0.270	-2.028	-0.311	0.495	2022Q1	-1.567	2.445	1.691	4.600	0.436	-0.263
2009Q4	-1.100	2.617	-1.229	-1.156	1.050	0.442	2022Q2	0.971	0.075	4.010	1.751	0.748	0.832
2010Q1	-0.547	4.043	-0.365	0.101	0.541	-0.529	2022Q3	-0.620	-1.716	1.252	0.731	-0.046	0.499
2010Q2	-1.793	2.187	-1.542	0.340	1.206	-0.427	2022Q4	0.061	0.022	2.266	-0.851	-1.192	0.783
2010Q3	-1.201	0.342	-2.299	1.191	0.827	-0.692	2023Q1	0.425	-2.110	-0.233	-1.455	-0.299	0.544
2010Q4	-0.391	-1.727	0.589	0.701	-0.308	-1.172	2023Q2	1.577	1.298	-0.291	-0.675	-0.050	0.461
2011Q1	-1.331	0.363	1.399	-0.658	-1.095	-0.099	2023Q3	1.539	1.242	1.520	-0.536	0.161	-0.297
2011Q2	-0.730	-0.687	-0.538	0.029	-0.157	0.701	2023Q4	1.048	0.435	0.899	-0.270	-0.424	-0.188
2011Q3	-0.203	-1.352	-1.804	0.296	-0.066	1.104	2024Q1	0.842	0.337	-0.529	0.444	0.033	0.626
2011Q4	0.018	1.573	-1.238	0.427	0.630	0.917	2024Q2	-0.247	0.424	0.783	-0.223	-0.158	0.282
2012Q1	-0.126	0.036	0.146	0.636	0.852	-0.040	2024Q3	0.782	0.385	-1.250	0.431	-0.998	0.592
2012Q2	0.015	-0.246	-0.363	0.651	-0.031	0.116	2024Q4	-0.305	-0.930	1.131	-0.654	-0.087	0.134
2012Q3	-0.102	-0.273	-0.343	0.565	-0.255	-0.094	2025Q1	0.137	2.872	-0.082	-0.225	-0.450	0.525
2012Q4	-0.437	-1.354	0.483	-0.054	-1.141	-0.042	2025Q2	-0.484	-4.304	0.769	0.008	1.735	0.053
2013Q1	-0.110	0.293	0.905	0.480	0.372	0.237	2025Q3	0.291	-0.695	0.087	0.685	0.269	-0.188
2013Q2	-0.469	-0.074	0.390	0.485	-0.334	0.566	2025Q4	0.098	0.060	-0.218	0.848	0.977	0.045
2013Q3	-0.705	1.261	1.858	-0.084	0.373	0.110	2026Q1	0.329	-2.018	0.999	0.525	0.674	-0.324
2013Q4	-0.177	0.026	0.382	1.025	0.425	-0.122							

Appendix E Statistical tests and model diagnostics

This appendix reports, for each country, the diagnostics used to choose between the three estimators of the factor structure: the sparse dynamic factor model (Sparse DFM) retained in Appendix D, an unrestricted dynamic factor model (DFM), and principal components (PCA). All three are fitted to the same seven series over the same window as in Appendix D.

The table for each country compares the three estimators on fit (log-likelihood, BIC, parameter count and the number of nonzero loadings), on Bartlett's test of sphericity of the correlation matrix, which is not available on this estimation design and is reported as a dash, on the average Ljung–Box and Breusch–Godfrey p-values across the seven residual series, and on the spectral radius of the transition matrix, which is below one whenever the estimated factor process is stationary.

Three figures accompany each country. The residual boxplots show the dispersion of the sparse model's residuals series by series. The second figure reports the information criterion IC_2 of Bai and Ng (2002) and the share of variance explained, both against the number of factors, with the minimiser marked; in every country it lies at the upper boundary of the admissible range, which with seven series extends only to six factors, so the criterion cannot identify a factor count here; the count is instead fixed at six throughout, as set out in Section 5. The third traces the cold-started BIC across the LASSO tuning parameter α on a $\log_{10}$ scale, whose minimiser over the converged fits sets the degree of sparsity imposed on the loadings. It is the curve that selected the penalty: each point is a separate fit, started from scratch at that α, over the thirteen-point grid the selection uses, and the dashed line marks the chosen value. Penalties at which no converged fit is available carry no comparable criterion and take no part in the minimisation; they are marked on the horizontal axis rather than plotted. How many converge varies by country, from nine of thirteen in Italy to all thirteen in seven countries. Of the seventeen penalties excluded across the panel, ten are fits that ran without converging; the remaining seven are the largest penalty on the grid, at which the estimator declined to return a fit because one of the six factors had had all of its penalised loadings shrunk to zero.

Bartlett's test of sphericity is tabulated but carries no value in any country. The estimation samples run to the ends of each country's coverage and therefore contain missing quarters at the edges, which the Kalman filter of the state-space form absorbs but which propagate into the residual series; the routine that computes the statistic forms the residual correlation matrix without a missing-value rule and so returns none. The degrees of freedom, which do not depend on the data, are reported. Nothing else in the paper rests on this test: the factor structure is identified by the

criteria of Section 5 and compared across estimators by the Bayesian information criterion reported in the same table.

E.1 Austria

Table E1 compares the three estimators for Austria. The sparse dynamic factor model uses 37 nonzero loadings against 42 for the unrestricted model, and attains a Bayesian information criterion of 2122.098; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.752, below one, so the estimated factor process is stationary.

Figure E1 shows the residual boxplots, Figure E2 the information criterion and variance explained by number of factors, and Figure E3 the BIC across the LASSO tuning parameter.

Table E1: Austria: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-730.220	-729.611	-51888.773
Bayesian information criterion	2122.098	2153.636	1.04×10^5
Number of parameters	101	106	49
Nonzero loadings	37	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.023	0.023	0.019
Breusch–Godfrey average p-value	0.669	0.650	0.516
Stability and residuals			
Spectral radius	0.752	0.752	0.724
Residual degrees of freedom	599	594	651

Figure E1: Austria: residual boxplots by series

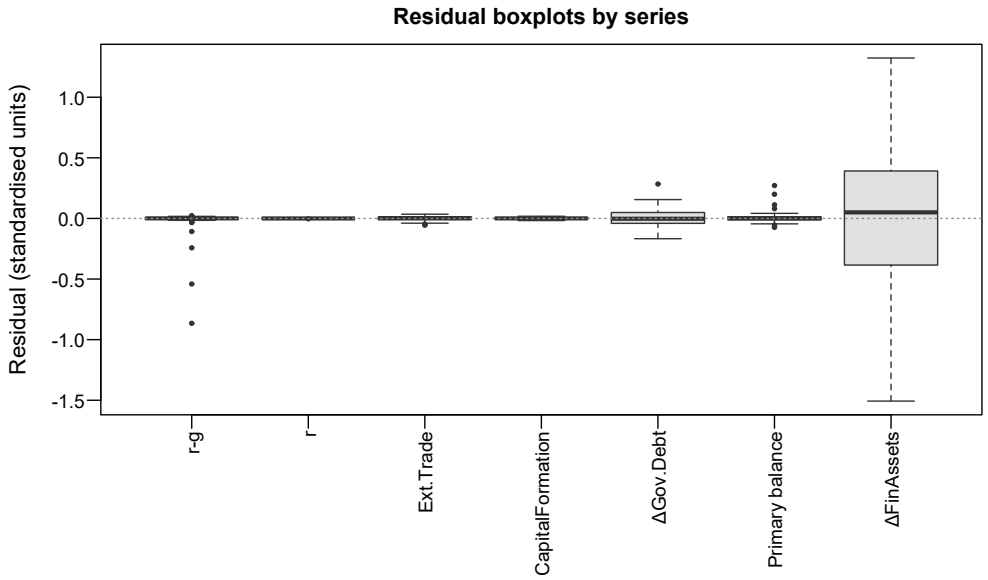


Figure E2: Austria: information criterion and variance explained by number of factors

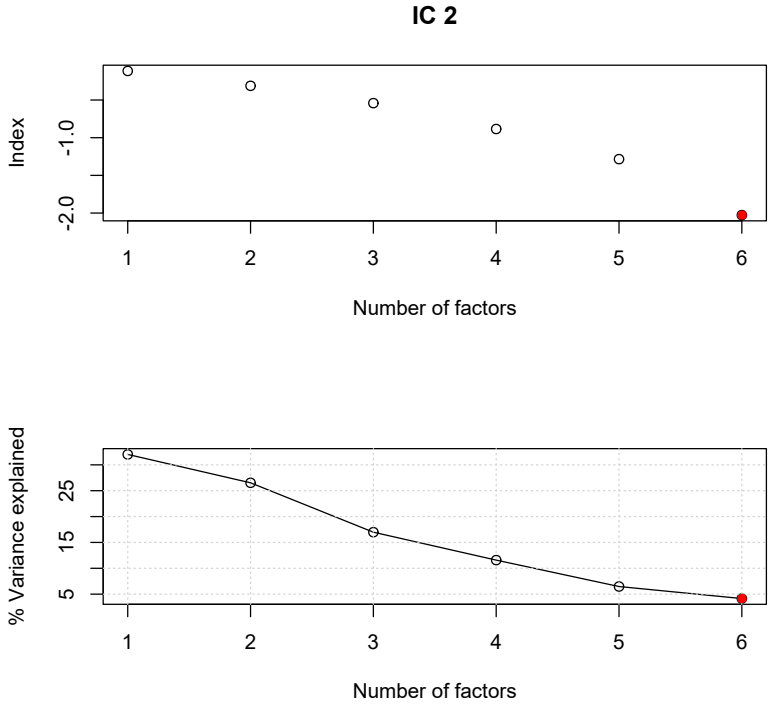
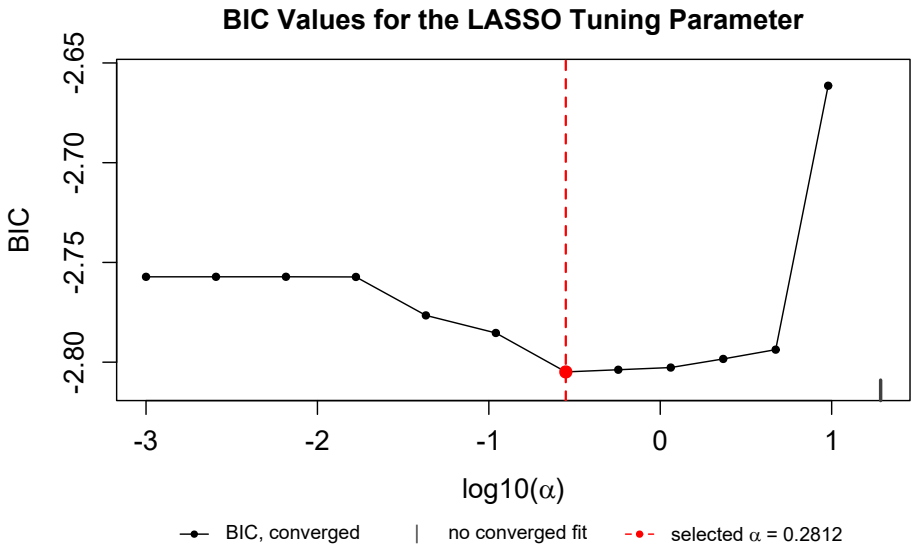


Figure E3: Austria: cold-started BIC across the LASSO tuning parameter



E.2 Belgium

Table E2 compares the three estimators for Belgium. The sparse dynamic factor model uses 33 nonzero loadings against 42 for the unrestricted model, and attains a Bayesian information criterion of 1997.053; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.792, below one, so the estimated factor process is stationary.

Figure E4 shows the residual boxplots, Figure E5 the information criterion and variance explained by number of factors, and Figure E6 the BIC across the LASSO tuning parameter.

Table E2: Belgium: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-680.799	-678.074	-2.01×10^5
Bayesian information criterion	1997.053	2050.562	4.02×10^5
Number of parameters	97	106	49
Nonzero loadings	33	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.004	0.014	0.013
Breusch–Godfrey average p-value	0.377	0.376	0.360
Stability and residuals			
Spectral radius	0.792	0.789	0.787
Residual degrees of freedom	603	594	651

Figure E4: Belgium: residual boxplots by series

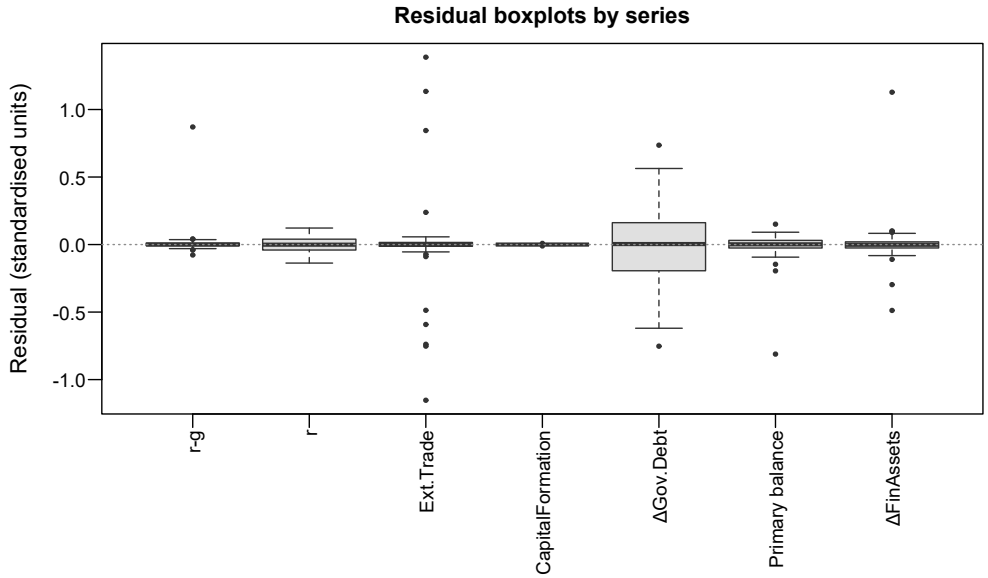


Figure E5: Belgium: information criterion and variance explained by number of factors

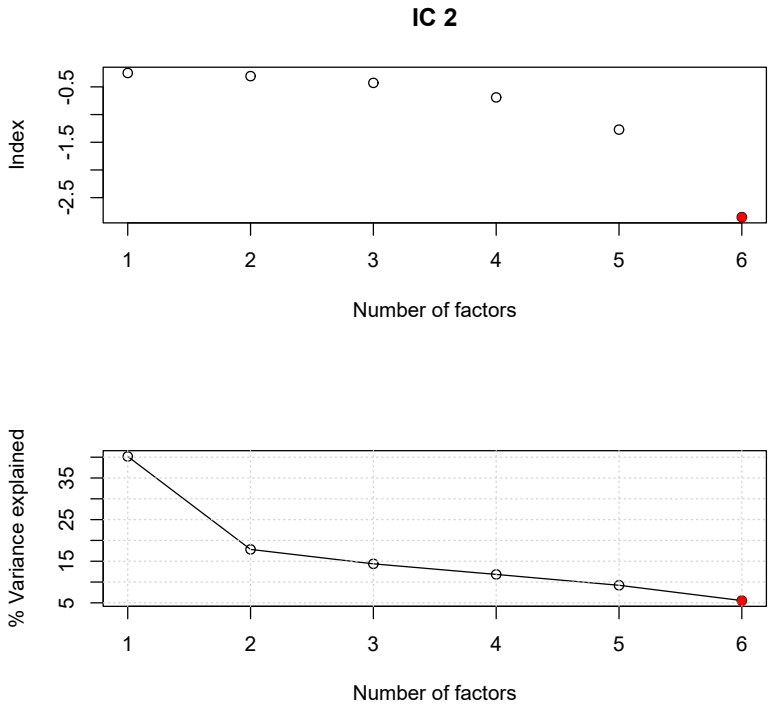
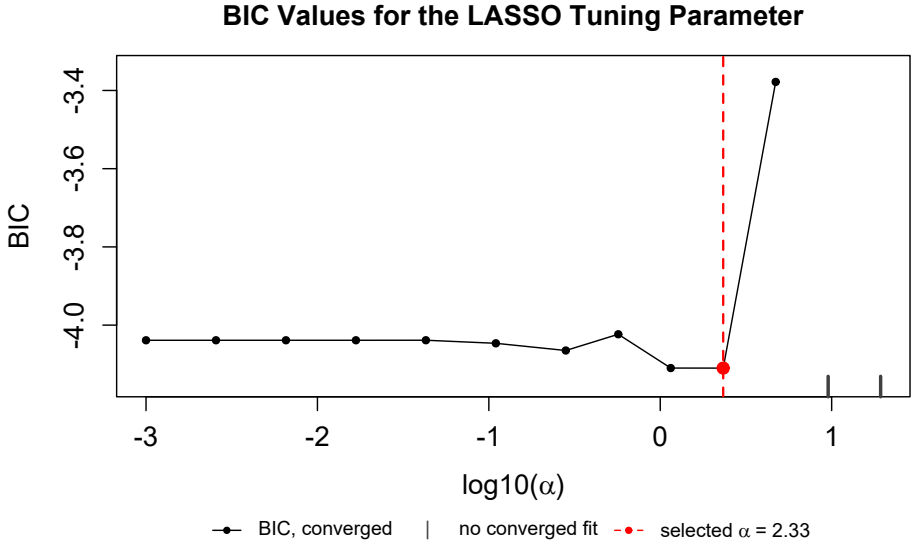


Figure E6: Belgium: cold-started BIC across the LASSO tuning parameter



E.3 Finland

Table E3 compares the three estimators for Finland. The sparse dynamic factor model uses 36 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 1994.983; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.855, below one, so the estimated factor process is stationary.

Figure E7 shows the residual boxplots, Figure E8 the information criterion and variance explained by number of factors, and Figure E9 the BIC across the LASSO tuning parameter.

Table E3: Finland: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-669.938	-667.928	-3.67×10^7
Bayesian information criterion	1994.983	2030.271	7.34×10^7
Number of parameters	100	106	49
Nonzero loadings	36	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.013	0.016	0.023
Breusch–Godfrey average p-value	0.293	0.289	0.317
Stability and residuals			
Spectral radius	0.855	0.848	0.840
Residual degrees of freedom	600	594	651

Figure E7: Finland: residual boxplots by series

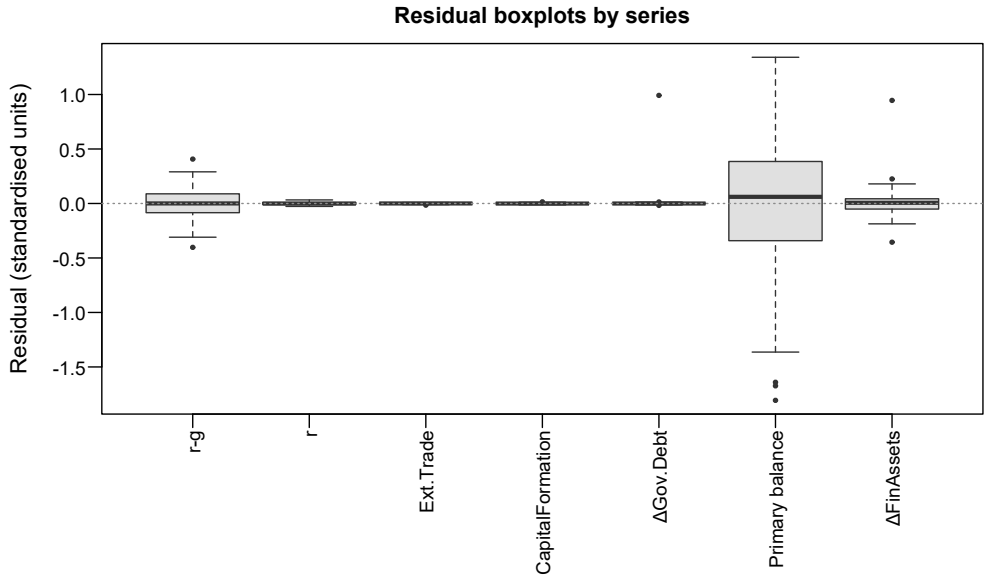


Figure E8: Finland: information criterion and variance explained by number of factors

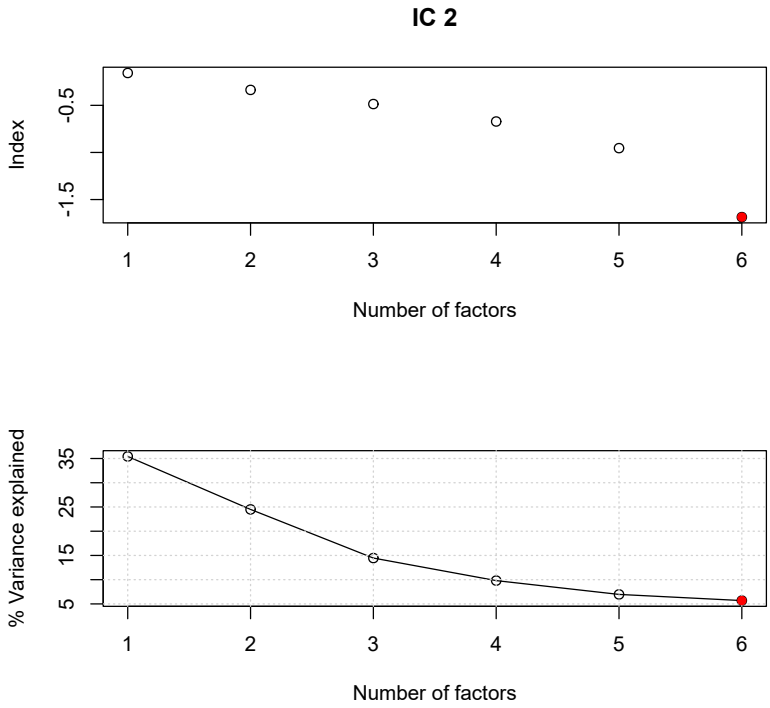
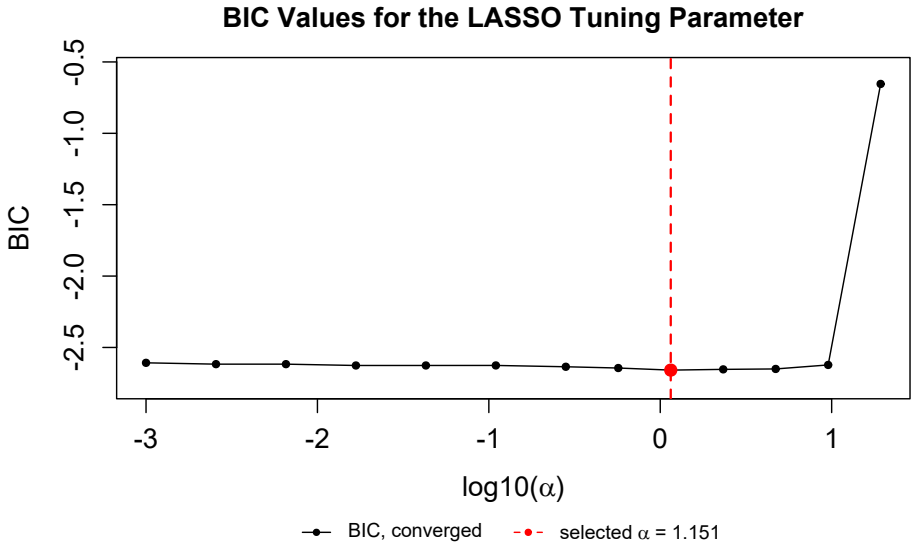


Figure E9: Finland: cold-started BIC across the LASSO tuning parameter



E.4 France

Table E4 compares the three estimators for France. The sparse dynamic factor model uses 31 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 1894.056; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.812, below one, so the estimated factor process is stationary.

Figure E10 shows the residual boxplots, Figure E11 the information criterion and variance explained by number of factors, and Figure E12 the BIC across the LASSO tuning parameter.

Table E4: France: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-633.989	-631.389	-1.04×10^6
Bayesian information criterion	1894.056	1961.349	2.09×10^6
Number of parameters	95	106	49
Nonzero loadings	31	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.036	0.049	0.048
Breusch–Godfrey average p-value	0.428	0.374	0.419
Stability and residuals			
Spectral radius	0.812	0.811	0.811
Residual degrees of freedom	633	622	679

Figure E10: France: residual boxplots by series

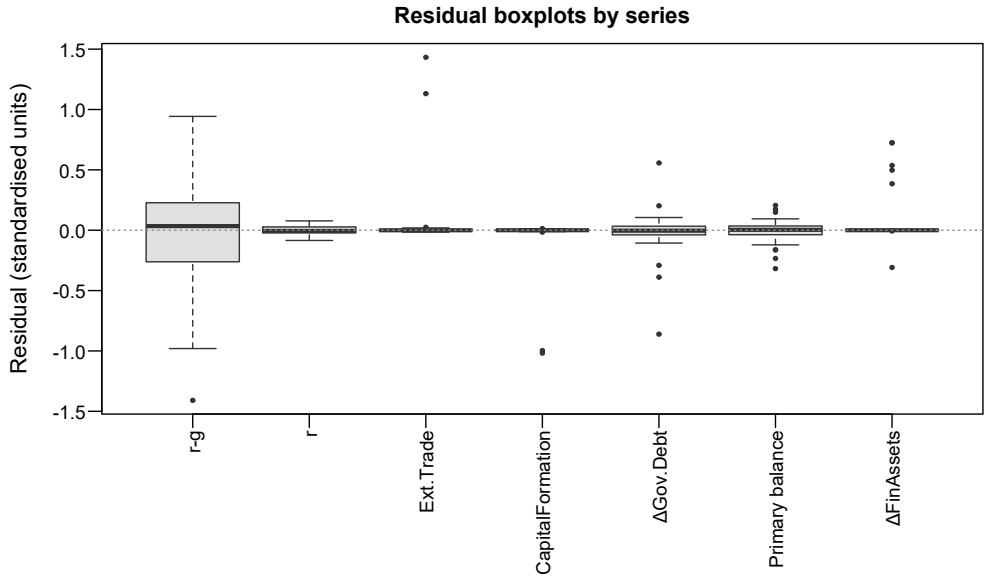


Figure E11: France: information criterion and variance explained by number of factors

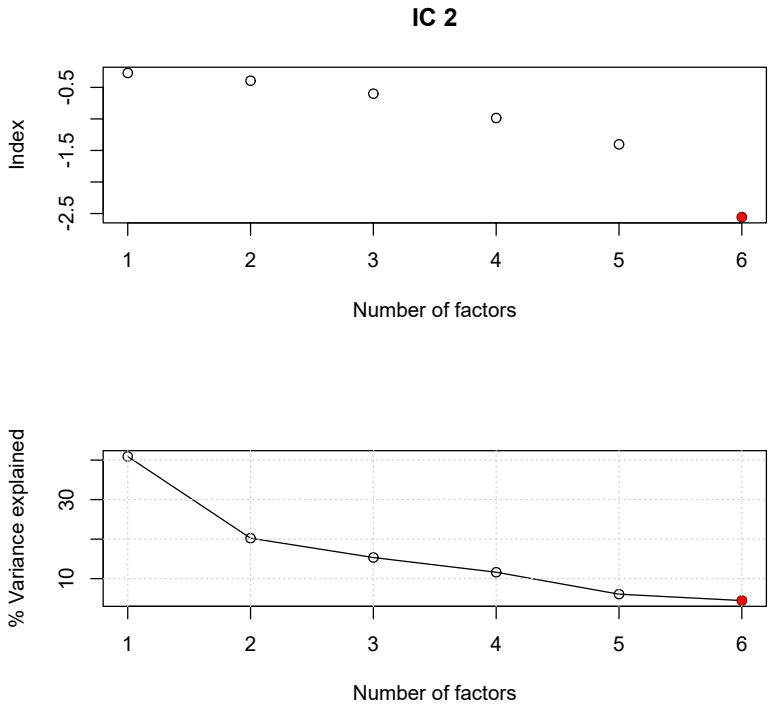
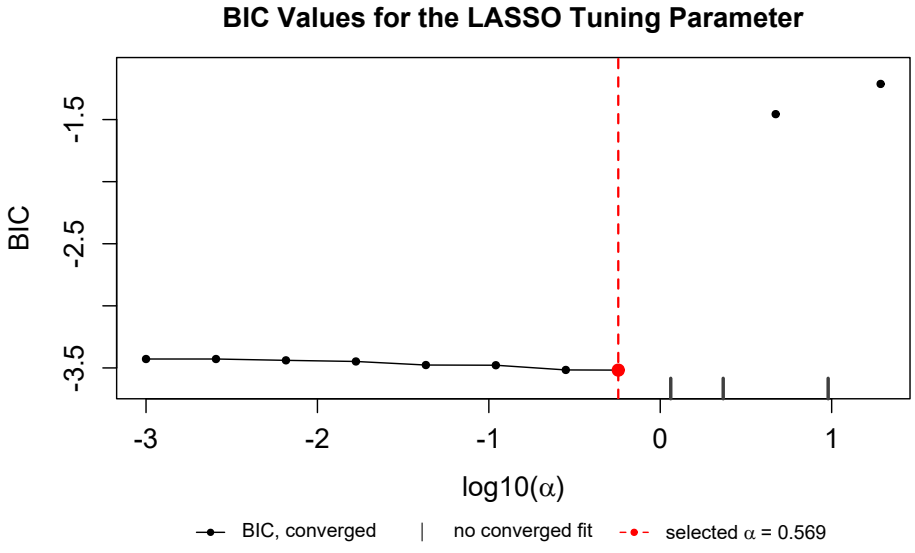


Figure E12: France: cold-started BIC across the LASSO tuning parameter



E.5 Germany

Table E5 compares the three estimators for Germany. The sparse dynamic factor model uses 35 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 1718.847; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.846, below one, so the estimated factor process is stationary.

Figure E13 shows the residual boxplots, Figure E14 the information criterion and variance explained by number of factors, and Figure E15 the BIC across the LASSO tuning parameter.

Table E5: Germany: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-535.145	-534.335	-1.80×10^8
Bayesian information criterion	1718.847	1763.084	3.60×10^8
Number of parameters	99	106	49
Nonzero loadings	35	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.047	0.048	0.061
Breusch–Godfrey average p-value	0.269	0.290	0.429
Stability and residuals			
Spectral radius	0.846	0.845	0.813
Residual degrees of freedom	601	594	651

Figure E13: Germany: residual boxplots by series

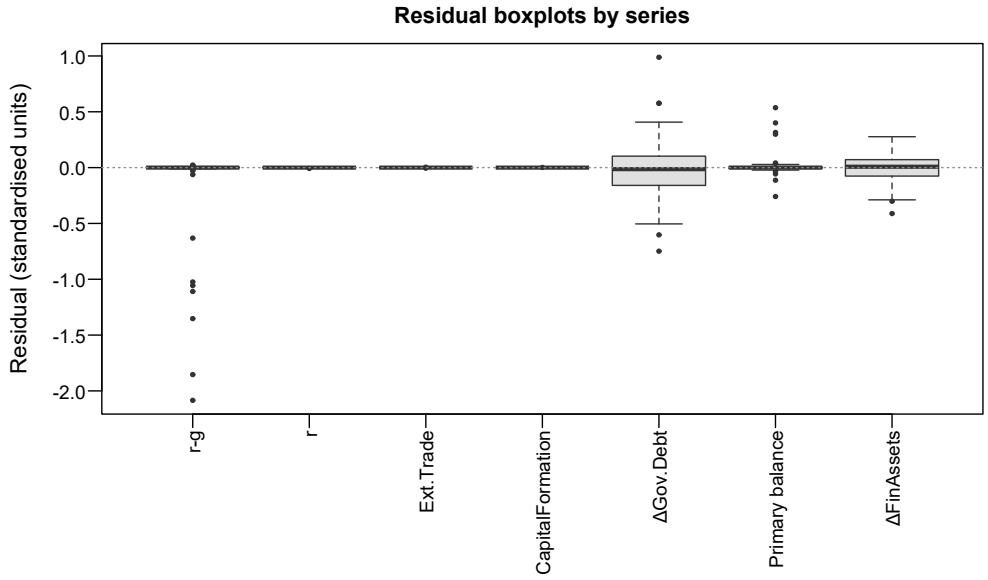


Figure E14: Germany: information criterion and variance explained by number of factors

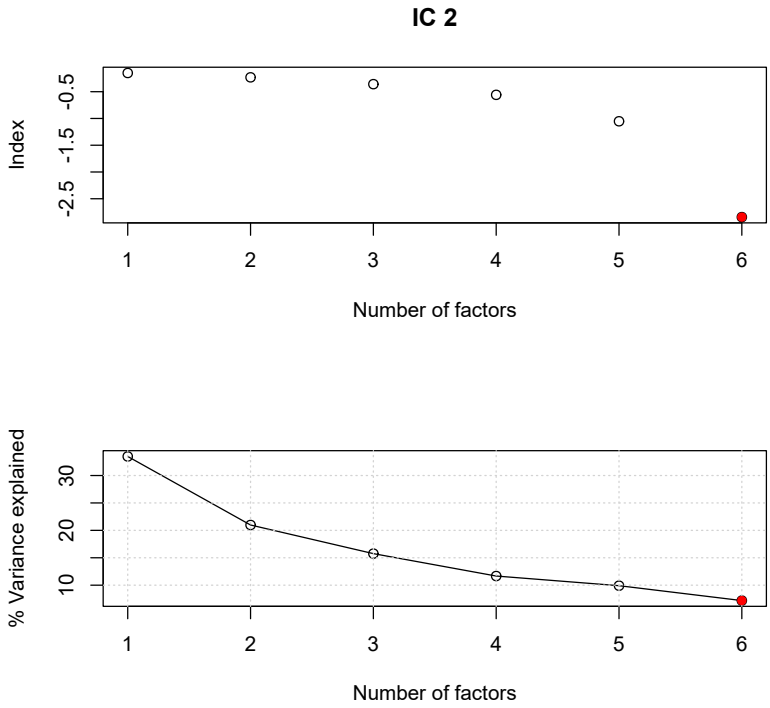
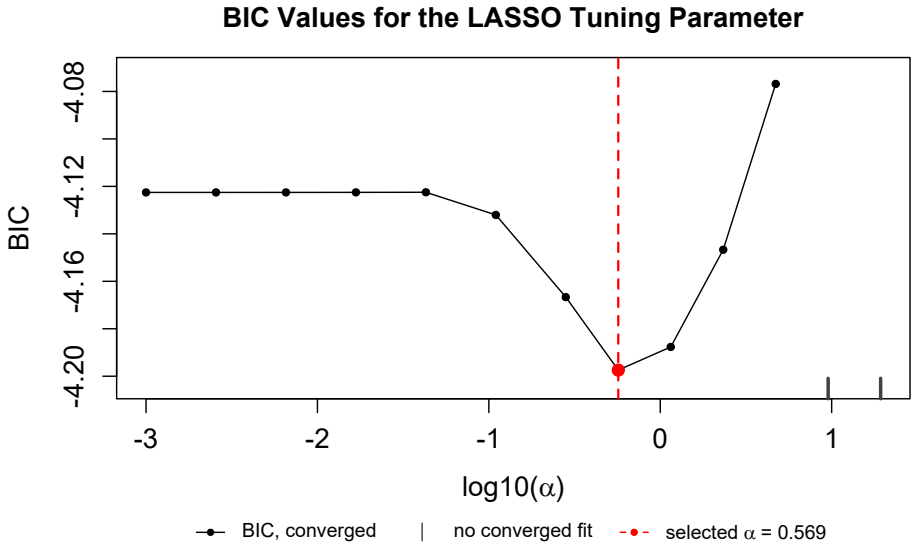


Figure E15: Germany: cold-started BIC across the LASSO tuning parameter



E.6 Greece

Table E6 compares the three estimators for Greece. The sparse dynamic factor model uses 38 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 2232.822; on that criterion the preferred specification is the DFM. The spectral radius of the sparse model is 0.628, below one, so the estimated factor process is stationary.

Figure E16 shows the residual boxplots, Figure E17 the information criterion and variance explained by number of factors, and Figure E18 the BIC across the LASSO tuning parameter.

Table E6: Greece: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-782.306	-765.166	-2.68×10^5
Bayesian information criterion	2232.822	2224.747	5.37×10^5
Number of parameters	102	106	49
Nonzero loadings	38	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.000	0.000	0.000
Breusch–Godfrey average p-value	0.360	0.551	0.365
Stability and residuals			
Spectral radius	0.628	0.652	0.615
Residual degrees of freedom	598	594	651

Figure E16: Greece: residual boxplots by series

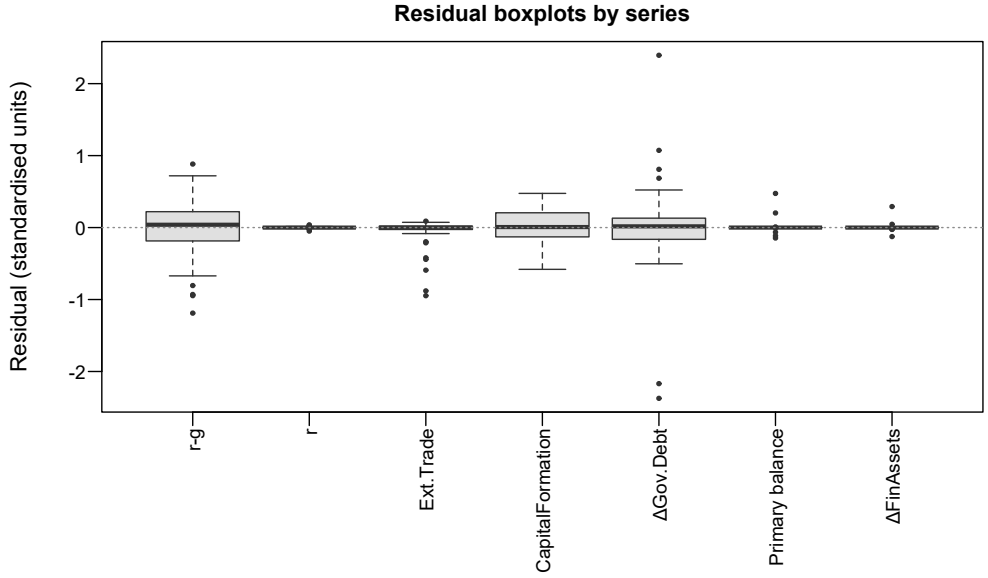


Figure E17: Greece: information criterion and variance explained by number of factors

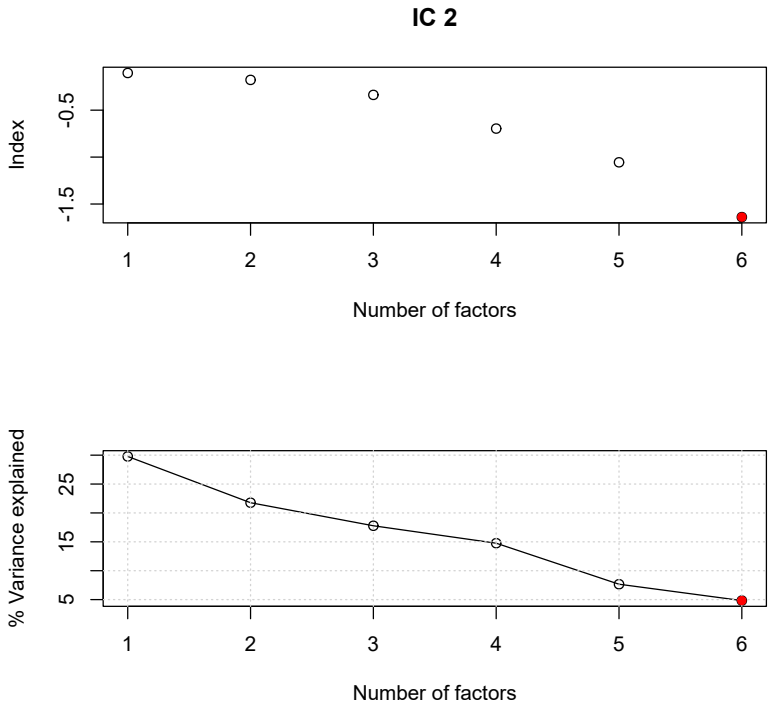
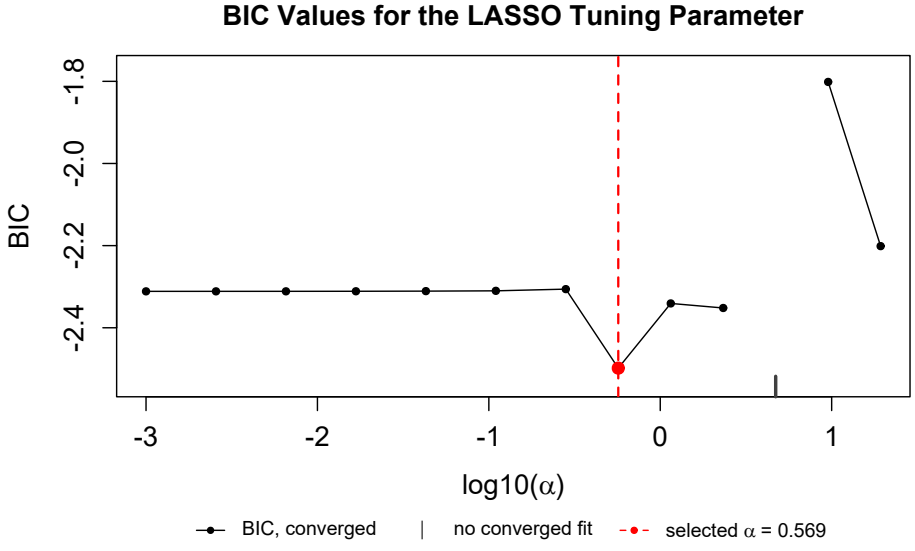


Figure E18: Greece: cold-started BIC across the LASSO tuning parameter



E.7 Ireland

Table E7 compares the three estimators for Ireland. The sparse dynamic factor model uses 29 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 1889.658; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.586, below one, so the estimated factor process is stationary.

Figure E19 shows the residual boxplots, Figure E20 the information criterion and variance explained by number of factors, and Figure E21 the BIC across the LASSO tuning parameter.

Table E7: Ireland: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-640.204	-637.088	-2.40×10^{11}
Bayesian information criterion	1889.658	1968.591	4.80×10^{11}
Number of parameters	93	106	49
Nonzero loadings	29	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.012	0.012	0.012
Breusch–Godfrey average p-value	0.285	0.295	0.295
Stability and residuals			
Spectral radius	0.586	0.586	0.591
Residual degrees of freedom	607	594	651

Figure E19: Ireland: residual boxplots by series

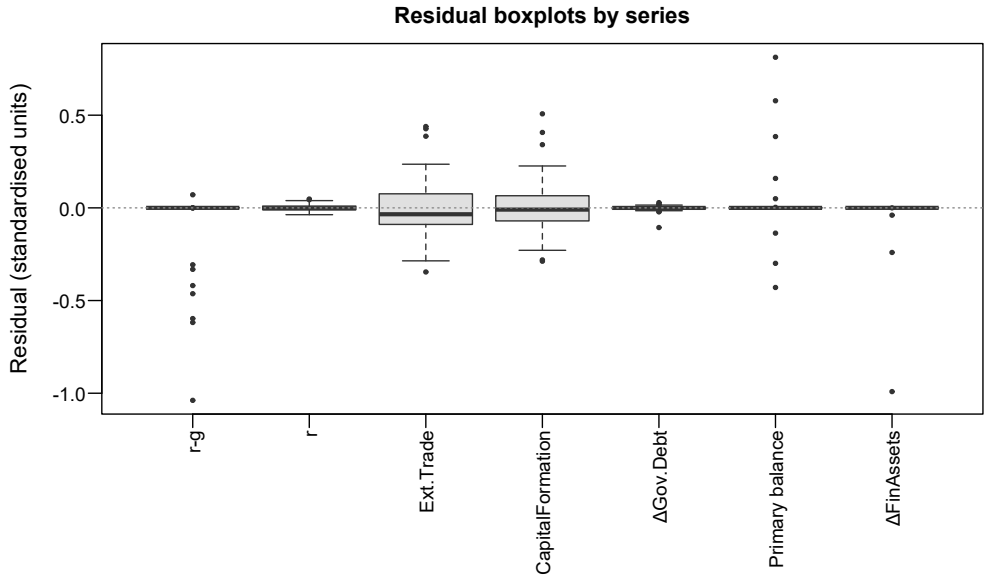


Figure E20: Ireland: information criterion and variance explained by number of factors

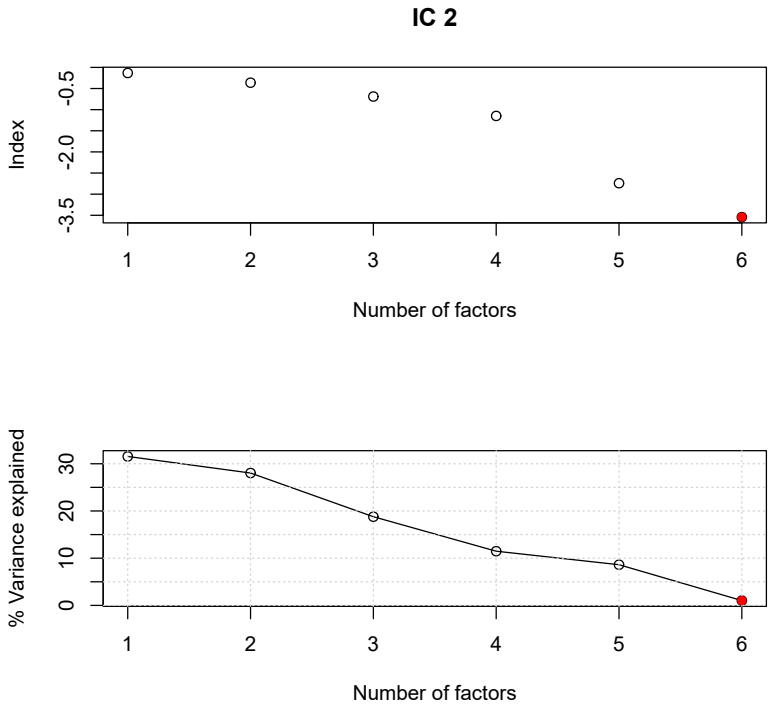
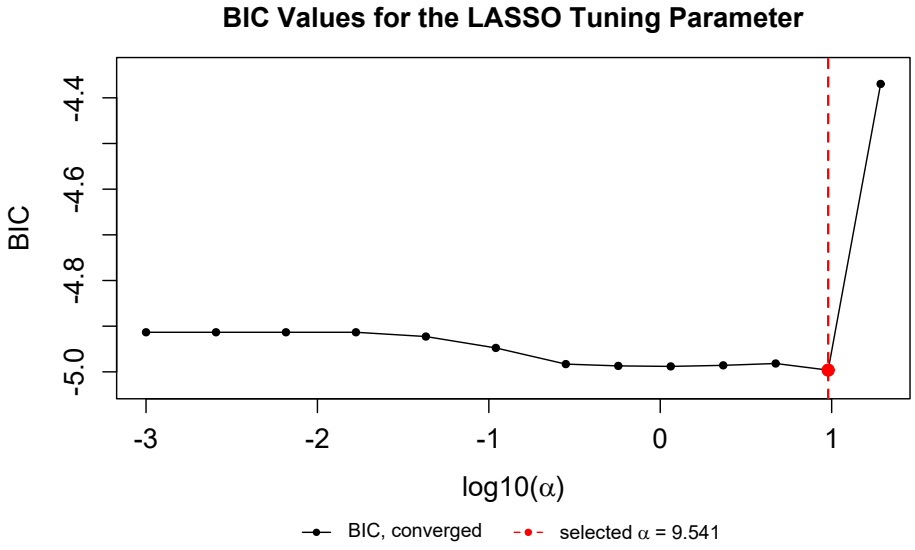


Figure E21: Ireland: cold-started BIC across the LASSO tuning parameter



E.8 Italy

Table E8 compares the three estimators for Italy. The sparse dynamic factor model uses 30 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 1851.771; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.772, below one, so the estimated factor process is stationary.

Figure E22 shows the residual boxplots, Figure E23 the information criterion and variance explained by number of factors, and Figure E24 the BIC across the LASSO tuning parameter.

Table E8: Italy: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-617.985	-606.252	-2.75×10^5
Bayesian information criterion	1851.771	1906.918	5.50×10^5
Number of parameters	94	106	49
Nonzero loadings	30	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.008	0.011	0.008
Breusch–Godfrey average p-value	0.583	0.693	0.675
Stability and residuals			
Spectral radius	0.772	0.755	0.770
Residual degrees of freedom	606	594	651

Figure E22: Italy: residual boxplots by series

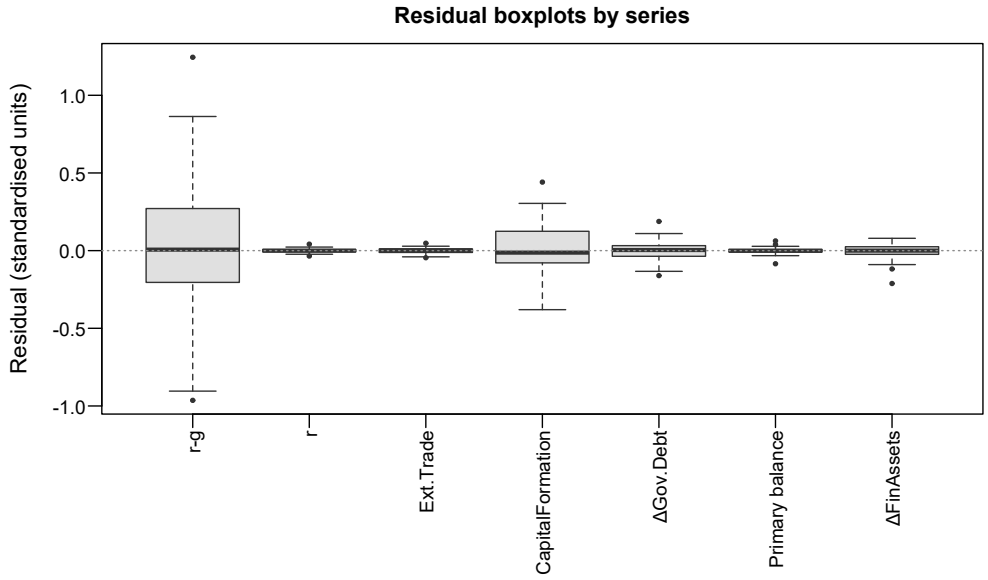


Figure E23: Italy: information criterion and variance explained by number of factors

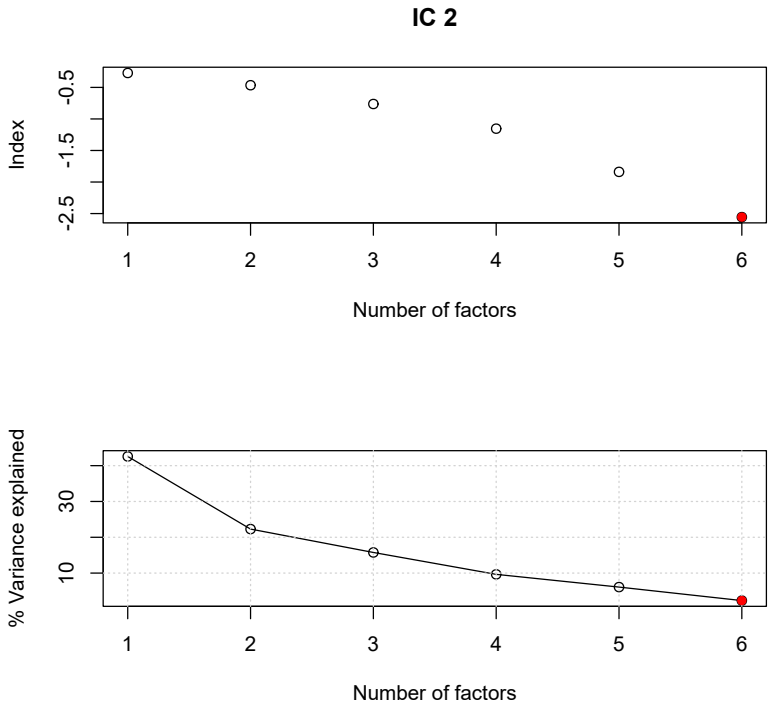
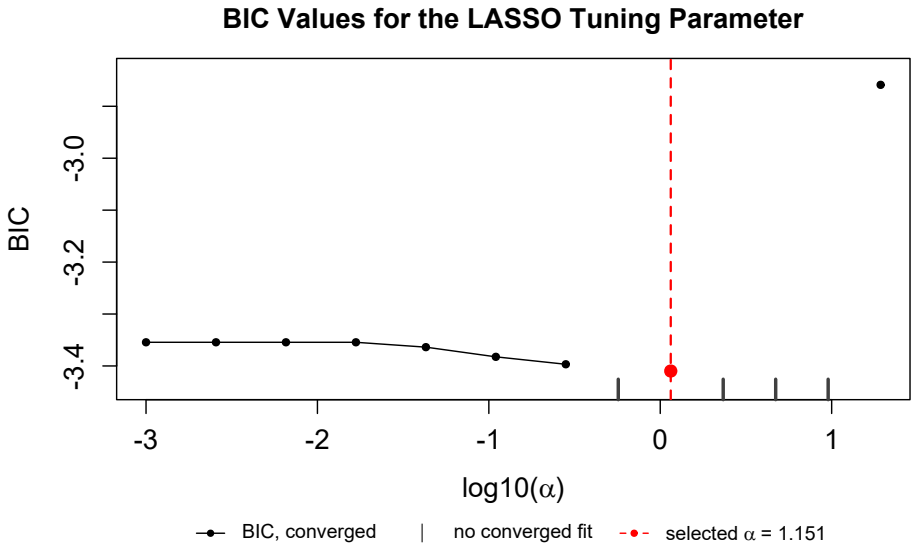


Figure E24: Italy: cold-started BIC across the LASSO tuning parameter



E.9 Latvia

Table E9 compares the three estimators for Latvia. The sparse dynamic factor model uses 36 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 2002.860; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.758, below one, so the estimated factor process is stationary.

Figure E25 shows the residual boxplots, Figure E26 the information criterion and variance explained by number of factors, and Figure E27 the BIC across the LASSO tuning parameter.

Table E9: Latvia: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-673.876	-673.870	-1.78×10^5
Bayesian information criterion	2002.860	2042.154	3.57×10^5
Number of parameters	100	106	49
Nonzero loadings	36	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.001	0.001	0.008
Breusch–Godfrey average p-value	0.431	0.431	0.419
Stability and residuals			
Spectral radius	0.758	0.758	0.776
Residual degrees of freedom	600	594	651

Figure E25: Latvia: residual boxplots by series

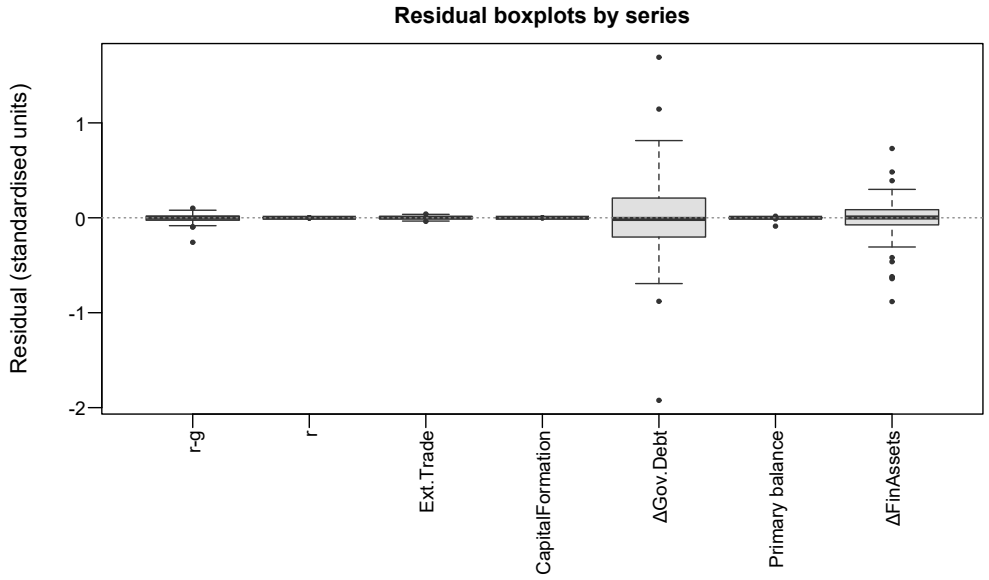


Figure E26: Latvia: information criterion and variance explained by number of factors

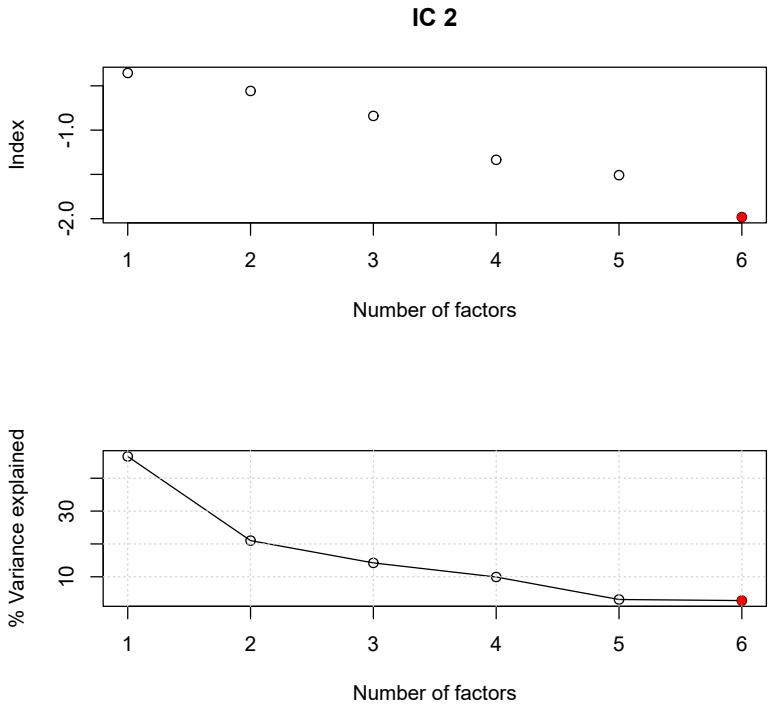
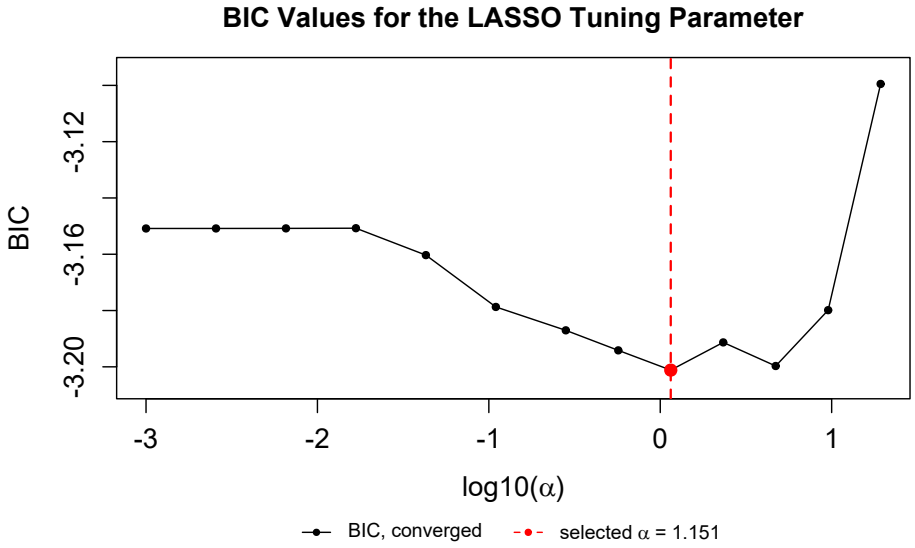


Figure E27: Latvia: cold-started BIC across the LASSO tuning parameter



E.10 Luxembourg

Table E10 compares the three estimators for Luxembourg. The sparse dynamic factor model uses 35 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 2227.951; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.797, below one, so the estimated factor process is stationary.

Figure E28 shows the residual boxplots, Figure E29 the information criterion and variance explained by number of factors, and Figure E30 the BIC across the LASSO tuning parameter.

Table E10: Luxembourg: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-789.697	-787.978	-1.92×10^5
Bayesian information criterion	2227.951	2270.370	3.84×10^5
Number of parameters	99	106	49
Nonzero loadings	35	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.000	0.001	0.001
Breusch–Godfrey average p-value	0.242	0.213	0.253
Stability and residuals			
Spectral radius	0.797	0.793	0.595
Residual degrees of freedom	601	594	651

Figure E28: Luxembourg: residual boxplots by series

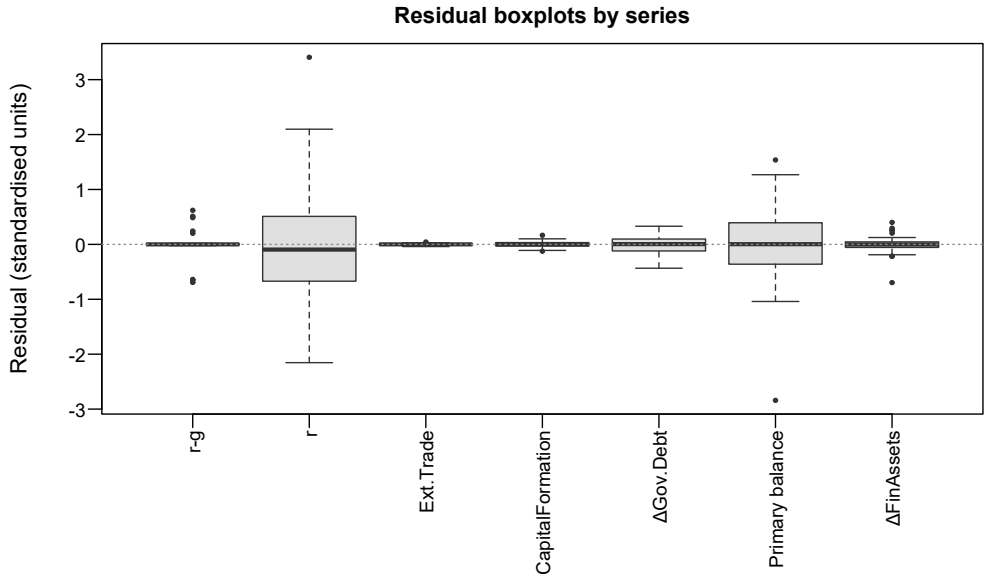


Figure E29: Luxembourg: information criterion and variance explained by number of factors

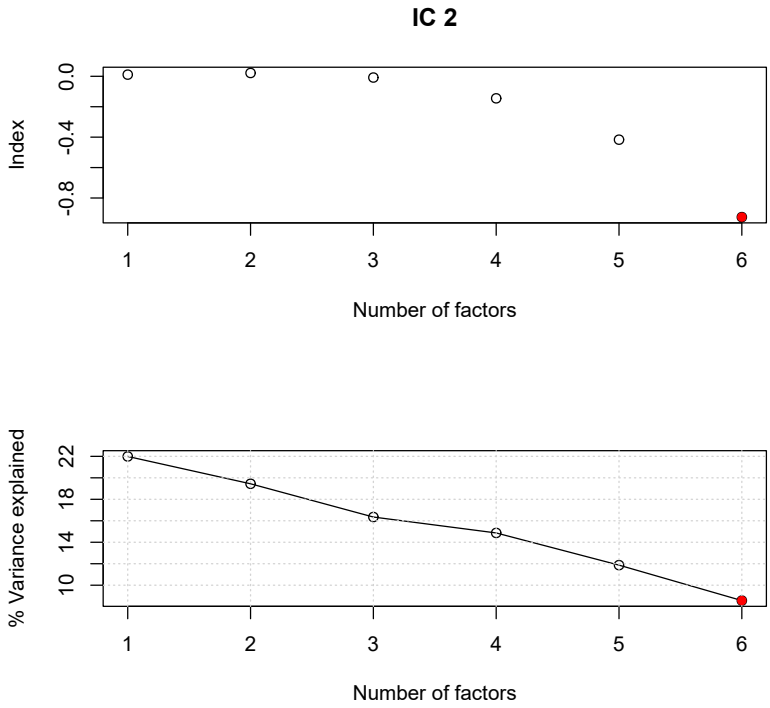
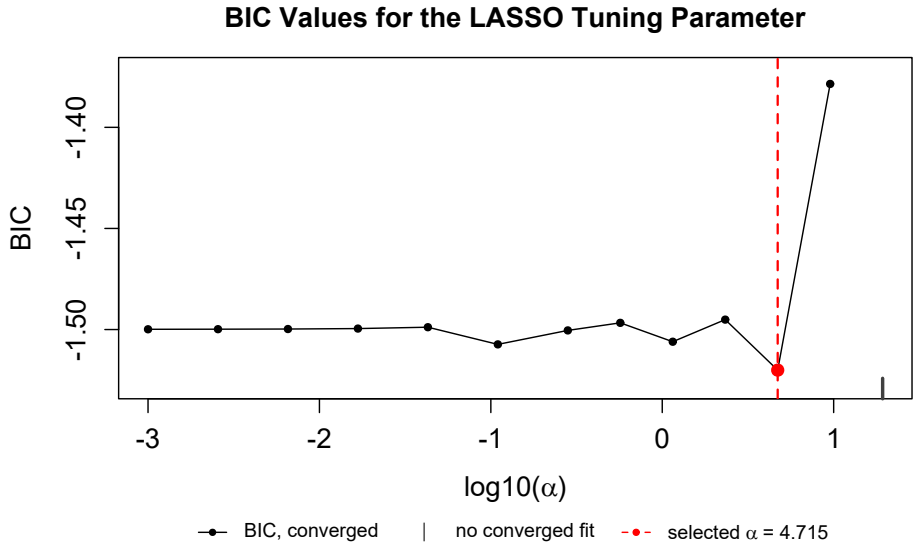


Figure E30: Luxembourg: cold-started BIC across the LASSO tuning parameter



E.11 Netherlands

Table E11 compares the three estimators for Netherlands. The sparse dynamic factor model uses 34 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 2003.290; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.683, below one, so the estimated factor process is stationary.

Figure E31 shows the residual boxplots, Figure E32 the information criterion and variance explained by number of factors, and Figure E33 the BIC across the LASSO tuning parameter.

Table E11: Netherlands: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-680.642	-679.022	-1.42×10^5
Bayesian information criterion	2003.290	2052.458	2.84×10^5
Number of parameters	98	106	49
Nonzero loadings	34	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.023	0.023	0.036
Breusch–Godfrey average p-value	0.633	0.635	0.587
Stability and residuals			
Spectral radius	0.683	0.685	0.657
Residual degrees of freedom	602	594	651

Figure E31: Netherlands: residual boxplots by series

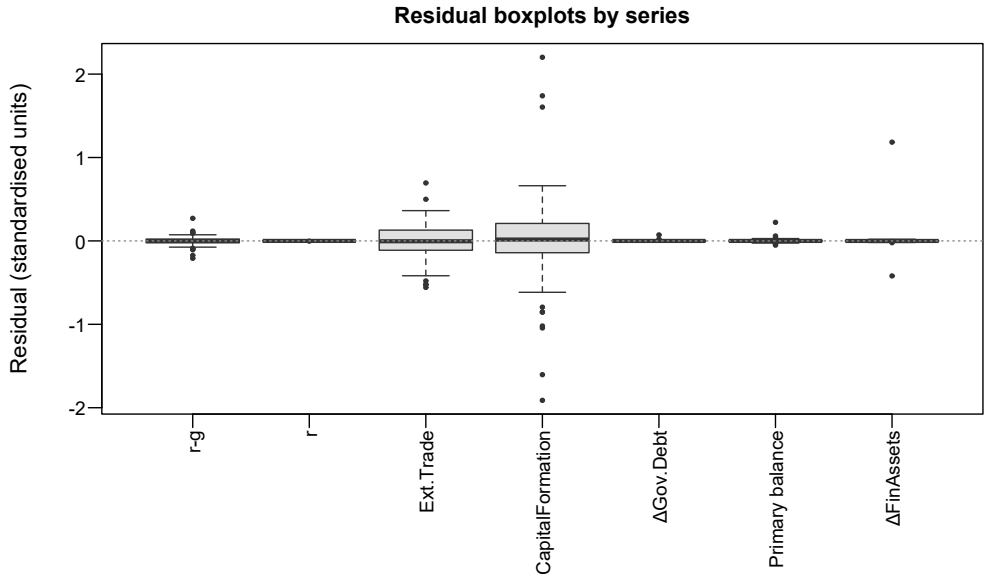


Figure E32: Netherlands: information criterion and variance explained by number of factors

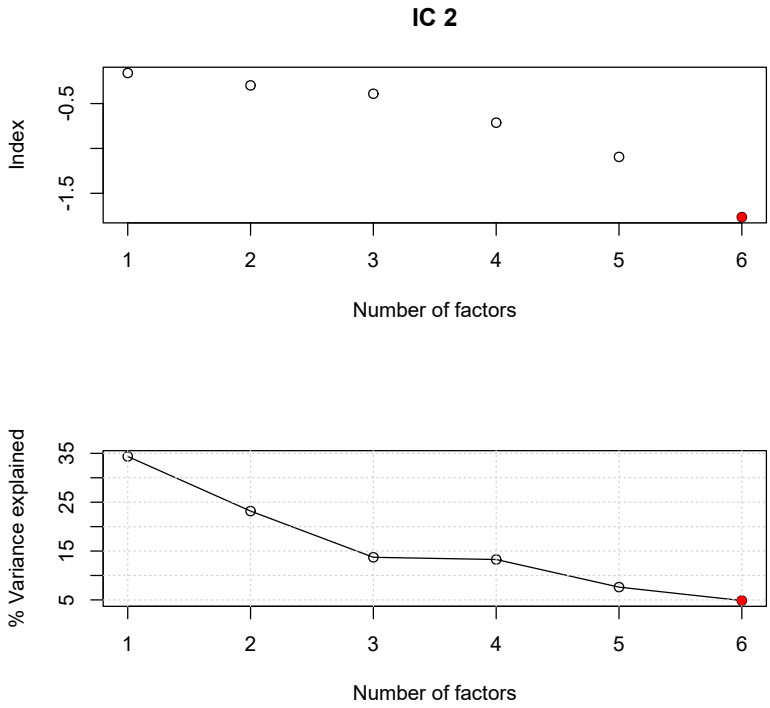
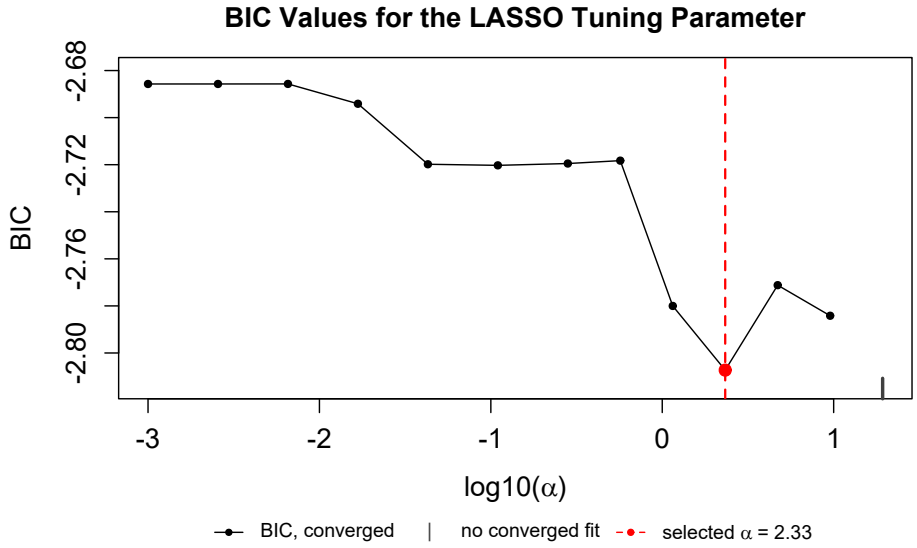


Figure E33: Netherlands: cold-started BIC across the LASSO tuning parameter



E.12 Portugal

Table E12 compares the three estimators for Portugal. The sparse dynamic factor model uses 35 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 2080.639; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.715, below one, so the estimated factor process is stationary.

Figure E34 shows the residual boxplots, Figure E35 the information criterion and variance explained by number of factors, and Figure E36 the BIC across the LASSO tuning parameter.

Table E12: Portugal: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-716.538	-715.261	-8.87×10^5
Bayesian information criterion	2080.639	2123.870	1.77×10^6
Number of parameters	99	106	49
Nonzero loadings	35	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.030	0.020	0.004
Breusch–Godfrey average p-value	0.633	0.535	0.548
Stability and residuals			
Spectral radius	0.715	0.707	0.703
Residual degrees of freedom	594	587	644

Figure E34: Portugal: residual boxplots by series

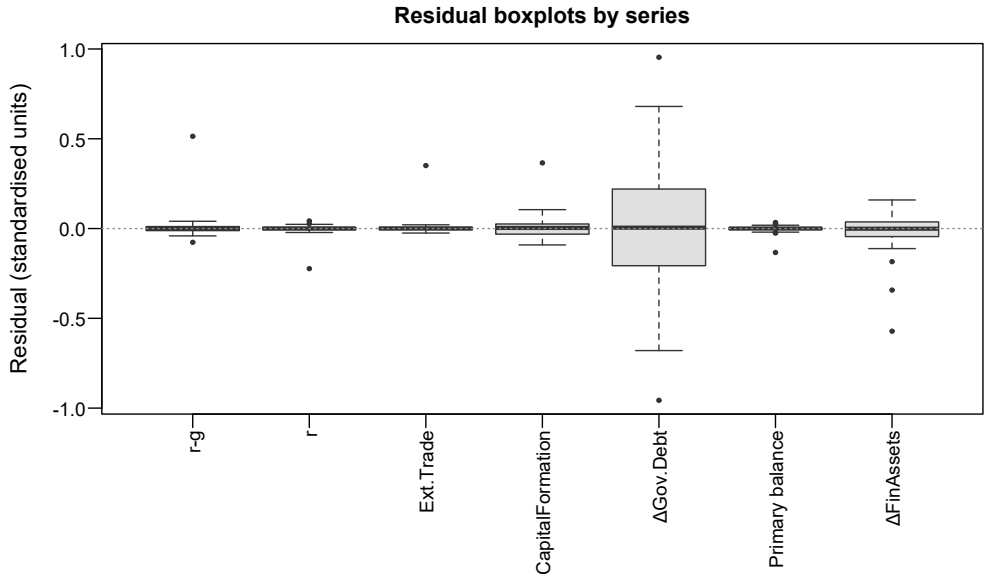


Figure E35: Portugal: information criterion and variance explained by number of factors

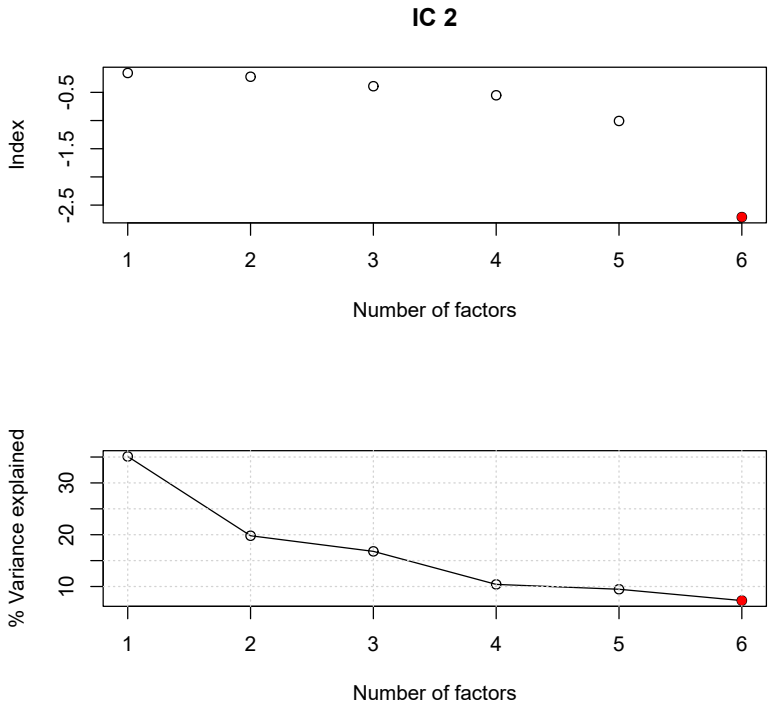
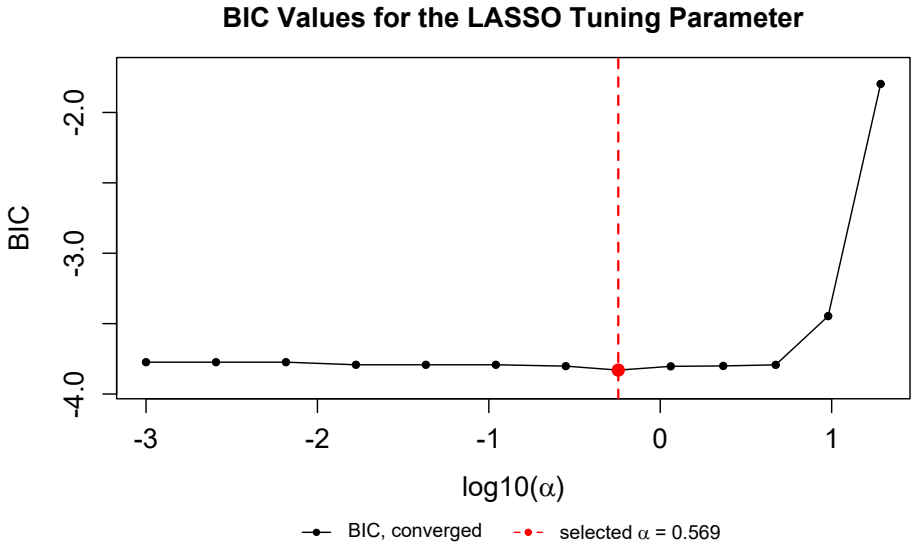


Figure E36: Portugal: cold-started BIC across the LASSO tuning parameter



E.13 Slovakia

Table E13 compares the three estimators for Slovakia. The sparse dynamic factor model uses 34 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 1908.480; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.825, below one, so the estimated factor process is stationary.

Figure E37 shows the residual boxplots, Figure E38 the information criterion and variance explained by number of factors, and Figure E39 the BIC across the LASSO tuning parameter.

Table E13: Slovakia: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-633.237	-633.255	-56242.644
Bayesian information criterion	1908.480	1960.924	1.13×10^5
Number of parameters	98	106	49
Nonzero loadings	34	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.065	0.068	0.007
Breusch–Godfrey average p-value	0.384	0.341	0.218
Stability and residuals			
Spectral radius	0.825	0.827	0.748
Residual degrees of freedom	602	594	651

Figure E37: Slovakia: residual boxplots by series

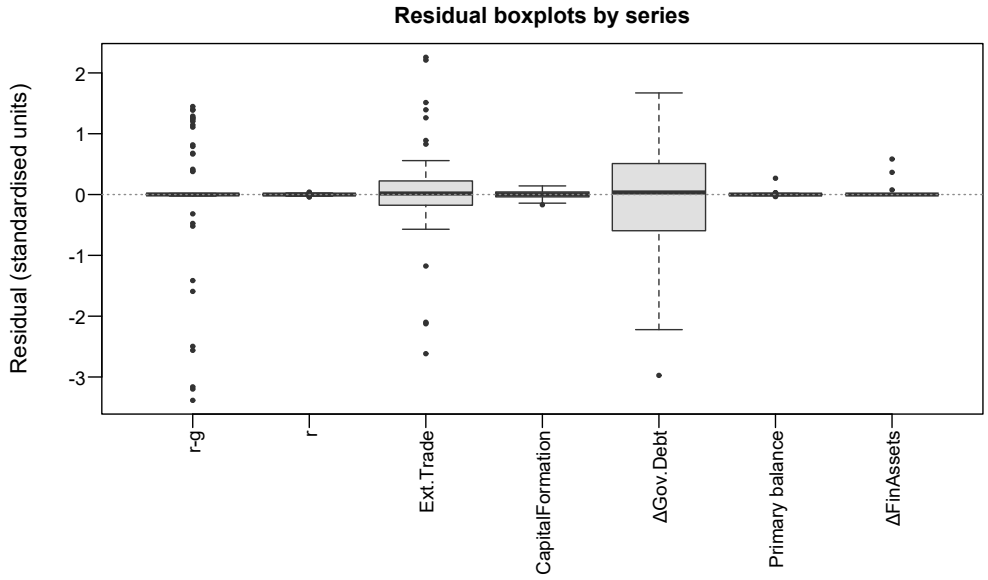


Figure E38: Slovakia: information criterion and variance explained by number of factors

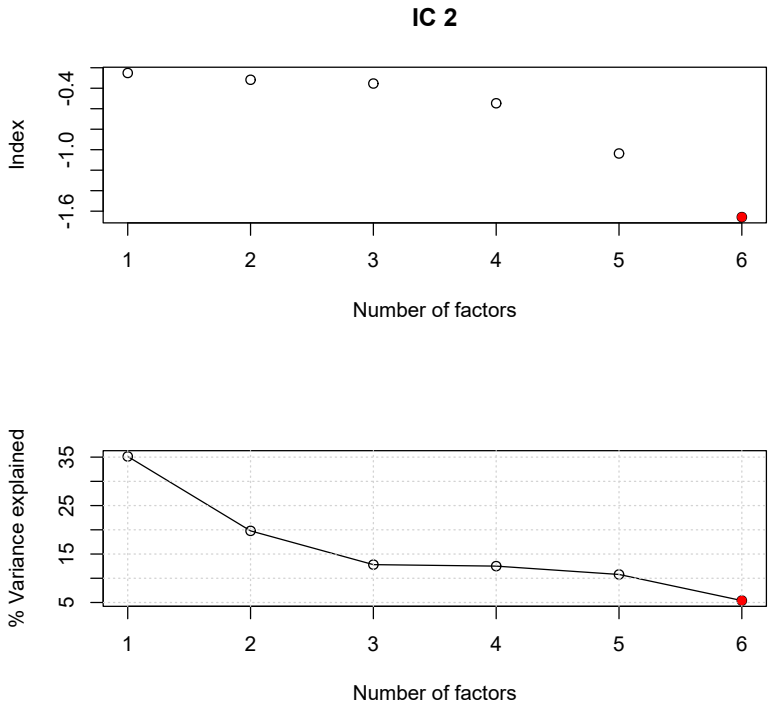
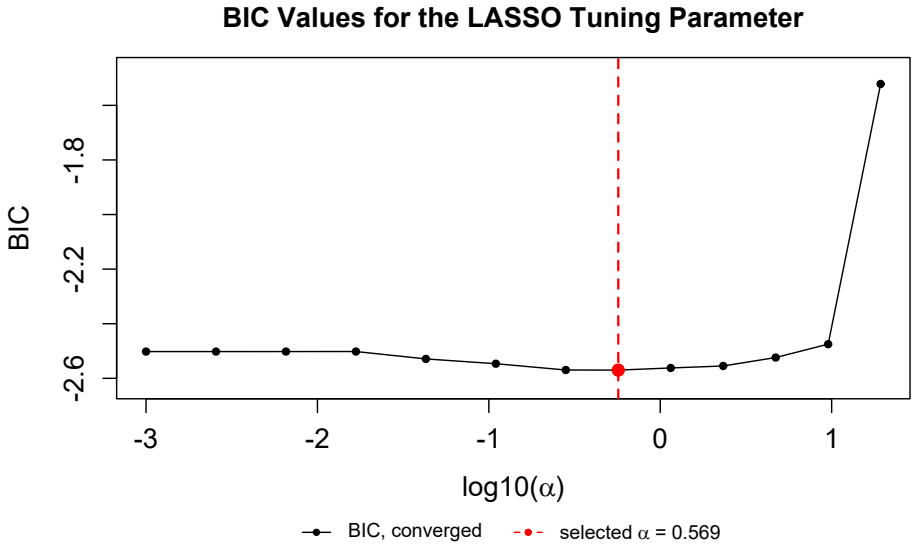


Figure E39: Slovakia: cold-started BIC across the LASSO tuning parameter



E.14 Slovenia

Table E14 compares the three estimators for Slovenia. The sparse dynamic factor model uses 33 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 2085.517; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.658, below one, so the estimated factor process is stationary.

Figure E40 shows the residual boxplots, Figure E41 the information criterion and variance explained by number of factors, and Figure E42 the BIC across the LASSO tuning parameter.

Table E14: Slovenia: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-725.031	-723.232	-73893.384
Bayesian information criterion	2085.517	2140.878	1.48×10^5
Number of parameters	97	106	49
Nonzero loadings	33	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.004	0.004	0.004
Breusch–Godfrey average p-value	0.483	0.461	0.564
Stability and residuals			
Spectral radius	0.658	0.656	0.661
Residual degrees of freedom	603	594	651

Figure E40: Slovenia: residual boxplots by series

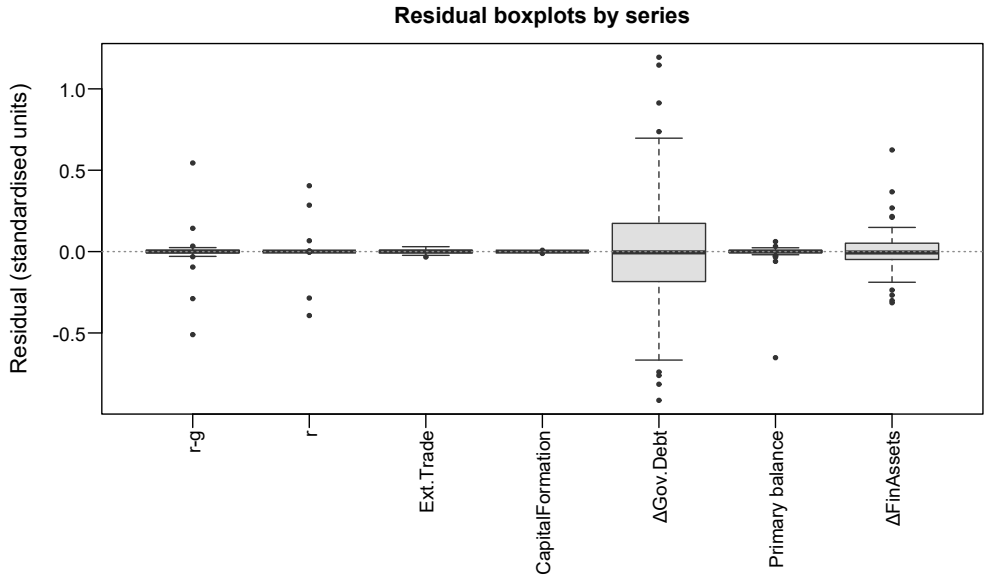


Figure E41: Slovenia: information criterion and variance explained by number of factors

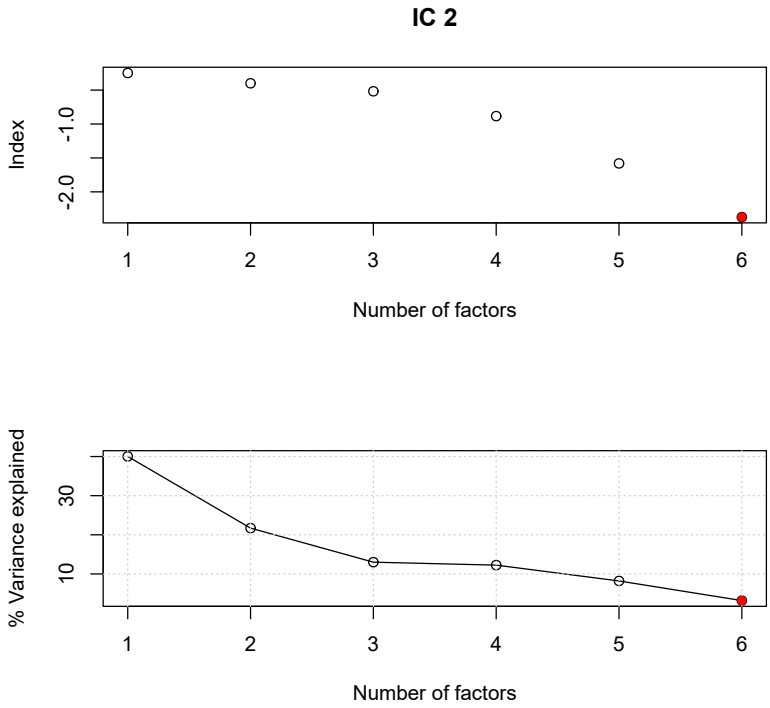
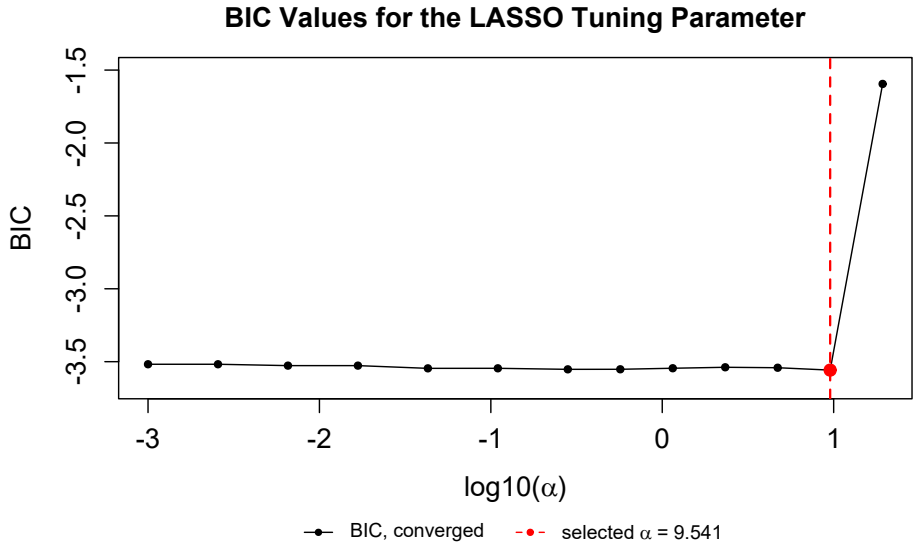


Figure E42: Slovenia: cold-started BIC across the LASSO tuning parameter



E.15 Spain

Table E15 compares the three estimators for Spain. The sparse dynamic factor model uses 30 nonzero loadings against 42 for the unrestricted model, and attains a

Bayesian information criterion of 1996.038; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.476, below one, so the estimated factor process is stationary.

Figure E43 shows the residual boxplots, Figure E44 the information criterion and variance explained by number of factors, and Figure E45 the BIC across the LASSO tuning parameter.

Table E15: Spain: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-690.590	-685.507	-3.11×10^7
Bayesian information criterion	1996.038	2064.364	6.22×10^7
Number of parameters	94	106	49
Nonzero loadings	30	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.001	0.001	0.002
Breusch–Godfrey average p-value	0.573	0.509	0.545
Stability and residuals			
Spectral radius	0.476	0.474	0.468
Residual degrees of freedom	599	587	644

Figure E43: Spain: residual boxplots by series

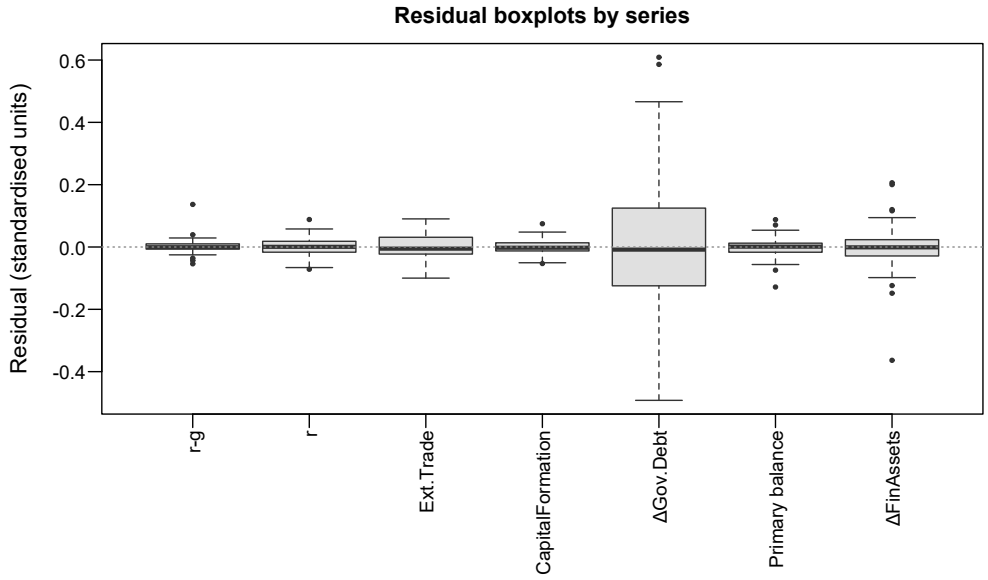


Figure E44: Spain: information criterion and variance explained by number of factors

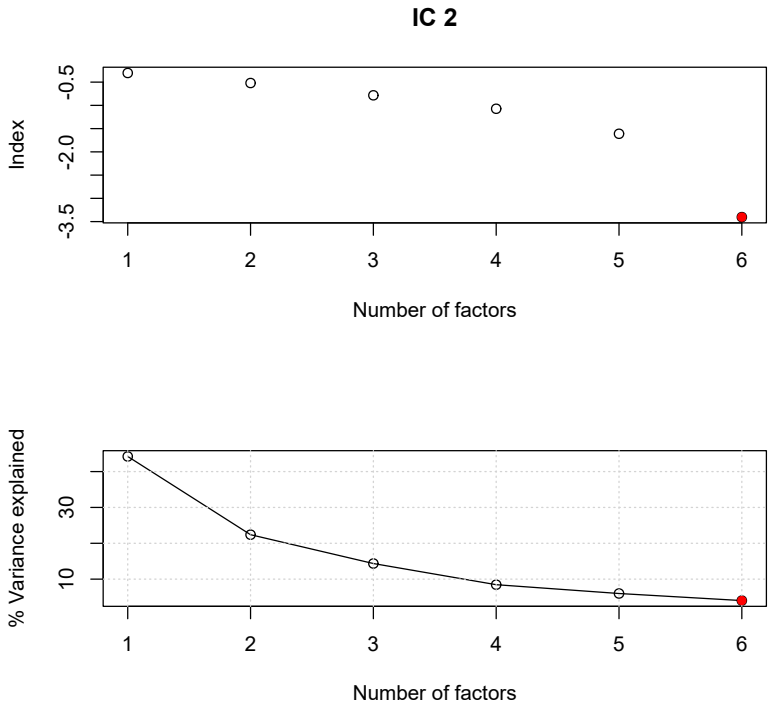
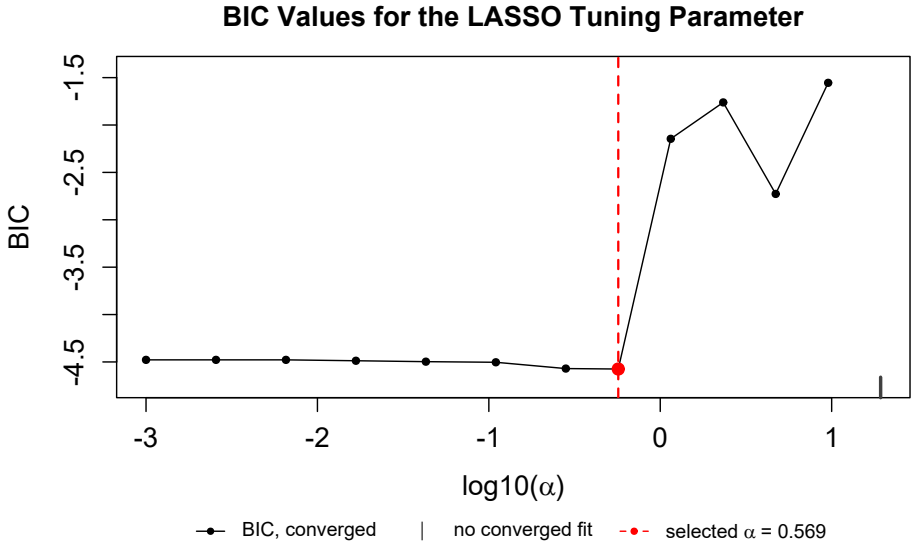


Figure E45: Spain: cold-started BIC across the LASSO tuning parameter



E.16 United Kingdom

Table E16 compares the three estimators for United Kingdom. The sparse dynamic factor model uses 30 nonzero loadings against 42 for the unrestricted model,

and attains a Bayesian information criterion of 2116.798; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.529, below one, so the estimated factor process is stationary.

Figure E46 shows the residual boxplots, Figure E47 the information criterion and variance explained by number of factors, and Figure E48 the BIC across the LASSO tuning parameter.

Table E16: United Kingdom: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-750.498	-746.864	-3.98×10^5
Bayesian information criterion	2116.798	2188.143	7.96×10^5
Number of parameters	94	106	49
Nonzero loadings	30	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.024	0.029	0.030
Breusch–Godfrey average p-value	0.388	0.374	0.334
Stability and residuals			
Spectral radius	0.529	0.525	0.529
Residual degrees of freedom	606	594	651

Figure E46: United Kingdom: residual boxplots by series

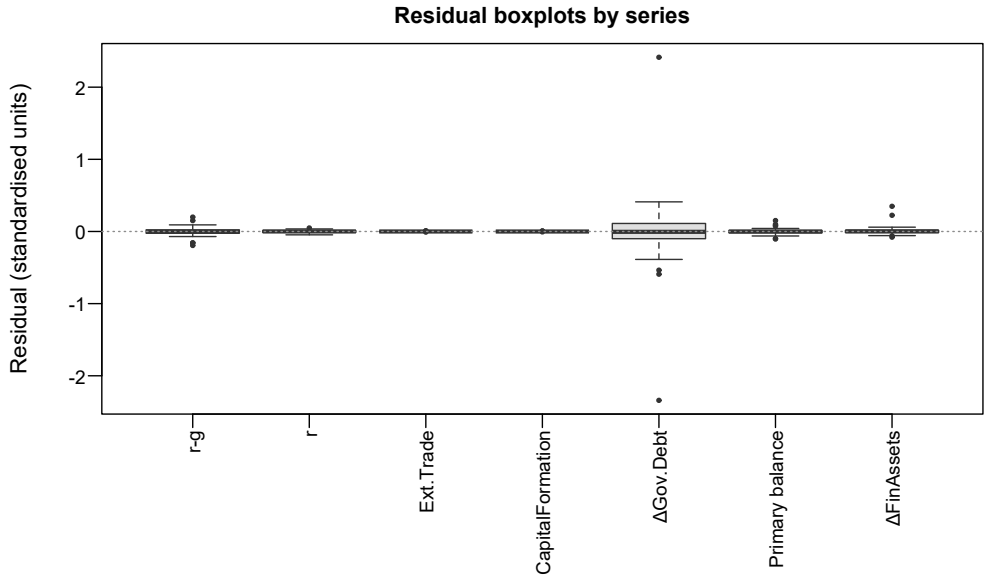


Figure E47: United Kingdom: information criterion and variance explained by number of factors

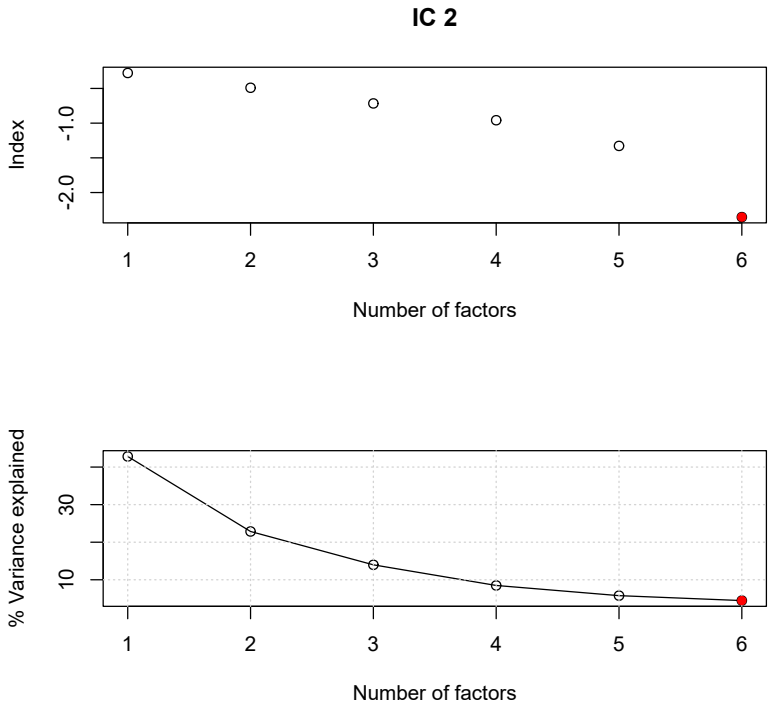
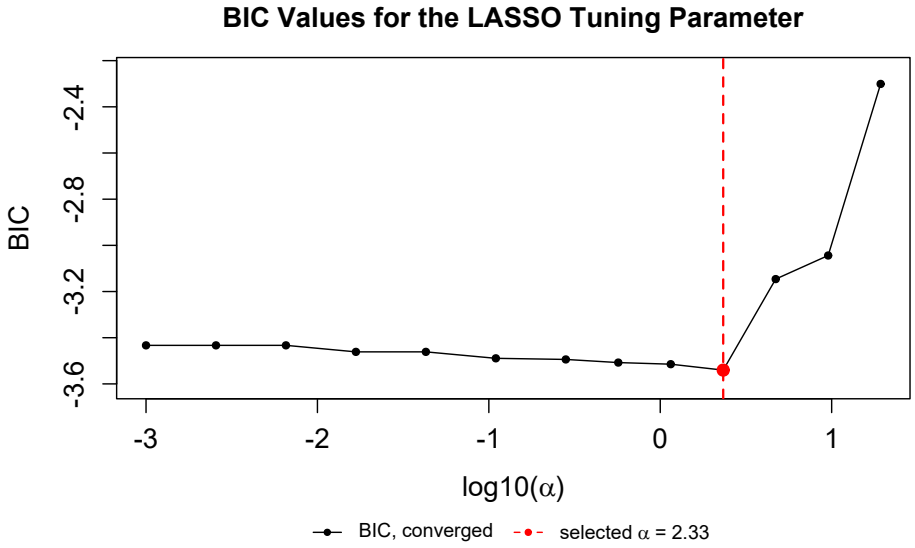


Figure E48: United Kingdom: cold-started BIC across the LASSO tuning parameter



E.17 United States

Table E17 compares the three estimators for United States. The sparse dynamic factor model uses 30 nonzero loadings against 42 for the unrestricted model, and

attains a Bayesian information criterion of 2078.479; on that criterion the preferred specification is the Sparse DFM. The spectral radius of the sparse model is 0.693, below one, so the estimated factor process is stationary.

Figure E49 shows the residual boxplots, Figure E50 the information criterion and variance explained by number of factors, and Figure E51 the BIC across the LASSO tuning parameter.

Table E17: United States: model comparison diagnostics

Diagnostic	*Sparse DFM*	*DFM*	*PCA*
Model fit			
Log-likelihood	-731.811	-722.514	-1.83×10^6
Bayesian information criterion	2078.479	2138.378	3.67×10^6
Number of parameters	94	106	49
Nonzero loadings	30	42	42
Sphericity and residual autocorrelation			
Bartlett χ^2	–	–	–
Bartlett d.f.	21	21	21
Bartlett p-value	–	–	–
Ljung–Box average p-value	0.021	0.021	0.021
Breusch–Godfrey average p-value	0.404	0.403	0.434
Stability and residuals			
Spectral radius	0.693	0.701	0.664
Residual degrees of freedom	599	587	644

Figure E49: United States: residual boxplots by series

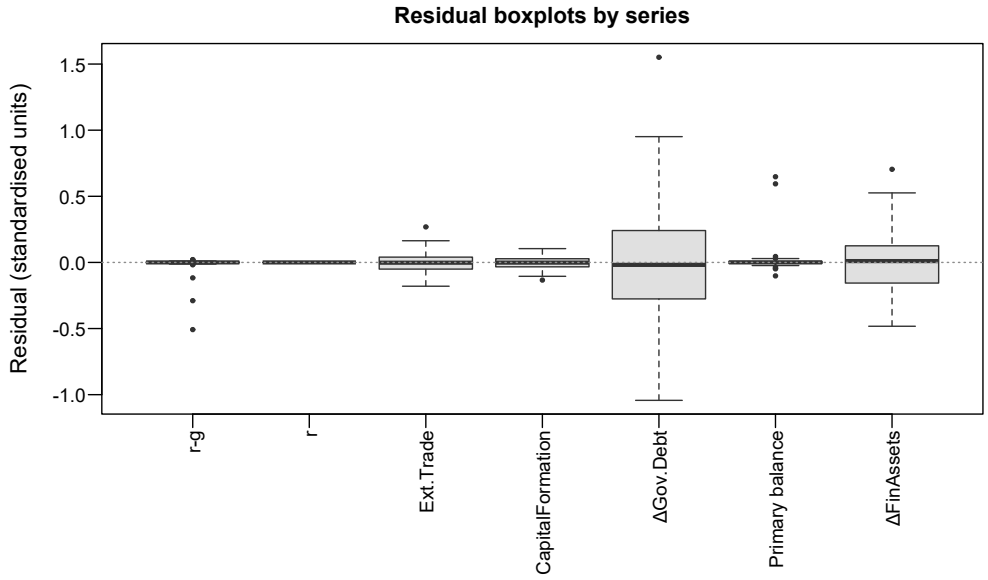


Figure E50: United States: information criterion and variance explained by number of factors

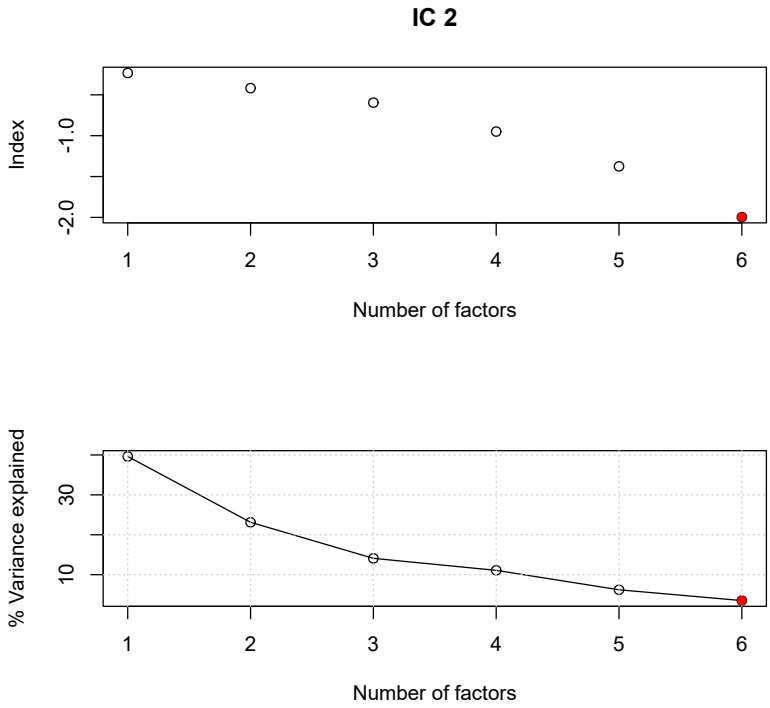
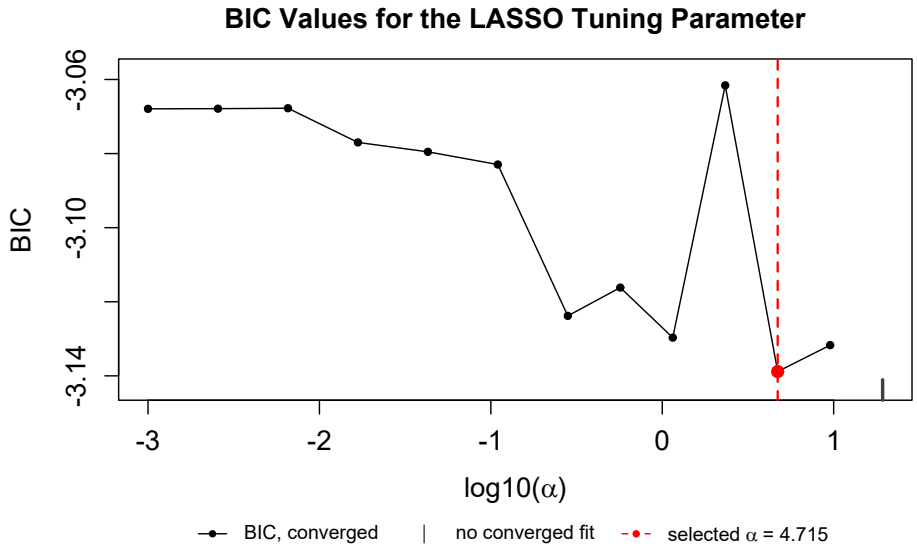


Figure E51: United States: cold-started BIC across the LASSO tuning parameter



Appendix F Pooled specification

The pooled estimates reported in Section 6 come from a single model fitted to all seventeen countries at once. The seven series of each country are stacked into one panel, each series first transformed by its own stationarity recipe (Appendix C) and then standardised. The specification this paper identifies from stops before the pandemic: $N = 119$ series observed over $T = 79$ quarters, 2000Q2–2019Q4, with $r = 3$ factors. The full sample, $T = 104$ quarters to 2026Q1 at $r = 4$, is reported throughout as a companion and never the other way round; what that choice costs – the pandemic, the energy shock and the rule reforms that followed them – is discussed in Section 7 rather than left for a reader to notice. The panel is the *union* of the seventeen country samples, with a missing entry wherever a country has no observation for a quarter; the Kalman filter of the estimator absorbs those ragged edges, and they amount to 7.5% of the 9,401 cells. The alternative of a balanced panel – the quarters in which every one of the 119 series is observed – would discard 38.0% of the quarters, cutting T from 79 to 49 for every country: it runs from 2007Q4, the quarter from which the series of Slovakia are all observed, to 2019Q4, which is the end of the sample rather than a country's last observation. That loss buys nothing, and it is worth being exact about what it does not buy: the two panels do *not* agree on the count of countries in which financial assets and $r - g$ have absolute correlations above 0.3 with the same factor, which is 2 of the 17 on the union panel and 3 on the balanced one (12 and 14 on the companion). The union panel is preferred on efficiency; the disagreement is one of four reasons, set out in the second paragraph below and in the note to Table F2, why that count carries no weight in this paper. The number of factors is not imposed. Six criteria are computed on each space over $k = 1, \ldots, 15$ – the two information criteria of Bai and Ng (2002), the eigenvalue and growth ratios of Ahn and Horenstein, the edge distribution of Onatski and a Marchenko–Pastur edge count – and they agree on the specification the paper identifies from and disagree on the companion. On the pre-2020 space four of the six select three (Bai–Ng IC_{p1}, Bai–Ng IC_{p2}, Onatski ED and the Marchenko–Pastur edge count), with the Bai–Ng margins for $k = 4$ against $k = 3$ at +0.0035 and +0.0134, both favouring three; the dissenters are Ahn–Horenstein GR and the eigenvalue ratio ER, and it returns one on every space, which is the documented symptom of a fat bulk in a panel with cross-correlated idiosyncratic errors rather than a reading of the factor structure. On the full sample the same battery splits Bai–Ng IC_{p1} at four, Bai–Ng IC_{p2} at four, Ahn–Horenstein GR at one, Onatski ED at two, the Marchenko–Pastur edge count at four and the eigenvalue ratio ER at one: the two Bai–Ng criteria both select four. Because the estimator normalises each loading column to approximately unit norm across the

N series, the equal-share loading is $1/\sqrt{N}$ and therefore falls from 0.378 at $N = 7$ to 0.092 at $N = 119$; the fixed 0.3 cut of the country-by-country design cannot be carried over literally, and the scale-free criterion $|\text{corr}(\text{series}, \text{factor})| > 0.3$ is used in its place, with the N-rescaled loading cut $0.3\sqrt{7/N} = 0.073$ reported alongside it. Table F1 profiles the factors of both specifications, Table F2 gives each country's correlations with the identifying ones, Table F3 reports what the common component explains variable by variable on both, and Tables F4, F7 and F5 test the restriction that the country-by-country design cannot test: that every country's loading block is the same. The dispersion test applies a deliberately strict composite rule: D_R and D_C, the two statistics free of the scale of Λ, of which the second is also free of its rotation and the first only of the signed permutations the penalty leaves open, must both reject in every cell of the null, so the composite p-value is the largest over the cells. That form was fixed before any cell of the identifying specification was computed. The null generator it is evaluated under, and the sweep described next, were fixed only after the cells of Table F4 were known, because the original generator itself carried loading dispersion (see the note to Table F4); the rebuilt test is a disclosed repair of that defect, not a blind one. The size of the common block, a nuisance parameter the restriction leaves free, is swept over a confidence set in the manner of Berger and Boos (1994), the composite then being the largest p-value over the cells inside that set plus $\gamma = 0.01$ (Table F7). On that null the composite is $p = 0.2313$, so the restriction is recorded as not rejected; under the original generator it is $p = 0.2773$, with the same verdict. Under the original generator the individual statistics nonetheless reject almost uniformly. The scale-free dispersion of the correlation blocks across countries, $D_R = 20.39$, exceeds the largest of 10,000 simulated values under either Gaussian calibration – by a factor of 2.5 against the null as estimated and of 1.9 against its signal-to-noise-matched counterpart, or 18.4 and 20.2 null standard deviations above the two null means – and rejects at the $1/(B+1)$ bootstrap floor, $p < 0.0001$, under both. It rejects under the third calibration too, at $p = 0.0030$, with 29 of the 10,000 block-resampled draws at or above it – and that is the widest of the three nulls on this statistic, so the rejection is not bought by a narrow one. Its rotation-invariant companion, $D_C = 18.13$, gives $p = 0.0002$ under the null as estimated, $p = 0.0120$ under the block-resampled null and $p = 0.2773$ under the signal-to-noise-matched null, where 2,772 of the 10,000 draws matched or exceeded the observation. The two raw-loading statistics are less clear-cut, D_Λ at $p = 0.0060$, $p = 0.0105$, $p = 0.1342$ under calibrations A, B and C in that order and $D_\Lambda / \sum_c \|\Lambda_c\|_F^2$ at $p = 0.0190$, $p = 0.0208$, $p = 0.0770$ in the same order. Nine of the twelve cells of Table F4 therefore reject at the 5% level, and its composite turns on D_C under the signal-to-noise-matched null. At $B = 10{,}000$ draws the Monte Carlo standard error of a p-value of that size is 0.0045, so that cell lies about 50.8 of those

standard errors on the far side of the 5% level: its failure to reject is resolved by the bootstrap rather than an artefact of its length. It remains a failure to reject and not evidence that the seventeen loading blocks are common; at the ten per cent level, at which this paper also reads its tests, it does not reject either. Variable by variable, the coefficient-equality test rejects for all seven series once the χ^2 approximation is replaced by a block bootstrap, each at the same Holm-adjusted $p = 0.0035$. On the companion the same test rejects for seven. Heterogeneous transmission is the direction in which the evidence of the pooled specification points – nine of the twelve dispersion cells under the original generator, nineteen of the twenty-two under the restricted one in Table F7, and all seven coefficient tests reject – rather than an assumption imported from the estimation design, although the composite criterion for declaring it established is not met under either generator and nothing stronger than that is claimed for it here. Figure F1 plots the three factor paths of the identifying specification, which are common to all seventeen countries rather than country-specific.

The pooled specification imposes two restrictions that Section 5 states but does not test, and both are rejected. The first is that the idiosyncratic covariance R is diagonal, which requires each idiosyncratic component to be uncorrelated with its own past as well as with every other series. Ljung–Box tests at four lags on the 119 idiosyncratic residual series of the identifying fit reject serial independence for 103 series at the 5% level and 91 at the 1%; 78 still reject after a Holm correction across the 119 tests, the median p-value is 1.2×10^{-4}, and only 4 series return a p-value above 0.5 where a uniform distribution would give about 60. The first-order residual autocorrelation has a median absolute value of 0.229 and exceeds 0.2 in absolute value in 67 of the 119 series, against a standard error of $1/\sqrt{T} = 0.113$, and the rejection is not an artefact of the series the common component fails to reach: it holds for 48 of the 57 series whose common component explains at least a quarter of their variance and for 21 of the 28 above one half. On the full-sample companion the corresponding counts are 67 of 75 and 33 of 40, so nothing here turns on the sample window. Cross-sectionally the picture is the same. Of the 7,021 off-diagonal residual pairs of the identifying fit, 18.7% carry an absolute correlation above 0.2 and 5.8% above 0.3, where independent residuals at the median pairwise overlap of $T = 73$ would give 9.0% and 1.0%; the Pesaran cross-sectional dependence statistic is 2.465 ($p = 0.014$), against 5.624 ($p = 1.86 \times 10^{-8}$) on the companion, and the first principal component of the residual correlation matrix accounts for 9.0% of residual variance where a diagonal R implies $1/N = 0.84\%$. The violations sit where accounting identities bind rather than at random: of the 42 within-country pairs among the 200 largest, 23 are one of just two combinations, the current account balance with capital formation or ΔGov. Debt with ΔFinAssets, and the three largest are the

current account balance with capital formation (-0.94 in Ireland) and ΔGov. Debt with ΔFinAssets (0.89 in Spain and 0.88 in Germany). Across countries the same variable in two different countries is over-represented without dominating – 53 of the 158 cross-country pairs among the 200 largest, against the one in seven that random pairing of the seven variables would give, led by y^{10} in Greece and Portugal at 0.68; the within-country blocks average 0.173 in absolute value against 0.119 across countries. The second restriction, the single lag in the factor transition, is also rejected: a likelihood-ratio test of VAR(1) against VAR(2) on the three estimated factor paths gives 143.6 on 9 degrees of freedom ($p = 1.83 \times 10^{-26}$), and 169.1 on 16 ($p = 1.29 \times 10^{-27}$) on the companion. That statistic is a property of the estimated common space rather than of the rotation the estimator happens to return: replacing the factor paths by any non-singular linear transformation of them multiplies the residual covariance determinants of the restricted and the unrestricted fit by the same factor and leaves the difference of their logarithms unchanged. Ljung–Box statistics on the transition residuals of the individual factors are computed and are deliberately not reported, on the rule this paper applies in Section 6.3: which factor carries the serial dependence is a property of the rotation and not of the data, so it is not a fact about the transition that survives the estimator's indeterminacy. No multivariate portmanteau statistic is computed on these residuals, so none is reported here and the likelihood-ratio test carries the restriction on its own. Two qualifications keep this from disturbing the results reported above. First, the pair the co-loading count is about is barely touched by the violations: the residual correlation of $r - g$ with ΔFinAssets within a country averages 0.151 in absolute value and exceeds 0.3 in one of the seventeen, reaching 0.314 in the Netherlands, so Table F2 is not reading a residual comovement the diagonal restriction had nowhere else to put. Second, half of the first restriction can be lifted directly. The estimator's AR(1) option relaxes the requirement that each idiosyncratic component be uncorrelated with its own past, but it leaves the contemporaneous covariance across series diagonal, so the cross-sectional violations just documented go untested. Re-estimating the identical panel with the identical number of factors, shocks and seed but an AR(1) idiosyncratic process – the estimator's `err` option set to `AR1`, which appends the 119 idiosyncratic components to the state vector and raises its dimension to 122 – confirms the serial dependence: its idiosyncratic AR(1) coefficients have a median absolute value of 0.288 and exceed 0.2 in 71 of the 119 series, independently confirming the Ljung–Box verdict. The search that returns it is a cold start at every point of a 21-point penalty grid, so that no fit inherits the state the previous one left the algorithm in, and it is clean on every convergence diagnostic: the minimiser $\alpha = 0.0869$ is interior rather than on a boundary, 0 of the 21 fits reach the iteration cap and 0 fail to converge. What matters is not the penalty but what it leaves: the AR(1) and IID specifications select penalties

about 43 times apart (0.0869 against 3.73) and recover the same factor space, at canonical correlations of 0.9996, 0.9967, 0.9698. The span is the only identified object here, and on the identifying specification it is invariant to the error assumption, which is what justifies reporting the IID fit as a quasi-maximum-likelihood estimator (Doz et al. (2011)) with AR(1) as a co-equal column. On the companion the same comparison splits: the first three canonical correlations are 0.9994, 0.9973, 0.9793 but the fourth is 0.9215, so there the error specification does move what is estimated, and it moves precisely the dimension that is otherwise suspect. The one thing the refit does not settle is the co-loading count, and it should not be read as a check on it: the count is 3 of the 17 under AR(1) against 2 under IID on the same space, the two agreeing on the classification of sixteen of the seventeen, with Latvia the countries that switch, and at a fixed penalty five perturbed EM starts return counts ranging 2 to 5. A statistic that moves with the error specification, with the optimisation start and with the choice of panel is not one a finding can rest on, and this paper rests none on it. What the serial-independence and VAR(1) restrictions cost, on this evidence, is the calibration of the standard errors – which is why the tests of Tables F4 and F5 are bootstrapped rather than read off an asymptotic χ^2 – and, on the companion alone, the identity of the fourth factor. What a non-diagonal R would cost is not established here; that remains the robustness check Section 5 states is not attempted.

Table F1: Pooled model: profile of the common factors, both specifications

	Breadth		*Active series, by variable*							
Factor	***Series***	***Countries***	$r-g$	y^{10}	***Ext.***	***Cap.***	***ΔDebt***	***P. bal.***	***ΔFA***	***Var. share***
Panel A. Identifying: pre-2020, $r=3$, $T=79$										
1	53	17	15	1	4	11	7	13	2	0.161
2	42	16	2	11	3	11	3	8	4	0.186
3	34	15	2	4	3	9	5	6	5	0.168
Sum of the three	–	–	–	–	–	–	–	–	–	0.514
All three jointly	–	–	–	–	–	–	–	–	–	0.292
Panel B. Companion: full sample, $r=4$, $T=104$										
1	61	17	16	0	5	5	12	15	8	0.194
2	37	15	3	15	2	3	3	6	5	0.144
3	35	14	14	1	3	3	8	0	6	0.091
4	40	16	12	3	5	10	1	8	1	0.049
Sum of the four	–	–	–	–	–	–	–	–	–	0.479
All four jointly	–	–	–	–	–	–	–	–	–	0.369

Notes: Panel A is the specification the paper identifies from, Panel B the full-sample companion. A series is counted as active on a factor when $|\text{corr}(\text{series}, \text{factor})| > 0.3$, out of 119 series and 17 countries. Variance share is $\lambda_j'\lambda_j \text{var}(F_j)/N$, the share of total panel variance the factor accounts for on its own. The estimated factors are correlated, so those shares do not add up to the share the factors account for together: the two rows at the foot of each panel give the sum and the joint share $\text{tr}(\Lambda \text{var}(F)\Lambda')/N$. The joint share is the one reported in Section 6 and the one to set against the series-by-series decomposition of Table F3. Individual factors are identified only up to rotation, so no factor of either panel is given an economic name anywhere in this paper.

Table F2: Identifying specification: correlation of $r - g$ and ΔFinAssets with each factor, by country

	$r-g$			Δ*FinAssets*				
Country	***1***	***2***	***3***	***1***	***2***	***3***	***Shared***	***Co-loading***
Austria	0.83	**-0.09**	0.07	0.02	**-0.46**	0.41	2	No
Belgium	0.79	0.24	**-0.23**	0.17	0.15	**-0.27**	3	No
Finland	**0.92**	-0.16	0.06	**-0.52**	-0.49	0.56	1	Yes
France	0.89	0.11	**-0.12**	0.04	-0.50	**0.58**	3	No
Germany	0.87	0.10	**-0.19**	0.01	0.07	**-0.11**	3	No
Greece	**0.12**	0.08	-0.07	**0.17**	0.05	0.02	1	No
Ireland	**0.31**	0.06	-0.15	**0.18**	0.23	-0.31	1	No
Italy	**0.87**	0.11	-0.18	**0.22**	-0.44	0.45	1	No
Latvia	**0.78**	-0.24	0.20	**0.30**	-0.11	-0.08	1	No
Luxembourg	0.57	0.19	**-0.29**	0.15	-0.10	**0.20**	3	No
Netherlands	**0.91**	-0.20	0.08	**0.67**	-0.21	0.12	1	Yes
Portugal	0.62	0.22	**-0.40**	0.04	0.02	**-0.15**	3	No
Slovakia	**0.15**	-0.87	0.83	**0.22**	0.07	-0.11	1	No
Slovenia	**0.90**	-0.37	0.26	**0.24**	-0.09	0.23	1	No
Spain	**0.85**	0.00	-0.07	**0.24**	0.03	-0.04	1	No
United Kingdom	**0.76**	0.06	-0.06	**0.25**	-0.14	0.05	1	No
United States	**0.71**	0.24	-0.22	**0.21**	0.06	-0.09	1	No
Countries co-loading							2 of 17	

Notes: correlations of each country's $r - g$ and ΔFinAssets with the three common factors of the identifying specification. *Shared* is the factor on which the weaker of the two correlations is largest; its two entries are set in bold. A country co-loads when both exceed 0.3 in absolute value on that factor. This table is given for the identifying specification only, and the count at its foot carries no weight in the paper, for four independent reasons, each measured rather than asserted. It is conditional on a threshold: under the N-rescaled loading rule $|\lambda| > 0.3\sqrt{7/N} = 0.073$ the count is 2 of 17 and under the unadjusted $|\lambda| > 0.3$ it is 0, which is an arithmetic consequence of the change in N and not an economic finding. It is conditional on the panel: the balanced sub-panels, of 49 and 69 quarters, give 3 of 17 on this specification and 14 on the companion, against 12 on the companion's union panel. It is conditional on the error specification: the AR(1) re-estimation of the same panel gives 3 of 17. And it is conditional on the optimisation: at a fixed penalty, five perturbed EM starting points return counts ranging 2 to 5. The columns headed 1 to 3 are the estimator's factor columns in the order it returns them and are not names: individual factors are identified only up to rotation, so no factor is named here or anywhere else in this paper, and which column a country shares is a property of that rotation rather than of the data. The dispersion and coefficient statistics of Tables F7 and F5 are what the conclusions rest on; of the two dispersion statistics the composite uses, D_C is a property of the estimated common space and D_R is a property of the representation the penalty selects.

Table F3: Pooled model: variance explained by the common component, by variable

		Identifying: pre-2020, $r = 3$						*Companion*
Variable	***Series***	***Mean*** R^2	***Median***	***Min***	***Max***	***At min***	***At max***	***Mean*** R^2
$r - g$	17	0.637	0.715	0.000	0.863	GRC	FRA	0.721
y^{10}	17	0.575	0.726	0.000	0.982	GRC	FIN	0.630
Capital form.	17	0.305	0.248	0.000	0.773	IRL	FRA	0.270
Primary balance	17	0.261	0.213	0.000	0.695	IRL	FIN	0.347
ΔGov. Debt	17	0.217	0.212	0.000	0.536	GRC	FIN	0.432
ΔFinAssets	17	0.141	0.069	0.000	0.689	DEU	FIN	0.261
Ext. Trade	17	0.105	0.000	0.000	0.484	AUT	LVA	0.131
All 119 series	119	0.320	–	–	–			0.399

Notes: R^2 of each standardised series against its common component $\Lambda F'$, averaged over the 17 countries within each variable. The first six columns are the identifying specification; the last is the mean on the full-sample companion, given for comparison. *At min* and *at max* name the country attaining the extreme. On the identifying specification the common component is financial rather than fiscal or real: it accounts for 57.5% of the variance of interest rates and 63.7% of $r - g$, but 30.5% of capital formation, 26.1% of the primary balance, 21.7% of the change in gross debt and only 10.5% of the current account balance. Capital formation therefore ranks above all three fiscal flows on this specification, which it does not on the companion.

The null of Table F4 is simulated under three calibrations, all three fixed before any cell of the identifying specification was computed. Two of them are Gaussian. They redraw the fitted state space with normal factor innovations and a normal, serially independent and cross-sectionally independent idiosyncratic term, the first at the estimated idiosyncratic variances and the second rescaling those variances so that each series keeps its fitted signal-to-noise ratio once its block is replaced by the common one. Every one of those assumptions is false on this panel, and measurably so. The fitted idiosyncratic residuals of the identifying specification have a median first-order autocorrelation of 0.215 and exceed 0.2 in absolute value in 67 of the 119 series; Jarque–Bera rejects normality for 53 of them and their median excess kurtosis is 0.853; and their absolute correlations average 0.173 within a country and 0.119 across countries, where a diagonal R implies zero. A null that assumes all of that away is not self-evidently the right yardstick for a dispersion statistic, which is why a third calibration was included alongside the other two. It keeps the state-space skeleton, the null, the pattern of missing quarters and the standardisation of the first, and replaces its two random draws by resampling. One moving-block index of 8 quarters is drawn once per replication over the whole quarter axis and read by all 119 series at the same source quarter, so the simulated idiosyncratic panel is a block of the fitted residuals $E = Z - F\Lambda'$ and carries their cross-sectional dependence, serial dependence, heteroskedasticity and non-normality without parameterising any of it; a second and independent block index resamples the fitted transition innovations $U_t = F_t - AF_{t-1}$ jointly across the three components. Where a source quarter leaves a cell unobserved that the simulated panel needs, that cell is drawn from the series' own

observed residuals, which happens for 3.8% of observed cells per replication and is the one place the construction falls back on independence. Its rejections are not bought by a narrow null, and the direction of the error is the conservative one: the resampled residuals overshoot the dependence in the data, averaging 0.195 within a country and 0.147 across against the data's 0.173 and 0.119, where the Gaussian calibrations produce 0.095 and 0.096, which is what independent series look like at this sample size. What it does not carry in full is stated with the same care. Blocks break serial dependence at their joins and a resample of a window is thinner-tailed than the pool it came from, so a simulated panel has a median idiosyncratic autocorrelation of 0.191 against the data's 0.215 and a median excess kurtosis of 0.543 against 0.853; the resampled innovations are computed from the smoothed factor path, so they omit the smoothing uncertainty the estimated innovation covariance carries; and the construction is not signal-to-noise-matched, the resampled residuals having 1.00 times the first calibration's idiosyncratic scale at the median against 1.30 times the second's. It is therefore a dependence-robust counterpart of the first calibration, and there is no counterpart of the second. The rule is a maximum over cells, so a third calibration can raise the composite and cannot lower it, and this one does not raise it: the composite of Table F4 is $p = 0.2773$ on the identifying specification and $p = 0.0636$ on the companion, both carried by D_C under the signal-to-noise-matched null. What the third calibration does settle is the condition the design attached to it, that the dispersion statistics should reject under it. On the identifying specification two of the four do – D_R at $p = 0.0030$, D_C at $p = 0.0120$, D_Λ at $p = 0.1342$ and $D_\Lambda / \sum_c \|\Lambda_c\|_F^2$ at $p = 0.0770$. On the companion D_R and D_C reject, at $p = 0.0011$ and $p = 0.0034$, and D_Λ and $D_\Lambda / \sum_c \|\Lambda_c\|_F^2$ do not, at $p = 0.6979$ and $p = 0.3413$; the two that reject there are the two the composite rule turns on and the two that do not are the raw-loading statistics the rule excludes, which is the split the rule was written to respect and not a result selected after the fact. Because the third calibration inherits the first's idiosyncratic scale and not the second's, it says nothing about the one cell on which the composite of Table F4 turns, D_C under the signal-to-noise-matched null. That cell has no counterpart under the restricted generator of Table F7: once the null is written on the standardised scale the idiosyncratic scale cancels, and the signal-to-noise matching the second calibration was meant to supply becomes one size of the common block, $\kappa = 1.24$, inside the range swept, between the $\kappa = 1.20$ and $\kappa = 1.35$ rows of that table.

Table F4: Test of the null that all seventeen countries share one loading block: block dispersion, under the original null generator (see the note below)

Statistic	*Observed*	*Calibration of H_0*	*Null mean*	*Null max*	*p*
Panel A. Identifying: pre-2020, $r = 3$, $T = 79$; 10,000 draws per calibration					
$D_R = \sum_c \|R_c - \bar{R}\|_F^2$	20.388	A. As estimated	4.755	8.090	< 0.0001
		B. SNR-matched	7.295	10.718	< 0.0001
		C. Block-resampled	9.175	26.284	0.0030
$D_C = \sum_c \|C_c - \bar{C}\|_F^2$	18.133	A. As estimated	4.550	23.263	0.0002
		B. SNR-matched	15.248	42.567	0.2773
		C. Block-resampled	9.204	43.820	0.0120
$D_\Lambda = \sum_c \|\Lambda_c - \bar{\Lambda}\|_F^2$	0.536	A. As estimated	0.201	0.673	0.0060
		B. SNR-matched	0.255	0.758	0.0105
		C. Block-resampled	0.371	1.227	0.1342
$D_\Lambda / \sum_c \|\Lambda_c\|_F^2$	0.336	A. As estimated	0.176	0.516	0.0190
		B. SNR-matched	0.208	0.493	0.0208
		C. Block-resampled	0.220	0.762	0.0770
Verdict	Homogeneity not rejected (composite rule: 5 of its 6 cells reject)				
Panel B. Companion: full sample, $r = 4$, $T = 104$; 10,000 draws per calibration					
$D_R = \sum_c \|R_c - \bar{R}\|_F^2$	22.113	A. As estimated	5.405	8.483	< 0.0001
		B. SNR-matched	7.194	10.006	< 0.0001
		C. Block-resampled	11.280	24.509	0.0011
$D_C = \sum_c \|C_c - \bar{C}\|_F^2$	20.448	A. As estimated	5.406	12.151	< 0.0001
		B. SNR-matched	15.528	33.782	0.0636
		C. Block-resampled	9.797	65.500	0.0034
$D_\Lambda = \sum_c \|\Lambda_c - \bar{\Lambda}\|_F^2$	0.626	A. As estimated	0.359	0.928	0.0231
		B. SNR-matched	0.282	0.845	0.0154
		C. Block-resampled	0.715	1.426	0.6979
$D_\Lambda / \sum_c \|\Lambda_c\|_F^2$	0.311	A. As estimated	0.195	0.387	0.0174
		B. SNR-matched	0.156	0.389	0.0058
		C. Block-resampled	0.291	0.583	0.3413
Verdict	Homogeneity not rejected (composite rule: 5 of its 6 cells reject)				

Notes: the null is $\Lambda_c = \Lambda_0$ for all $c = 1, \ldots, 17$, that is, every country's $7 \times r$ loading block in the pooled model is the same. Panel A is the identifying specification and Panel B the full-sample companion. Each compares the observed between-country dispersion of the blocks with its distribution when the null holds: the fitted state space is simulated with every block replaced by the average block $\bar{\Lambda}$, the same T, the same pattern of missing quarters and the same idiosyncratic variances, and the identical estimation call (LASSO penalty re-selected by BIC on every draw) is re-run on each simulated panel. Λ_c is the $7 \times r$ block, R_c the corresponding block of correlations between series and factors and $C_c = \Lambda_c \Sigma_F \Lambda_c'$ the country's common-component covariance; D_Λ and D_R are invariant to the sign and to the permutation of the factor columns and D_C to any rotation of them, so no alignment step is required. The three calibrations of the null are described in the paragraph above. The first (A. As estimated) draws a Gaussian panel at the estimated idiosyncratic variances; the second (B. SNR-matched) rescales those variances so that each series keeps its fitted signal-to-noise ratio once its block is replaced by the common one; the third (C. Block-resampled) replaces the Gaussian idiosyncratic and innovation draws by moving blocks of the fitted residuals, so that no row of its cells rests on a diagonal R or on Gaussian white noise. All three share one feature: each imposes the common block on the series in their original units, keeps each series' own idiosyncratic variance, and standardises every simulated column afterwards, so a country with a larger idiosyncratic variance ends with a smaller standardised loading although the block imposed on it is identical. What is simulated is therefore equality of the unstandardised blocks with country-specific noise, not the equality of standardised blocks the restriction is written on, and the null carries a dispersion of its own: evaluated at the fitted parameters it is 0.047 under A. As estimated and 0.176 under B. SNR-matched, against 0.536 observed. The second figure belongs to the calibration whose cell decides the composite verdict, and it is part of why that calibration's cutoff is the largest of the three. Table F7 evaluates the composite under a restricted generator that satisfies the written null after every transformation, and the verdict is read from it: $p = 0.2313$, with nineteen of its twenty-two cells rejecting at five per cent. The cells above apply the composite rule to a null looser than the one stated. $p = (1 + \#\{D_b \geq D_{\text{obs}}\})/(B + 1)$, so at $B = 10{,}000$ its floor is $1/10{,}001 = 0.0001$: a cell printed as an inequality is at that floor, no simulated draw having reached the observation, and a cell printed as a number is that number, so the 0.0231 printed for D_Λ in the first calibration row of Panel B is a genuine $231/(B + 1)$: 230 of those 10,000 draws did reach the observation, so it is not a second floor. The composite verdict rests on D_R and D_C, the two statistics free of the scale of Λ – of which only the second is also free of a general rotation of the factors, the first being invariant to the signed permutations the penalty leaves open but not beyond them: on the identifying fit it is 20.39 and five random orthogonal rotations which leave the fitted common component numerically unchanged move it to between 18.73 and 20.52, where D_C does not move at all – and requires both to reject under all three calibrations; it is the maximum over those six cells, so a further calibration can raise it and cannot lower it. On the identifying specification five of those six cells reject; D_C under the signal-to-noise-matched null does not, at $p = 0.2773$ with 2,772 of the 10,000 draws at or above the observation, so the composite is not met and the null is recorded as not rejected. The Monte Carlo standard error of a bootstrap p-value of that size at $B = 10{,}000$ is 0.0045, so the cell that decides the verdict lies about 50.8 standard errors from the 5% level, so it is resolved by the bootstrap rather than an artefact of its length; it remains a failure to reject and not evidence that the blocks are common, and it does not reject at the ten per cent level either. The companion returns the same verdict, on the same cell, at $p = 0.0636$, which is below the ten per cent level. The two raw-loading statistics are less clear-cut. They are listed for completeness only, because the magnitude of Λ depends on how the estimator divides scale between Λ and F and that division is not held fixed across simulated panels; a correlation is free of that dependence. The coefficient-equality test is in Table F5.

Table F5: Test of the null that all seventeen countries share one loading block: equal factor coefficients, by variable

	Identifying: pre-2020, $r = 3$			*Companion: full sample,* $r = 4$		
Variable	W	$W/$***d.f.***	***Holm*** p	W	$W/$***d.f.***	***Holm*** p
y^{10}	580.9	12.1	0.0035	566.8	8.9	0.0035
Capital form.	419.2	8.7	0.0035	371.4	5.8	0.0100
$r - g$	416.1	8.7	0.0035	1037.5	16.2	0.0035
ΔGov. Debt	385.7	8.0	0.0035	813.7	12.7	0.0035
Ext. Trade	331.9	6.9	0.0035	307.4	4.8	0.0270
ΔFinAssets	214.8	4.5	0.0035	500.6	7.8	0.0035
Primary balance	183.4	3.8	0.0035	278.9	4.4	0.0035
Rejected at 5%		7 of 7			7 of 7	

Notes: each standardised series is regressed on the estimated factors country by country with a Newey–West covariance and the equality of the seventeen coefficient vectors is tested; the truncation is $\lfloor 4(T/100)^{2/9} \rfloor$, which is 3 lags at $T = 79$ and 4 at $T = 104$. The test has $16r$ degrees of freedom, sixteen contrasts across the seventeen countries on each of the r factors: 48 on the identifying specification and 64 on the companion. The χ^2 approximation is not used and is not reported: it is grossly optimistic at these sample sizes – the simulated null mean of W/d.f. runs from 1.48 to 2.59 on the identifying specification and from 1.56 to 2.79 on the companion, against a nominal 1 – so the p-values above are those of a moving-block residual bootstrap that imposes the common coefficient vector (block length 8 quarters, 2,000 draws, resampled on the panel axis), Holm-corrected across the seven variables. On the identifying specification equality is rejected for 7 of the seven ($r - g$, y^{10}, Ext. Trade, Capital form., ΔGov. Debt, Primary balance, ΔFinAssets), with no exception; on the companion, for 7.

A verdict of not rejected is not a result until the reader is told what the test could have detected, so the composite rule of Table F4 was put through a power analysis after it had been run and before anything further was claimed for it. Panels are simulated from the fitted state space with the common block imposed and the seventeen country blocks then moved by s times their fitted deviation from it, the same pattern of missing quarters re-imposed, the identical estimation call re-run on every draw and the rule judged at the production critical values; 840 refits at 120 replications a point, all of which converged. Everything in this paragraph describes the original null of Table F4 except the next four sentences, in which the restricted rule of Table F7 is measured on its own terms. Its rejection frequency, taken from the 42,000 draws of that table by treating each in turn as the observation and running the rule against the rest, is between 0.000 and 0.021 across the eighteen null points, 0.0041 pooled and 0.0021 over the eleven points inside the observed confidence set, against a nominal 0.05. Its power, on 7,920 fresh panels in which the seventeen blocks are moved s times their fitted deviation from the common one, is 0.000 to 0.010 at $s = 0.70$, somewhat below the scale the attenuation correction on the original generator implied, 0.84 (recorded below); 0.000 to 0.083 at $s = 1$, the fitted dispersion itself; and 0.37 to 0.52 at $s = 1.35$, the largest deviation simulated, where the exact intervals, $[0.325, 0.422]$ under A* and $[0.475, 0.575]$ under C*, exclude eighty per cent; at that deviation a panel can be drawn only at the smallest size, $\kappa = 0.70$, because at $\kappa = 0.90$ the implied communality of a series would exceed one. At $s = 0$ the

rejection frequency reproduces the leave-one-out size at every point, which is the check that the two are of one rule. The restricted rule therefore rejects on its own null on average between a twelfth and a twenty-fourth as often as the level it is read at allows, and at its worst null point about two-fifths as often, and is essentially blind to dispersion at $s = 0.70$. Table F6 is the curve, and three things follow from it and from the null draws Table F4 already rests on. First, the composite is not a five per cent test. Because it is a maximum over six cells it rejects only when every one of them rejects, which requires both statistics to clear the largest of the three calibrations' cutoffs; counting that event directly in the 30,000 null draws behind Table F4 gives 0 of 10,000 under the null as estimated, 0 of 10,000 under the signal-to-noise-matched null and 3 of 10,000 under the block-resampled null, a rejection frequency of 0.0003 under its own fitted nulls against the nominal 0.05 at which the rule is read, and of 0.0000 and 0.0007 at the one and ten per cent levels – frequencies under those generators, not the supremum of the rejection probability over the composite null. The three cutoffs on D_C are 7.41, 23.84 and 14.26, a spread of 3.2 to one across nulls meant to be three readings of one hypothesis, and the observed 18.13 clears two of them and fails the third.Second, the rule attains eighty per cent power only at $s = 1.24$, with a 95% interval of $[1.17, 1.32]$, while a method-of-moments correction for the estimation error carried in the fitted spread puts this panel's own heterogeneity at $s = 0.84$: the minimum detectable effect is 1.48 times the dispersion the panel supports, and the rule had 0.50 power against that dispersion when it declined to reject. In the units of the loading matrix it detects a panel in which the average country's block lies 2.40 panel-mean row norms from the common block against the fitted 1.93, and the most deviant lies 3.61 where Latvia, the most heterogeneous country in the fit, lies at 2.91.Third, against a single country's block it did not reject once. Moving the block of Finland, the third most heterogeneous country in the fit (after Latvia and Greece), alone leaves the composite rejecting 0 of 60 at every size simulated, up to 18.34 panel-mean row norms, which is 6.3 times the most extreme heterogeneity in the fit — none in sixty bounds that rejection probability at 0.049 and does not establish that it is nil; turning that same block through a quarter turn to fully orthogonal at unchanged amplitude never lifts the composite above 0 rejection in 60, and leaves it at 0 of 60 at five of the five angles simulated; and moving the financial-asset row alone across all seventeen countries leaves it rejecting 19 of 60 at 12.31 row norms.The mechanism is arithmetic and it was diagnosed rather than inferred. The panel is standardised and every simulated draw is re-standardised before the refit, so amplitude divides out: multiplying Finland's loading rows by 8.42, which takes its true common-component R^2 from 0.33 to 0.82, raises its estimated row norms by a factor of only 1.16 and makes D_R fall, from 8.73 to 7.16, rather than rise. What remains visible to the rule is the shape of the loading pattern, and

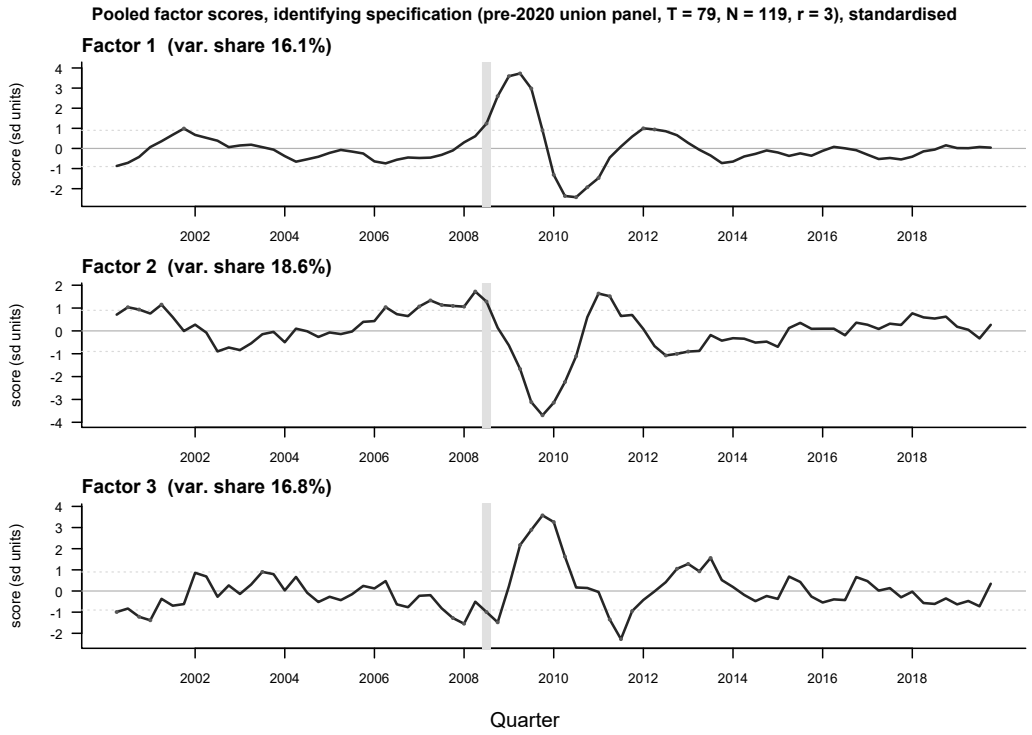
on shape D_C is bounded: it is built from a common-component covariance, so on a standardised panel the largest population value that any pure change of loading direction can produce, at any angle and on any number of countries, is 7.71 against a binding cutoff of 23.84; and its mean is not monotone in the deviation it is meant to track, rising from 8.96 to 20.45 as the blocks turn through forty-five degrees and then falling to 19.17 at ninety. Because the composite is the minimum of the two statistics' verdicts it inherits that ceiling and discards D_R's power, which reaches 1.000 on the same alternative.This also accounts for the asymmetry between Tables F4 and F5 that this appendix otherwise records without explaining. The seven Wald rows reject on every variable because they reach eighty per cent power at $s = 0.44$ to 0.58, below the scale the data shows, while the composite reaches it at $s = 1.24$, about one and a half times that scale; the two families are reading the same evidence through a rule that requires six simultaneous rejections and a rule that requires one. None of this revises the verdict of Table F4, which stands as the rule returned it, and nothing here licenses re-aggregating its six p-values now that they have been seen. What it establishes is the reach of the rule: the composite can detect a panel about a quarter more dispersed than this one, its average block 2.40 panel-mean row norms from the common block against the 1.93 now fitted and its most deviant 3.61 against Latvia's 2.91; and it cannot detect one country's block moving, which it rejects 0 of 60 times at every magnitude simulated and not once in sixty at any angle of rotation. Its failure to reject is therefore a statement about the rule and not about the seventeen countries, and it is not reported here as evidence that the blocks are common; nor, since a test that cannot see an alternative is uninformative about it, is it read the other way.

Table F6: What the composite rule of Table F4 could have detected: power against block dispersion

s	*Mean deviation*	*Composite rule*		D_R *vs C*	D_C *vs B*
	(row norms)	*power*	*MC s.e.*	*power*	*power*
0.00	0.00	0.000	0.000	0.050	0.000
0.50	0.97	0.083	0.025	0.292	0.142
0.75	1.45	0.442	0.045	0.883	0.442
1.00	1.93	0.617	0.044	1.000	0.617
1.25	2.42	0.775	0.038	1.000	0.775
1.50	2.90	0.875	0.030	1.000	0.875
2.00	3.87	0.983	0.012	1.000	0.983
80 per cent power	2.40	at $s = 1.24$, interval $[1.17, 1.32]$			
Spread in the data	1.93	$s = 1$ fitted, $s = 0.84$ corrected; power 0.50			

Notes: the data-generating process is the fitted state space with the common block $\bar{\Lambda}$ imposed and every country block then set to $\bar{\Lambda} + s(\Lambda_c - \bar{\Lambda})$, so $s = 0$ is the null, $s = 1$ is the fitted panel and the second column states the same thing in multiples of the panel-mean row norm. The idiosyncratic and innovation draws are the block-resampled calibration; the estimation call, the missingness mask, the standardisation and the penalty selection are those of the production run, and the rule is judged at the production critical values, so the table measures the rule as it was run and not a reconstruction of it. 120 replications a point, 840 refits, all converged; the Monte Carlo standard error of each power is printed beside it. The composite is the maximum of the six cell p-values, so it rejects only when all of them do: counted in the 30,000 null draws of Table F4, that event has probability 0.0003 under the block-resampled null and is not observed at all under the other two, against the nominal 0.05. The last two columns give the two cells that bind. Against narrower alternatives the rule has less power still: with Finland's block alone moved to 18.34 panel-mean row norms it rejects 0 of 60, and with that block turned towards a direction orthogonal to the common one at unchanged amplitude it rejects at most 0 of 60 at any of the five angles simulated and 0 of 60 at five of them, because the panel is standardised and D_C cannot exceed 7.71 in population under any pure change of loading direction. The verdict of Table F4 is unchanged by any of this; what the table records is what that verdict is evidence for.

Figure F1: Identifying specification: the three common factor score paths



Notes: smoothed factor scores of the identifying specification, standardised to unit variance, 2000Q2–2019Q4. Dotted lines mark the ±0.9 activation cut used in the main text and filled points the quarters that clear it; the shaded vertical marks 2008Q3. Under the pooled specification these three paths are common to all seventeen countries, so countries differ only through their loadings, not through their scores. The paths are identified only up to rotation, so the panels are labelled by number and not by name.

Table F7: Identifying specification: the restriction of (23) re-tested on a null that satisfies it. Cell p-values by calibration, statistic and the size of the common block.

	A* Gaussian		C* resampled		in C_γ	
κ	D_R	D_C	D_R	D_C	A*	C*
0.60	0.0003	0.0003	0.4165	0.0173	no	no
0.70	0.0003	0.0003	0.3059	0.0097	no	no
0.80	0.0003	0.0003	0.2213	0.0127	no	yes
0.90	0.0003	0.0003	0.1236	0.0103	yes	yes
1.00	0.0003	0.0003	0.0810	0.0123	yes	yes
1.10	0.0003	0.0003	0.0440	0.0117	yes	yes
1.20	0.0010	0.0010	0.0180	0.0160	yes	yes
1.35	0.0010	0.0010	0.0040	0.0180	yes	yes
1.50	0.0010	0.0020	0.0010	0.0190	no	no

Notes: the null is written on the standardised scale, $z_{ij,t} = F_t'\lambda_j + \sqrt{1-q_j}e_{ij,t}$ with $\lambda_j = \kappa\Lambda_{\mathrm{RLS},j}$, so the standardised loading blocks are equal by construction: the population dispersion is 2.4×10^{-31} at every κ. Λ_{RLS} is the pooled restricted least-squares block. κ is the size of the common block, which (23) does not identify, so it is swept: the block of Table F4 sits at $\kappa = 0.93$ and the signal-to-noise-matched intent of that table's second calibration at $\kappa = 1.24$. A* draws Gaussian idiosyncratic errors and Gaussian transition innovations; C* resamples the fitted idiosyncratic residuals and transition innovations in moving blocks, the latter rescaled so that the simulated factor path carries the model's own stationary covariance. The last two columns mark the κ inside the confidence set built from a rotation-invariant matching statistic at $\gamma = 0.01$; the composite is the supremum of the cell p-values over that set plus γ, after Berger and Boos (1994). $B = 3{,}000$ at $\kappa \leq 1.10$ and $B = 1{,}000$ above, 42,000 draws in all, every one converged.

Verdict: homogeneity not rejected, composite $p = 0.2313$ — the supremum over the confidence set plus γ — with 19 of its 22 cells rejecting at five per cent.

Table F7 repeats the exercise of Table F4 on a null generator that satisfies the restriction. The generator of Table F4 imposed the common block on the series in their original units and standardised each simulated column afterwards, so a country with a larger idiosyncratic variance received a smaller standardised loading although the block imposed on it was identical; the blocks that null made equal are therefore not the blocks (23) equates. Written on the standardised scale the dispersion the null carries falls to 2.4×10^{-31}. Two consequences follow. The idiosyncratic scale cancels under the standardisation, so the signal-to-noise-matched calibration of Table F4 was not a second reading of the hypothesis but a null of no hypothesis, and it has no counterpart here. And the size of the common block is not identified by (23), so it is swept rather than plugged in. The repair does not remove the spread between the cutoffs noted above. The widest of them, 23.84, belongs to the calibration just set aside, but the Gaussian and the block-resampled calibrations share the idiosyncratic scale, so the defect never accounted for the gap between them, and that gap survives: at $\kappa = 1.00$ the 95 per cent cutoffs on D_C are 4.99 under A* and 14.64 under C*, about 2.9 to one, against 7.41 and 14.26, about 1.9 to one, under the first and third calibrations of Table F4. At every size inside both confidence sets the ratio of the two cutoffs lies between 2.0 and 3.6 to one. On the restricted null the composite is $p = 0.2313$, attained by D_R, the statistic that depends on the representation the penalty selects, under C* at $\kappa = 0.80$, against 0.2773 under the original generator. Nineteen of the twenty-two cells entering the composite reject at five per cent: D_C in every one of its cells, and D_R in every Gaussian cell. What does not reject is D_R

under C* at the lower sizes, and because the rule is a maximum those cells carry the verdict. A rule that fails while nineteen of twenty-two cells reject is reporting its own strictness and not the homogeneity of the seventeen blocks; neither reading is available from it. One further defect of the block-resampled calibration of Table F4 is recorded: its blocked transition innovations are serially correlated, and passing them through the transition equation again inflated the simulated factor variance to about 1.7 times the model's stationary value on the first two components. Neither finding revises a number in Table F4, which stands as produced; both change what may be read from it. The power analysis of Table F6 describes the original null; the restricted rule is measured separately: its rejection frequency is 0.0041 pooled over the eighteen null configurations and 0.0021 over the eleven inside the observed confidence set, against the nominal 0.05 at which it is read; neither is the size, which is the largest rejection probability over the whole null, and the largest frequency at any single configuration is 0.021. Its power at $s = 0.70$, somewhat below the scale the attenuation correction on the original generator implied, 0.84, is 0.000 under A* and at most 0.010 under C*, rising to 0.37 at $s = 1.35$ under A* ($[0.325, 0.422]$) and 0.52 under C*, the largest deviation simulated, which at the larger sizes already implies a communality above one. On its own null the restricted rule therefore rejects on average between a twelfth and a twenty-fourth as often as the level it is read at allows, and at its worst null point about two-fifths as often, and it is essentially blind to dispersion at $s = 0.70$; Section 6 reports the exercise in full.

Appendix G Regime classification across specifications

Country by country, the five configurations listed in the notes to Table G1 are assigned from seventeen separately estimated models, and the classification does not carry over unchanged to a pooled four-factor fit of all 119 series. It also does not carry over in one single way, because the pooled model admits two defensible thresholds and they disagree. Under the scale-free rule, a series is taken to load on a factor when $|\text{corr}(\text{series}, \text{factor})| > 0.3$; the same cut then means the same thing at $N = 7$ and at $N = 119$, and applied to the seventeen country-by-country fits re-estimated for this appendix, it agrees with the paper's own $|\lambda| > 0.3$ verdict on those same fits – whether $r - g$ and ΔFA attach to a shared factor at all – in sixteen of the seventeen countries. It is worth stating why a substitute threshold is needed: the rule the paper states, $|\lambda| > 0.3$ applied to loadings, cannot simply be carried over, because on the pooled matrix it classifies all seventeen countries as having no association, an arithmetic consequence of N rather than a finding. Under the N-rescaled loading rule, the cut is instead placed at $0.3\sqrt{7/N} = 0.073$, which holds it at the same multiple of the equal-share loading $1/\sqrt{N}$ that 0.3 occupied at $N = 7$. Neither is obviously right, and the choice is consequential: France, which the country-by-country re-estimate places in management orthogonal to both, moves into buffer accumulation under the correlation rule and into co-movement with capital formation or the current account balance under the rescaled loading rule, while the buffer accumulation group falls from one country under the country-by-country fits to five under the correlation rule and to none at all under the rescaled one. Reporting either figure alone would present a threshold convention as a result. Table G1 therefore gives all three classifications side by side and records, in the final column, whether a country carries the same label under all of them. Only three of the seventeen countries do: Spain, the United Kingdom and the United States. The remaining fourteen change configuration under at least one of the two pooled criteria. Taken pairwise, ten countries change between the country-by-country classification and the pooled correlation rule, fourteen change between the country-by-country classification and the pooled rescaled-loading rule, and the two pooled rules agree with each other on only six. Table G2 gives the resulting group sizes. Only crisis liquidity provision and co-movement with capital formation or the current account balance retain any members under all three criteria, and even they are unstable in size, the three columns giving eight, five and four for crisis liquidity provision and six, four and one for co-movement with capital formation or the current account balance. What is not robust is the assignment of most individual countries to one of them, and still less the existence of configuration I, buffer accumulation, which

the rescaled-loading rule empties outright while the correlation rule does not, or of configuration IV, management orthogonal to both, which the correlation rule empties outright while the rescaled-loading rule does not. The classification is reported here as a description of the country-by-country estimates that does not survive as a finding of the pooled four-factor fit. That fit, from which the two pooled columns come, is estimated on the full-sample design of the paper's companion specification, not on the pre-pandemic window the paper identifies from, and over the balanced panel – the quarters in which all 119 series are observed, 2007Q4–2024Q4, $T = 69$ – rather than the union panel ($T = 104$); but alignment is not the source of the instability: the two panels deliver the same label in fourteen of seventeen countries under the correlation rule.

Table G1: Regime classification under the three specifications

	Country by country	*Pooled, 119 series*		
Country	***Published specification***	***Correlation***	***Rescaled loading***	***Stable***
Austria	II Crisis liquidity	I Buffer accumulation	IV Autonomous	
Belgium	II Crisis liquidity	III Capital/external	V Weak association	
Finland	III Capital/external	I Buffer accumulation	IV Autonomous	
France	IV Autonomous	I Buffer accumulation	III Capital/external	
Germany	II Crisis liquidity	V Weak association	V Weak association	
Greece	III Capital/external	V Weak association	V Weak association	
Ireland	I Buffer accumulation	V Weak association	V Weak association	
Italy	III Capital/external	III Capital/external	V Weak association	
Latvia	IV Autonomous	III Capital/external	V Weak association	
Luxembourg	III Capital/external	I Buffer accumulation	V Weak association	
Netherlands	II Crisis liquidity	II Crisis liquidity	V Weak association	
Portugal	II Crisis liquidity	II Crisis liquidity	V Weak association	
Slovakia	III Capital/external	I Buffer accumulation	V Weak association	
Slovenia	III Capital/external	III Capital/external	II Crisis liquidity	
Spain	II Crisis liquidity	II Crisis liquidity	II Crisis liquidity	✓
United Kingdom	II Crisis liquidity	II Crisis liquidity	II Crisis liquidity	✓
United States	II Crisis liquidity	II Crisis liquidity	II Crisis liquidity	✓
Stable across all three				3 of 17

Notes: Configurations are I buffer accumulation, II crisis liquidity provision, III co-movement with capital formation or the current account balance, IV management orthogonal to both, V no association above the threshold. *Published specification* classifies each country from its own separate fit with $r = 6$ factors, a loading cut of 0.3 and an activation cut of 0.9 on the raw factor score – the specification behind Table H2 – re-estimated for this table, and its labels are derived from that re-estimate rather than read off Table H2. *Correlation* and *Rescaled loading* classify from the pooled $N = 119$, $r = 4$ fit on the balanced panel, 2007Q4–2024Q4 ($T = 69$), under a common rotation, using $|\text{corr}| > 0.3$ and $|\lambda| > 0.3\sqrt{7/N} = 0.073$ respectively, with activation measured on the standardised score. A country is *stable* when the three columns agree.

Table G2: Group sizes under the three specifications

	Country by country	*Pooled, 119 series*	
Configuration	***Published specification***	***Correlation***	***Rescaled loading***
I Buffer accumulation	1	5	0
II Crisis liquidity	8	5	4
III Capital/external	6	4	1
IV Autonomous	2	0	2
V Weak association	0	3	10
Total	17	17	17

Notes: Counts of the seventeen countries falling in each configuration under the three classifications of Table G1.

Appendix H Supplementary results and tables

Table H1: Financial assets co-movements with $r - g$

$$
\begin{cases}
\text{co-movement} & \begin{cases}
\text{economic growth} & \begin{array}{l} \text{if } \exists f \text{ s.t. } |\lambda_{f,i}^{\Delta FinAss}| > 0.3 \wedge \lambda_{f,i}^{r-g} < -0.3 \wedge \hat{F} > 0.9 \\ \vee \\ \text{if } \exists f \text{ s.t. } |\lambda_{f,i}^{\Delta FinAss}| > 0.3 \wedge \lambda_{f,i}^{r-g} > 0.3 \wedge \hat{F} < -0.9 \end{array} \\
\text{subdued growth} & \begin{array}{l} \text{if } \exists f \text{ s.t. } |\lambda_{f,i}^{\Delta FinAss}| > 0.3 \wedge \lambda_{f,i}^{r-g} > 0.3 \wedge \hat{F} > 0.9 \\ \vee \\ \text{if } \exists f \text{ s.t. } |\lambda_{f,i}^{\Delta FinAss}| > 0.3 \wedge \lambda_{f,i}^{r-g} < -0.3 \wedge \hat{F} < -0.9 \end{array}
\end{cases} \\
\text{weak association} & \text{if } \exists f \text{ s.t. } |\lambda_{f,i}^{\Delta FinAss}| < 0.3 \wedge |\lambda_{f,i}^{r-g}| < 0.3 \\
\text{orthogonality} & \text{if } \forall f^{\Delta FinAss}, \forall f^{r-g}, f^{\Delta FinAss} \perp f^{r-g}
\end{cases}
$$

Table H2: Regimes $r - g$

Regime A: Decreasing $r - g$	
negative contributions ($r - g$ loading $\times$ factor score) < 0	
Country (Loading $r - g$***; FA)***	***Quarters (Factor Scores)***
Loadings $r - g < -0.3$ **(Positive factor scores** > 0.9**)**	
FA expansion ($FA > 0.3$)	
Finland Factor 2: (-0.408; 0.595)	2004Q4 (1.325), 2009Q4 (1.269), 2010Q1 (2.754), 2010Q2 (4.616), 2010Q3 (3.861), 2010Q4 (3.575), 2011Q1 (1.370), 2015Q1 (1.094), 2015Q2 (1.286), 2020Q4 (1.336), 2021Q1 (2.417), 2021Q2 (3.482), 2021Q3 (2.950), 2024Q4 (1.369), 2025Q2 (1.891), 2025Q4 (1.124), 2026Q1 (1.144)
United Kingdom Factor 4: (-0.325; 0.782)	2008Q3 (0.946), 2010Q1 (3.591), 2010Q2 (1.182), 2011Q4 (1.073), 2012Q3 (1.025), 2014Q1 (1.094), 2016Q4 (1.367), 2017Q1 (1.041), 2018Q2 (0.900), 2021Q3 (1.387), 2021Q4 (1.138)

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Table H2 – continued from the previous page

Country (Loading $r-g$; FA)	***Quarters (Factor Scores)***
Greece Factor 3: (-0.327; 0.312)	2012Q2 (2.239), 2012Q3 (1.578), 2012Q4 (4.468), 2013Q3 (1.752), 2014Q1 (1.889), 2014Q2 (3.538), 2015Q3 (0.957), 2015Q4 (1.042), 2016Q2 (1.236), 2021Q2 (1.969), 2021Q3 (2.644), 2021Q4 (1.415), 2022Q1 (0.908)
Ireland Factor 4: (-0.421; 0.524)	2009Q1 (0.947), 2011Q1 (4.228), 2011Q2 (2.837), 2011Q3 (2.107), 2015Q2 (1.125), 2015Q3 (1.142), 2021Q4 (1.470), 2022Q2 (1.340)
Luxembourg Factor 1: (-0.432; 0.416)	2001Q2 (1.422), 2001Q4 (1.500), 2002Q1 (1.089), 2003Q1 (1.615), 2005Q4 (1.291), 2007Q2 (1.371), 2007Q3 (2.512), 2007Q4 (1.060), 2010Q1 (2.630), 2010Q2 (2.509), 2012Q3 (1.040), 2014Q1 (1.094), 2014Q4 (2.074), 2015Q2 (1.146), 2017Q1 (1.609), 2017Q2 (1.344), 2019Q1 (1.514), 2019Q2 (3.075), 2019Q3 (2.708), 2019Q4 (2.382), 2021Q1 (2.793), 2021Q2 (4.275), 2024Q4 (1.024)
FA contraction ($FA < -0.3$) – empty set	
Loadings $r-g > 0.3$ (Negative factor scores < -0.9)	
FA expansion ($FA < -0.3$)	
Belgium Factor 6: (0.778; -0.305)	2005Q1 (-1.446), 2021Q2 (-2.119), 2021Q3 (-1.466), 2023Q1 (-0.951)
Ireland Factor 5: (0.693; -0.318)	2010Q1 (-2.453), 2010Q3 (-1.319), 2011Q3 (-1.725), 2015Q1 (-1.773), 2015Q2 (-1.093), 2015Q3 (-1.260), 2015Q4 (-1.554), 2017Q2 (-1.661), 2024Q4 (-1.066)
Italy Factor 2: (0.334; -0.412)	2009Q3 (-0.933), 2009Q4 (-2.636), 2012Q4 (-2.655), 2013Q1 (-1.518), 2013Q2 (-1.542), 2013Q3 (-1.299), 2013Q4 (-1.216), 2014Q1 (-1.887), 2014Q2 (-1.517), 2014Q3 (-1.793), 2014Q4 (-1.367), 2016Q3 (-1.000), 2019Q3 (-1.735), 2019Q4 (-1.294), 2020Q3 (-1.350), 2020Q4 (-1.415), 2021Q1 (-2.326), 2021Q2 (-6.106), 2021Q3 (-1.405)
Luxembourg Factor 5: (0.356; -0.364)	2003Q4 (-1.478), 2007Q4 (-1.274), 2009Q3 (-2.632), 2009Q4 (-2.849), 2010Q3 (-1.096), 2012Q4 (-1.869), 2013Q1 (-1.476), 2013Q2 (-1.183), 2013Q3 (-1.492), 2013Q4 (-2.206), 2014Q2 (-1.162), 2019Q4 (-0.994), 2020Q1 (-1.157), 2020Q4 (-1.198), 2021Q1 (-1.487), 2021Q3 (-1.049), 2023Q2 (-0.934)
Portugal Factor 6: (0.713; -0.505)	2009Q4 (-1.163), 2010Q3 (-1.757), 2014Q1 (-1.157), 2014Q3 (-1.291), 2016Q2 (-1.776), 2021Q2 (-2.272), 2022Q1 (-1.817), 2024Q4 (-1.083), 2025Q3 (-1.008)
FA contraction ($FA > 0.3$)	
Austria Factor 1: (0.305; 0.430)	2006Q1 (-1.578), 2006Q2 (-2.162), 2006Q3 (-1.570), 2007Q4 (-0.931), 2008Q1 (-1.135), 2008Q2 (-2.696), 2008Q3 (-2.601), 2011Q1 (-1.131), 2016Q3 (-1.011), 2016Q4 (-0.913), 2017Q1 (-1.455), 2017Q2 (-1.032), 2017Q3 (-0.947), 2017Q4 (-1.314), 2018Q1 (-1.786), 2018Q2 (-1.437), 2018Q3 (-1.067), 2018Q4 (-1.609), 2019Q1 (-1.053), 2019Q2 (-1.249), 2019Q4 (-1.066), 2021Q2 (-1.240), 2021Q4 (-2.104), 2022Q1 (-3.733), 2022Q2 (-3.303), 2022Q3 (-2.151), 2022Q4 (-2.711), 2023Q1 (-1.579), 2023Q2 (-1.167)
Belgium Factor 1: (0.513; 0.391)	2004Q2 (-0.912), 2004Q3 (-1.468), 2004Q4 (-2.212), 2006Q1 (-2.125), 2007Q4 (-0.974), 2009Q4 (-1.000), 2010Q1 (-1.756), 2010Q2 (-1.817), 2010Q3 (-2.041), 2010Q4 (-1.198), 2013Q4 (-1.052), 2015Q1 (-1.089), 2015Q2 (-1.108), 2017Q1 (-1.830), 2017Q2 (-1.168), 2018Q3 (-1.117), 2021Q2 (-8.256), 2021Q3 (-1.256), 2021Q4 (-3.699), 2022Q1 (-3.899), 2022Q3 (-1.573), 2026Q1 (-1.451)
Germany Factor 1: (0.463; 0.504)	2004Q2 (-1.442), 2004Q4 (-0.939), 2006Q2 (-1.283), 2006Q3 (-1.104), 2006Q4 (-0.974), 2007Q1 (-1.514), 2007Q3 (-1.229), 2010Q1 (-1.496), 2010Q2 (-1.315), 2011Q1 (-1.870), 2011Q2 (-1.521), 2011Q4 (-5.564), 2013Q1 (-0.934), 2013Q2 (-1.785), 2015Q2 (-1.098), 2021Q2 (-6.621), 2021Q3 (-2.692), 2022Q1 (-2.138)

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Table H2 – continued from the previous page

Country (Loading $r-g$; FA)	***Quarters (Factor Scores)***
Spain Factor 1: (0.487; 0.430)	2010Q1 (-1.535), 2010Q2 (-1.651), 2010Q3 (-1.134), 2010Q4 (-1.569), 2013Q3 (-0.913), 2013Q4 (-5.646), 2014Q1 (-2.900), 2015Q2 (-1.713), 2021Q2 (-8.736), 2021Q3 (-3.139), 2021Q4 (-3.620), 2022Q1 (-4.327)
United Kingdom Factor 1: (0.504; 0.425)	2010Q2 (-4.529), 2010Q3 (-1.004), 2010Q4 (-2.174), 2011Q1 (-3.157), 2011Q3 (-1.490), 2013Q1 (-1.475), 2013Q3 (-1.183), 2021Q2 (-9.174), 2021Q3 (-3.275), 2021Q4 (-4.028), 2022Q1 (-4.533), 2022Q3 (-1.043)
Greece Factor 1: (0.462; 0.367)	2006Q1 (-1.283), 2006Q2 (-2.531), 2006Q3 (-1.712), 2010Q1 (-1.702), 2012Q1 (-3.065), 2013Q4 (-1.162), 2014Q2 (-1.248), 2014Q3 (-1.091), 2014Q4 (-1.516), 2015Q1 (-3.001), 2016Q3 (-1.317), 2021Q2 (-4.341), 2021Q3 (-3.013), 2021Q4 (-5.256), 2022Q1 (-3.503), 2022Q2 (-1.007)
Ireland Factor 1: (0.396; 0.466)	2010Q2 (-2.655), 2011Q1 (-1.773), 2011Q4 (-1.075), 2012Q1 (-2.298), 2012Q3 (-4.080), 2013Q3 (-2.440), 2014Q1 (-2.438), 2014Q2 (-2.011), 2014Q4 (-1.180), 2015Q1 (-1.867), 2015Q3 (-1.829), 2015Q4 (-1.733), 2017Q2 (-1.294), 2019Q2 (-2.705), 2019Q3 (-1.385), 2019Q4 (-1.714), 2020Q1 (-2.933), 2021Q2 (-2.443), 2022Q1 (-1.352), 2024Q3 (-1.451), 2025Q1 (-1.511), 2025Q2 (-0.930)
Luxembourg Factor 4: (0.442; 0.445)	2002Q4 (-0.941), 2003Q1 (-3.030), 2003Q2 (-1.703), 2004Q3 (-0.936), 2005Q4 (-1.470), 2008Q3 (-1.549), 2009Q4 (-2.003), 2010Q1 (-3.677), 2010Q2 (-1.966), 2010Q3 (-1.070), 2011Q3 (-0.983), 2011Q4 (-1.768), 2013Q1 (-1.225), 2013Q2 (-1.041), 2014Q3 (-0.986), 2020Q1 (-2.591), 2020Q2 (-2.259), 2020Q3 (-1.346), 2020Q4 (-1.453), 2022Q1 (-1.002), 2023Q2 (-0.943), 2023Q3 (-0.953), 2025Q3 (-0.969), 2025Q4 (-1.569)
Netherlands Factor 1: (0.529; 0.472)	2004Q3 (-1.033), 2006Q2 (-1.935), 2006Q4 (-0.920), 2010Q3 (-1.401), 2010Q4 (-0.986), 2011Q1 (-1.797), 2013Q3 (-0.985), 2015Q2 (-1.997), 2016Q1 (-1.573), 2017Q1 (-1.253), 2021Q2 (-5.632), 2021Q3 (-2.771), 2021Q4 (-2.263), 2022Q1 (-4.014), 2022Q2 (-2.169), 2022Q3 (-1.403), 2022Q4 (-0.951)
Portugal Factor 1: (0.480; 0.510)	2004Q3 (-1.169), 2006Q3 (-1.133), 2010Q1 (-1.037), 2012Q2 (-2.840), 2013Q1 (-1.515), 2013Q2 (-1.892), 2013Q3 (-2.453), 2013Q4 (-1.811), 2014Q2 (-1.066), 2015Q1 (-1.000), 2015Q3 (-1.912), 2016Q4 (-0.962), 2017Q2 (-1.464), 2017Q3 (-1.229), 2018Q1 (-1.127), 2021Q2 (-5.655), 2021Q3 (-4.419), 2021Q4 (-3.393), 2022Q1 (-1.972), 2022Q4 (-0.919), 2025Q4 (-2.032)
Slovakia Factor 4: (0.477; 0.357)	2002Q1 (-1.926), 2005Q1 (-0.955), 2005Q3 (-1.954), 2006Q2 (-1.425), 2006Q4 (-3.100), 2007Q1 (-2.129), 2007Q3 (-1.564), 2007Q4 (-1.074), 2008Q1 (-1.303), 2008Q2 (-1.706), 2008Q3 (-1.492), 2008Q4 (-1.061), 2022Q2 (-1.471), 2022Q3 (-1.309), 2023Q1 (-3.219), 2023Q2 (-2.136), 2023Q3 (-2.665), 2023Q4 (-3.499), 2024Q1 (-1.360), 2024Q2 (-1.343), 2024Q3 (-1.101)
Slovenia Factor 1: (0.472; 0.454)	2006Q3 (-0.990), 2007Q2 (-1.899), 2007Q3 (-0.917), 2008Q1 (-1.551), 2010Q2 (-2.485), 2010Q3 (-1.861), 2012Q1 (-1.503), 2014Q2 (-1.564), 2014Q4 (-1.421), 2015Q1 (-1.119), 2015Q2 (-1.248), 2015Q4 (-2.125), 2016Q3 (-1.202), 2021Q2 (-7.358), 2021Q3 (-1.929), 2021Q4 (-4.183), 2022Q1 (-3.804), 2022Q3 (-1.264), 2025Q4 (-1.284), 2026Q1 (-1.321)
United States Factor 1: (0.515; 0.478)	2004Q2 (-1.000), 2005Q1 (-1.293), 2009Q4 (-1.100), 2010Q2 (-1.793), 2010Q3 (-1.201), 2011Q1 (-1.331), 2017Q1 (-1.322), 2021Q2 (-11.081), 2021Q3 (-3.324), 2021Q4 (-2.560), 2022Q1 (-1.567)

Table H2: Regimes $r - g$ *(continued)*

Regime B: Increasing $r - g$	
positive contributions ($r - g$ *loading* $\times$ *factor score*) > 0	
Country (Loading $r - g$***; FA)***	***Quarters (Factor Scores)***
Loadings $r - g > 0.3$ **(Positive factor scores** > 0.9**)**	
FA expansion (FA > 0.3*)*	
Austria Factor 1: (0.305; 0.430)	2001Q4 (0.976), 2005Q1 (1.362), 2005Q2 (1.118), 2005Q3 (1.076), 2007Q3 (1.115), 2008Q4 (1.860), 2009Q1 (1.939), 2009Q2 (2.807), 2009Q3 (3.086), 2009Q4 (3.279), 2010Q1 (1.863), 2010Q2 (2.106), 2010Q3 (2.501), 2012Q1 (1.326), 2012Q2 (1.788), 2012Q3 (1.764), 2014Q4 (1.002), 2015Q1 (1.284), 2020Q2 (4.275), 2020Q3 (3.121), 2020Q4 (4.838), 2021Q1 (4.753), 2024Q1 (0.945), 2024Q3 (1.184)
Belgium Factor 1: (0.513; 0.391)	2001Q3 (0.903), 2005Q1 (1.098), 2008Q4 (4.177), 2009Q1 (2.673), 2009Q2 (3.095), 2009Q3 (1.909), 2011Q4 (1.297), 2018Q1 (0.902), 2020Q1 (1.218), 2020Q2 (6.971), 2020Q3 (0.958), 2020Q4 (2.987), 2023Q1 (1.710), 2023Q2 (1.361), 2023Q3 (2.596), 2024Q1 (1.457), 2024Q2 (1.020), 2025Q3 (1.554)
Germany Factor 1: (0.463; 0.504)	2001Q4 (1.123), 2002Q1 (1.701), 2007Q4 (1.059), 2008Q3 (1.464), 2008Q4 (0.973), 2009Q1 (2.469), 2009Q2 (2.018), 2009Q3 (1.386), 2010Q4 (3.469), 2012Q1 (1.612), 2012Q2 (1.140), 2015Q1 (0.909), 2019Q1 (1.096), 2020Q1 (1.338), 2020Q2 (6.362), 2020Q3 (2.869), 2020Q4 (1.376), 2023Q1 (1.550), 2024Q3 (1.233), 2026Q1 (0.914)
Spain Factor 1: (0.487; 0.430)	2008Q2 (1.260), 2008Q3 (1.992), 2008Q4 (3.554), 2009Q1 (3.519), 2009Q2 (2.841), 2009Q3 (1.717), 2011Q1 (0.911), 2012Q4 (3.520), 2013Q1 (1.200), 2019Q2 (1.003), 2020Q1 (1.378), 2020Q2 (6.726), 2020Q3 (2.545), 2020Q4 (3.279), 2023Q1 (1.423), 2023Q2 (1.605), 2024Q3 (1.226)
United Kingdom Factor 1: (0.504; 0.425)	2008Q3 (1.643), 2008Q4 (2.435), 2009Q1 (2.478), 2009Q2 (2.161), 2009Q3 (1.390), 2010Q1 (1.204), 2011Q2 (3.325), 2011Q4 (0.903), 2018Q2 (1.017), 2020Q2 (7.387), 2020Q3 (3.132), 2020Q4 (1.764), 2021Q1 (1.402), 2023Q1 (1.564), 2023Q2 (0.924), 2023Q4 (0.918), 2024Q1 (1.188)
Greece Factor 1: (0.462; 0.367)	2005Q2 (1.293), 2008Q4 (1.071), 2009Q1 (2.743), 2009Q2 (1.532), 2009Q3 (2.619), 2010Q4 (1.410), 2011Q1 (1.817), 2012Q2 (2.151), 2012Q4 (2.744), 2013Q1 (3.049), 2015Q3 (1.531), 2015Q4 (1.740), 2020Q2 (2.010), 2020Q3 (1.989), 2020Q4 (1.831), 2023Q1 (1.738), 2023Q2 (1.044)
Ireland Factor 1: (0.396; 0.466)	2001Q4 (1.225), 2007Q3 (1.819), 2008Q2 (2.095), 2008Q4 (3.331), 2009Q1 (2.110), 2009Q2 (1.167), 2010Q1 (1.936), 2010Q3 (1.878), 2011Q3 (2.509), 2016Q1 (3.316), 2016Q2 (1.835), 2016Q3 (2.422), 2016Q4 (1.860), 2018Q2 (2.353), 2018Q3 (1.261), 2020Q2 (3.817), 2020Q4 (2.448), 2021Q1 (2.815), 2023Q3 (1.363), 2023Q4 (1.395), 2025Q3 (1.064), 2025Q4 (1.075), 2026Q1 (2.330)
Luxembourg Factor 4: (0.442; 0.445)	2001Q4 (1.257), 2002Q1 (1.307), 2002Q2 (2.007), 2006Q4 (1.945), 2007Q1 (1.578), 2007Q2 (1.687), 2007Q3 (1.610), 2008Q1 (1.336), 2008Q2 (1.024), 2008Q4 (1.253), 2009Q1 (2.318), 2009Q3 (2.424), 2014Q2 (1.384), 2017Q4 (1.073), 2018Q4 (1.012), 2019Q1 (2.051), 2019Q2 (2.127), 2024Q2 (1.426), 2024Q3 (2.221), 2024Q4 (1.088)
Netherlands Factor 1: (0.529; 0.472)	2001Q4 (1.108), 2002Q3 (1.101), 2008Q4 (5.489), 2009Q1 (3.975), 2009Q2 (2.598), 2009Q3 (4.432), 2012Q3 (1.032), 2015Q1 (1.599), 2020Q1 (1.577), 2020Q2 (5.300), 2020Q3 (2.757), 2020Q4 (2.462), 2021Q1 (0.945), 2025Q4 (0.933)

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Table H2 – continued from the previous page

Country (Loading $r-g$; FA)	***Quarters (Factor Scores)***
Portugal Factor 1: (0.480; 0.510)	2003Q1 (1.206), 2005Q3 (0.918), 2009Q1 (2.641), 2009Q2 (2.404), 2009Q4 (1.406), 2010Q3 (1.032), 2011Q2 (3.084), 2011Q3 (1.478), 2012Q1 (2.764), 2014Q3 (3.354), 2016Q2 (2.401), 2017Q1 (0.907), 2018Q4 (0.918), 2020Q2 (4.870), 2020Q3 (2.532), 2020Q4 (2.814), 2022Q2 (1.316), 2023Q1 (0.937), 2024Q2 (1.976), 2024Q4 (2.210), 2025Q1 (1.003), 2025Q3 (1.979)
Slovakia Factor 4: (0.477; 0.357)	2002Q2 (1.440), 2002Q3 (1.097), 2002Q4 (1.744), 2003Q1 (2.518), 2003Q2 (2.002), 2003Q3 (3.049), 2003Q4 (2.129), 2004Q1 (1.970), 2004Q2 (1.679), 2004Q3 (1.181), 2004Q4 (1.475), 2009Q3 (1.429), 2009Q4 (1.575), 2010Q1 (2.471), 2010Q2 (2.330), 2010Q3 (1.864), 2010Q4 (1.030), 2013Q4 (1.618), 2014Q2 (0.915), 2014Q3 (1.211), 2014Q4 (1.000), 2015Q1 (0.957), 2015Q2 (2.014), 2015Q3 (0.915), 2016Q4 (1.056), 2017Q1 (1.330), 2020Q2 (1.069), 2020Q4 (0.938)
Slovenia Factor 1: (0.472; 0.454)	2001Q3 (1.257), 2002Q3 (1.372), 2009Q1 (3.684), 2009Q2 (5.348), 2009Q3 (4.817), 2009Q4 (3.124), 2012Q2 (1.328), 2012Q3 (1.545), 2012Q4 (2.213), 2013Q1 (1.375), 2013Q2 (1.700), 2015Q3 (0.953), 2018Q4 (1.037), 2019Q2 (0.932), 2020Q1 (2.160), 2020Q2 (4.159), 2020Q3 (0.984), 2020Q4 (2.410), 2023Q1 (1.417), 2023Q3 (1.310), 2024Q4 (1.479)
United States Factor 1: (0.515; 0.478)	2001Q3 (1.000), 2002Q2 (0.962), 2007Q1 (1.007), 2008Q3 (1.570), 2008Q4 (2.582), 2009Q1 (1.753), 2009Q2 (2.508), 2016Q1 (1.100), 2020Q1 (1.110), 2020Q2 (8.452), 2020Q3 (2.277), 2020Q4 (1.018), 2022Q2 (0.971), 2023Q2 (1.577), 2023Q3 (1.539), 2023Q4 (1.048)
FA contraction ($FA < -0.3$)	
Belgium Factor 6: (0.778; -0.305)	2006Q1 (1.346), 2022Q2 (1.321), 2023Q4 (1.483), 2024Q1 (1.310), 2024Q2 (1.037)
Ireland Factor 5: (0.693; -0.318)	2010Q2 (1.338), 2011Q1 (2.518), 2012Q3 (1.396), 2013Q3 (1.000), 2016Q1 (1.569), 2016Q2 (1.485), 2016Q3 (0.914), 2016Q4 (1.562), 2022Q1 (0.990), 2022Q2 (1.133), 2022Q4 (1.078), 2023Q2 (0.903), 2023Q3 (1.067)
Italy Factor 2: (0.334; -0.412)	2007Q4 (0.936), 2008Q3 (1.859), 2008Q4 (1.974), 2009Q1 (0.957), 2010Q4 (1.351), 2011Q4 (1.403), 2018Q3 (1.086), 2018Q4 (1.146), 2020Q2 (3.555), 2021Q4 (1.192), 2022Q1 (2.736), 2022Q2 (5.455), 2022Q3 (4.842), 2022Q4 (3.066), 2023Q2 (1.322), 2025Q1 (1.005)
Luxembourg Factor 5: (0.356; -0.364)	2003Q1 (1.522), 2004Q2 (1.000), 2004Q4 (1.745), 2005Q1 (0.974), 2008Q3 (1.841), 2008Q4 (1.148), 2011Q3 (1.398), 2011Q4 (1.636), 2014Q4 (1.566), 2016Q2 (1.137), 2019Q2 (1.323), 2019Q3 (0.964), 2022Q1 (1.778), 2022Q2 (1.214), 2022Q3 (1.724), 2022Q4 (2.542), 2024Q1 (1.170), 2024Q2 (1.711)
Portugal Factor 6: (0.713; -0.505)	2011Q3 (1.013), 2011Q4 (0.992), 2020Q1 (1.193), 2020Q2 (1.668), 2023Q4 (1.011), 2025Q4 (1.348)
Loadings $r-g < -0.3$ (Negative factor scores < -0.9)	
FA expansion ($FA < -0.3$) – empty set	
FA contraction ($FA > 0.3$)	
Finland Factor 2: (-0.408; 0.595)	2001Q2 (-1.584), 2001Q3 (-2.255), 2001Q4 (-2.097), 2002Q1 (-1.940), 2002Q2 (-1.346), 2008Q1 (-0.914), 2008Q2 (-2.352), 2008Q3 (-3.293), 2008Q4 (-4.245), 2009Q1 (-3.477), 2009Q2 (-3.530), 2009Q3 (-1.372), 2011Q4 (-1.847), 2012Q1 (-1.602), 2018Q4 (-1.462), 2022Q2 (-2.181), 2022Q3 (-2.292), 2022Q4 (-1.716), 2023Q1 (-1.583), 2023Q2 (-1.326), 2023Q3 (-1.315), 2023Q4 (-1.402), 2024Q1 (-1.119)

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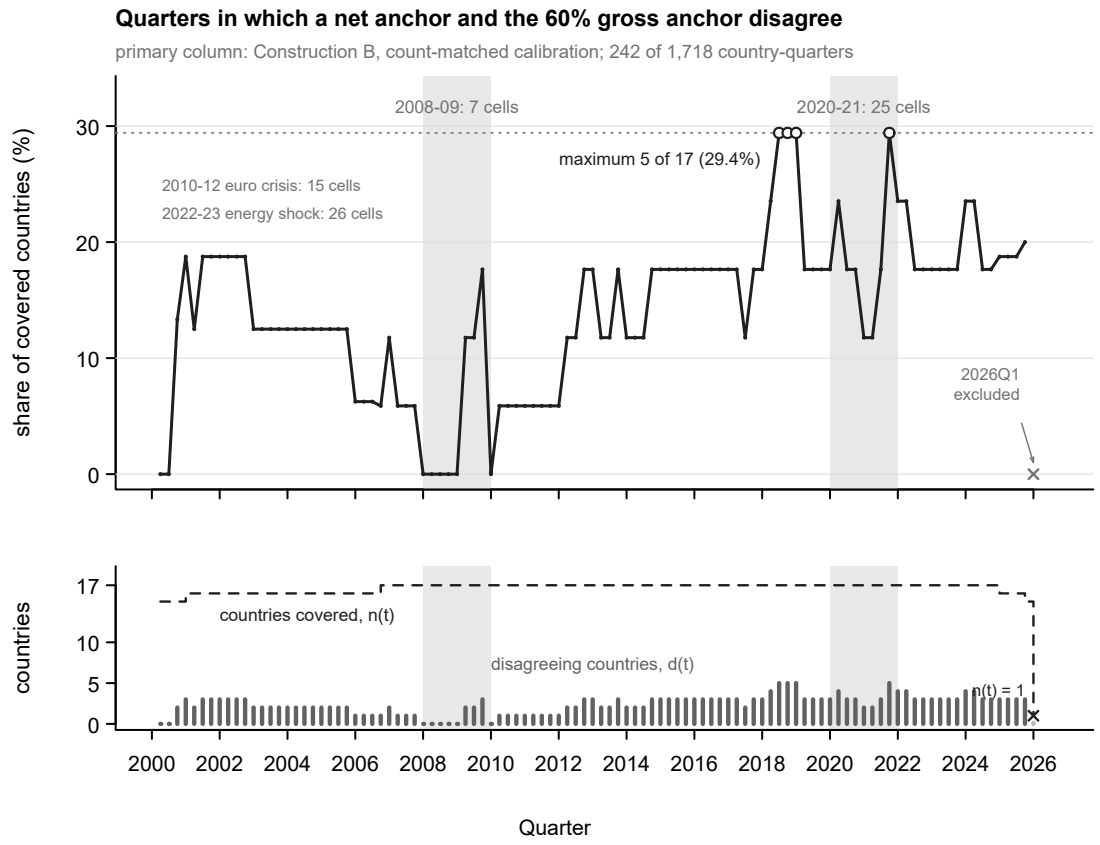
Table H2 – continued from the previous page

Country (Loading $r - g$; FA)	*Quarters (Factor Scores)*
United Kingdom Factor 4: (-0.325; 0.782)	2009Q1 (-1.189), 2009Q4 (-1.692), 2011Q1 (-2.773), 2011Q3 (-1.883), 2012Q2 (-0.940), 2013Q1 (-1.014), 2020Q4 (-1.050), 2022Q3 (-0.929)
Greece Factor 3: (-0.327; 0.312)	2007Q1 (-0.970), 2008Q2 (-1.090), 2008Q3 (-1.318), 2009Q1 (-1.032), 2011Q2 (-2.848), 2011Q3 (-1.070), 2012Q1 (-1.561), 2013Q2 (-2.535), 2014Q4 (-1.028), 2015Q1 (-2.122), 2020Q1 (-1.012), 2020Q2 (-3.170), 2020Q3 (-1.924), 2020Q4 (-2.443), 2021Q1 (-0.929), 2022Q3 (-1.081), 2022Q4 (-0.939)
Ireland Factor 4: (-0.421; 0.524)	2007Q4 (-0.962), 2008Q1 (-1.166), 2009Q3 (-1.558), 2010Q1 (-1.496), 2012Q1 (-2.569), 2012Q3 (-1.683), 2016Q1 (-1.192), 2016Q2 (-0.935), 2016Q3 (-1.222), 2019Q3 (-0.975), 2020Q4 (-0.915)
Luxembourg Factor 1: (-0.432; 0.416)	2004Q4 (-0.983), 2005Q1 (-1.861), 2005Q2 (-0.921), 2006Q2 (-1.103), 2006Q3 (-1.607), 2006Q4 (-1.452), 2009Q1 (-1.980), 2009Q3 (-0.930), 2011Q2 (-1.349), 2015Q4 (-1.736), 2016Q1 (-1.955), 2018Q1 (-1.280), 2018Q2 (-1.732), 2020Q2 (-1.937), 2022Q1 (-2.927), 2022Q2 (-4.622), 2022Q3 (-3.124), 2022Q4 (-2.790), 2023Q3 (-0.990)

Figure H1: Countries classified differently by the two anchors, by quarter



Notes: the upper panel plots, for each quarter, the number of covered countries whose breach status differs between the two anchors as a share of the countries covered in that quarter, on the primary construction and the count-matched calibration described in the text. The share and not the count is plotted because coverage moves between fifteen, sixteen and seventeen countries; the lower panel prints both the count $d(t)$ and the denominator $n(t)$, so the moving base is visible. The 2008–09 and 2020–21 windows are shaded and the 2010–12 and 2022–23 windows are annotated with their totals. The four quarters at the maximum, five of seventeen, are ringed. 2026Q1 covers the United Kingdom alone, and a single-country quarter cannot evidence agreement, so it was excluded from the ranking of largest quarters in advance and is drawn hollow. The plotted counts sum to the 242 changed country-quarters reported in Section 6.6.

The fifth probe of Section 5, the structural-break tests at known dates, is a Chow test of the asset and debt rows of one country on the pooled factors and on their interaction with a dummy for the quarters from the date onwards, with a Newey–West covariance and a null-imposed moving-block bootstrap of the residuals (block length eight, 2,000 draws per cell). On panels simulated without a break from the fitted state space of the identifying window, the test rejects in 5.70 per cent of 2,000 tests at its nominal five per cent (5.05 per cent on the full-sample space). At the eight programme entry and exit dates, the twelve countries without a programme reject in 4 of 96 cells on the identifying window (4.2 per cent) and in 17 of 96 on the full-sample space (17.7 per cent), more often by a one-sided test of two proportions at $p = 0.001$; the countries with a programme reject at their own dates in 2 of 8 cases on the window and in 1 of 8 on the full sample. These figures come from B1_break_tests.R in the replication package, run on 13 October 2026 on the database adopted here, under a decision rule fixed on 10 October 2026, before its first run.

Table H3 sets each headline result of the paper beside its value on the database of the first round, on which the interest–growth differential was the conventional measure (0b), and on the database adopted here (0c). Eight rows change sign or significance. The coefficient tests of the current account balance and of gross capital formation on the full sample become significant at five per cent, their Holm-adjusted p moving from 0.291 to 0.027 and 0.010. The regime difference in the financial-asset coefficient of the regression for $r - g$ changes sign in six cells of the smooth-transition fits: in the first specification in levels on the identifying window and on the full sample, untrimmed and trimmed, and in the second specification in levels on the full sample, untrimmed and trimmed, where, untrimmed, it also becomes significant at five per cent ($p = 0.042$). None of the six is a cell this paper reports as primary.

Table H3: Headline results on the Phase 0b and Phase 0c databases

Quantity	0b	0c	p 0b	p 0c	flag
Factor count					
Bai-Ng IC_p1, identifying (pre-2020)	3	3			
Bai-Ng IC_p2, identifying (pre-2020)	3	3			
Ahn-Horenstein GR, identifying (pre-2020)	3	1			CHANGED
Onatski ED, identifying (pre-2020)	3	3			
Marchenko-Pastur edge, identifying (pre-2020)	3	3			
eigenvalue ratio ER, identifying (pre-2020)	1	1			
Bai-Ng IC_p1, companion (full sample)	4	4			
Bai-Ng IC_p2, companion (full sample)	3	4			CHANGED
Ahn-Horenstein GR, companion (full sample)	1	1			
Onatski ED, companion (full sample)	2	2			
Marchenko-Pastur edge, companion (full sample)	4	4			
eigenvalue ratio ER, companion (full sample)	1	1			
Canonical correlations					
identifying penalty alpha (warm-start BIC)	5.9636	3.7276			CHANGED
pre-2020 r=4 against full-sample space, first	0.9999	0.9995			
pre-2020 r=4 against full-sample space, second	0.9899	0.9946			
pre-2020 r=4 against full-sample space, third	0.9744	0.9703			
pre-2020 r=4 against full-sample space, fourth	0.2767	0.3038			
pre-2020 r=3 against full-sample space, first	0.9999	0.9998			
pre-2020 r=3 against full-sample space, second	0.9907	0.9870			
pre-2020 r=3 against full-sample space, third	0.9714	0.9709			
AR(1) against independent errors, pre-2020, first	0.9996	0.9996			

Quantity	0b	0c	p 0b	p 0c	flag
AR(1) against independent errors, pre-2020, second	0.9950	0.9967			
AR(1) against independent errors, pre-2020, third	0.9839	0.9698			
cold against warm start, pre-2020, first	1.0000	1.0000			
cold against warm start, pre-2020, second	0.9996	0.9993			
cold against warm start, pre-2020, third	0.9988	0.9883			
Coefficient tests					
Wald W/df, r-g, pre-2020 (Holm block-bootstrap p)	23.9482	8.6687	0.0035	0.0035	
Wald W/df, r, pre-2020 (Holm block-bootstrap p)	11.5876	12.1031	0.0035	0.0035	
Wald W/df, Ext.Trade, pre-2020 (Holm block-bootstrap p)	7.4473	6.9138	0.0035	0.0035	
Wald W/df, CapitalFormation, pre-2020 (Holm block-bootstrap p)	7.1319	8.7344	0.0035	0.0035	
Wald W/df, dGov.Debt, pre-2020 (Holm block-bootstrap p)	7.9088	8.0348	0.0035	0.0035	
Wald W/df, PrimaryDeficit, pre-2020 (Holm block-bootstrap p)	4.1331	3.8210	0.0035	0.0035	
Wald W/df, dFinAssets, pre-2020 (Holm block-bootstrap p)	8.1964	4.4752	0.0035	0.0035	
composite p*, original null, pre-2020	0.2287	0.2773			
Wald W/df, r-g, full sample (Holm block-bootstrap p)	75.8889	16.2107	0.0035	0.0035	
Wald W/df, r, full sample (Holm block-bootstrap p)	7.6242	8.8558	0.0075	0.0035	
Wald W/df, Ext.Trade, full sample (Holm block-bootstrap p)	3.7659	4.8024	0.2909	0.0270	SIG5
Wald W/df, CapitalFormation, full sample (Holm block-bootstrap p)	3.4677	5.8036	0.2909	0.0100	SIG5
Wald W/df, dGov.Debt, full sample (Holm block-bootstrap p)	15.2010	12.7135	0.0035	0.0035	
Wald W/df, PrimaryDeficit, full sample (Holm block-bootstrap p)	7.4091	4.3576	0.0060	0.0035	
Wald W/df, dFinAssets, full sample (Holm block-bootstrap p)	15.6649	7.8218	0.0035	0.0035	
composite p*, original null, full sample	0.0942	0.0636			
composite p*, restricted null (P6), pre-2020	0.5248	0.2313			
Mardia skewness					
b1,p of the factor scores, pre2020_r3	8.8309	6.8327			
b1,p of the factor scores, full_r4	9.7231	11.1945			
b1,p of the factor scores, covidNA_r3	5.1914	3.9470			
b1,p of the factor scores, covidNA_r4	8.5454	14.3290			
Linearity					
pooled LM3 bootstrap p, S1, pre-2020	0.0030	0.0010			
countries rejecting at 5% (LM3 bootstrap), S1, pre-2020	11	12			CHANGED
pooled LM3 bootstrap p, S2, pre-2020	0.0060	0.0070			
countries rejecting at 5% (LM3 bootstrap), S2, pre-2020	14	13			CHANGED
pooled LM3 bootstrap p, S3, pre-2020	0.0020	0.0010			
countries rejecting at 5% (LM3 bootstrap), S3, pre-2020	7	13			CHANGED
pooled LM3 bootstrap p, S1, full sample	0.0010	0.0010			
countries rejecting at 5% (LM3 bootstrap), S1, full sample	13	13			
pooled LM3 bootstrap p, S2, full sample	0.0010	0.0030			
countries rejecting at 5% (LM3 bootstrap), S2, full sample	15	15			
pooled LM3 bootstrap p, S3, full sample	0.0010	0.0010			
countries rejecting at 5% (LM3 bootstrap), S3, full sample	14	12			CHANGED
Regime differences					
dFA regime difference, pre2020_rg_S1, LSTR	-0.0100	0.2571	0.9255	0.5659	SIGN
dFA regime difference, full_rg_S1, LSTR	0.4279	-0.0665	0.3946	0.7954	SIGN
dFA regime difference, pre2020_r_S1, LSTR	-0.2324	-0.2324	0.0345	0.0345	

Quantity	0b	0c	p 0b	p 0c	flag
dFA regime difference, full_r_S1, LSTR	0.2592	0.2592	0.1358	0.1358	
dFA regime difference, pre2020_rg_S2, LSTR	0.0975	0.1517	0.5750	0.1162	
dFA regime difference, full_rg_S2, LSTR	-0.0324	0.2202	0.8351	0.0418	SIGN SIG5
dFA regime difference, pre2020_r_S2, LSTR	0.0924	0.0924	0.4101	0.4101	
dFA regime difference, full_r_S2, LSTR	0.1114	0.1114	0.3501	0.3501	
dFA regime difference, pre2020_rg_S3, LSTR	0.3526	0.0320	0.4254	0.3397	
dFA regime difference, full_rg_S3, LSTR	0.1160	0.0630	0.2806	0.1892	
dFA regime difference, pre2020_r_S3, LSTR	0.1019	0.1019	0.0096	0.0096	
dFA regime difference, full_r_S3, LSTR	0.0846	0.0846	0.0119	0.0119	
dFA regime difference, pre2020_rg_S1, LSTR, 15% trimmed	-0.0100	0.3717	0.9255	0.8975	SIGN
dFA regime difference, full_rg_S1, LSTR, 15% trimmed	0.1672	-0.0665	0.6191	0.7954	SIGN
dFA regime difference, pre2020_r_S1, LSTR, 15% trimmed	-0.2525	-0.2525	0.0754	0.0754	
dFA regime difference, full_r_S1, LSTR, 15% trimmed	-0.1750	-0.1750	0.1218	0.1218	
dFA regime difference, pre2020_rg_S2, LSTR, 15% trimmed	0.0975	0.1517	0.5750	0.1162	
dFA regime difference, full_rg_S2, LSTR, 15% trimmed	-0.0324	0.1452	0.8351	0.0763	SIGN
dFA regime difference, pre2020_r_S2, LSTR, 15% trimmed	0.0924	0.0924	0.4101	0.4101	
dFA regime difference, full_r_S2, LSTR, 15% trimmed	0.1114	0.1114	0.3501	0.3501	
dFA regime difference, pre2020_rg_S3, LSTR, 15% trimmed	0.1763	0.0320	0.4707	0.3397	
dFA regime difference, full_rg_S3, LSTR, 15% trimmed	0.1395	0.0630	0.0782	0.1892	
dFA regime difference, pre2020_r_S3, LSTR, 15% trimmed	0.0981	0.0981	0.0197	0.0197	
dFA regime difference, full_r_S3, LSTR, 15% trimmed	1.1117	1.1117	0.0170	0.0170	
Co-loading counts					
countries co-loading, full sample r=4, union panel (of 17)	11	12			CHANGED
countries co-loading, full sample r=4, balanced panel (of 17)	13	14			CHANGED
countries co-loading, covidNA_r3 (of 17)	8	7			CHANGED
countries co-loading, covidNA_r4 (of 17)	6	7			CHANGED
countries co-loading, full_r4 (of 17)	11	12			CHANGED
countries co-loading, pre2020_r3 (of 17)	4	2			CHANGED
Break tests					
placebo rejection rate, window_r3	0.0729	0.0417			
size-test rejection rate, window_r3	0.0525	0.0570			
placebo rejection rate, full_r4	0.4479	0.1771			
size-test rejection rate, full_r4	0.0445	0.0505			
placebo rejection rate, window_r4_descriptive	0.0625	0.0625			
size-test rejection rate, window_r4_descriptive	0.0540	0.0535			
Net anchor					
country-quarters changing status, B_primary count-matched	242	242			
Spearman rank correlation, gross against net, B_primary	0.9240	0.9240			

Notes: 0b is the database of the first round, on which $r - g$ was the conventional measure, and 0c the database adopted here (Section 4); the p columns give the probability the row's own test reports. Rows name the specification as the replication files do: S1 and S2 are the two smooth-transition specifications in levels and S3 the one in first differences, rg and r the regressions for $r - g$ and for the ten-year yield, LSTR the smooth-transition fit, trimmed at 15 per cent where stated. Flags: SIGN, a change of sign; SIG5, a change of significance at five per cent; CHANGED, any other change in a count or a selection. No probability is shown for Mardia's statistic: the only reference available assumes independent draws, which these scores do not satisfy (Section 6.3).

Three further checks of the identifying factor space were run on the database

adopted here, and Table H4 reports them. The estimator exempts the first series of the panel, Austria's $r - g$, from the penalty (Section 5); refitted with no exemption, and again under five random orders of the seventeen country blocks, the specification selects the same penalty every time and recovers a factor space whose canonical correlations with the adopted one are all above 0.99, against a threshold of 0.95 fixed before the refits, so neither the exemption nor the ordering carries the space. Transforming every series of a variable in the same way in all seventeen countries, by the order of differencing most countries take, which is a first difference for all seven variables and changes 18 of the 119 transformations, recovers the space at 0.9998, 0.9959 and 0.9470; equal coefficients are still rejected at five per cent for $r - g$, the ten-year yield, gross capital formation and the primary balance, but not for the current account balance, the change in gross debt and the change in financial assets, so for those three the rejection is not invariant to the choice of transformation. Resampling the 79 quarters of the identifying window in blocks of eight and refitting 499 times gives the 90 per cent intervals of panel C: the common component explains between 0.387 and 0.765 of the variance of $r - g$ on average across countries, and the third canonical correlation with the full-sample space, at 0.9709, is the poorly determined one, its interval running from 0.6404 to 0.9842. These intervals measure the sampling uncertainty of the identifying fit only.

Superseded: the accumulation-state results. The analysis below is superseded by the test of Section 6.4 and is kept for the record. It places the transition on the lagged four-quarter change in the debt ratio rather than on the debt level, has country but not quarter fixed effects, and computes standard errors clustered by country conditional on the transition the grid search selects. Under the search-aware inference of Section 6.4, the same accumulation-state transition gives a difference in the yield's asset coefficient of $+0.100$ with $p = 0.065$ (S-A in Table 4). The figures below are on the database adopted here.

The model of Section 3 implies something sharper than state dependence in general: the response of $r - g$ to changes in financial assets should differ between low- and high-stress regimes, and in a particular direction. Taken literally, on the identifying window, that restriction finds no support. What survives, in the first-difference specification only, is the pricing channel that generates it, tested on a different interest rate, and the distance between those two statements is the subject of this analysis; we set them out in that order rather than lead with the more favourable one. What the first-difference specification estimates should be stated before either is read, because the transformation compounds. The dependent variable is the quarterly change in the rate under test, and the asset regressor is the quarterly change in a variable that is itself a four-quarter change in the assets-to-GDP ratio; the coefficient

Table H4: Checks of the identifying factor space

A. Exemption and ordering of the first block: canonical correlations with the adopted space			
Refit	first	second	third
no exemption ($q = 0$)	1.0000	0.9999	0.9990
country order 101, LVA first	1.0000	0.9998	0.9985
country order 102, SVN first	1.0000	0.9971	0.9909
country order 103, ITA first	1.0000	0.9997	0.9980
country order 104, SVK first	1.0000	0.9968	0.9913
country order 105, USA first	1.0000	0.9999	0.9988
Rule (threshold 0.95, fixed before the refits): robust; minimum 0.9909			
B. One transformation per variable in all seventeen countries			
canonical correlations	0.9998	0.9959	0.9470
Equal coefficients, variable	W uniform	Holm p uniform	Holm p adopted
$r - g$	388.0	0.0175	0.0035
ten-year yield y^{10}	568.9	0.0035	0.0035
current account balance	260.8	0.0585	0.0035
gross capital formation	296.5	0.0220	0.0035
change in gross debt	135.9	0.0720	0.0035
primary balance	264.6	0.0035	0.0035
change in financial assets	108.7	0.0925	0.0035
C. Block bootstrap of the identifying window, 499 refits			
Quantity	point	90% interval	
common share, $r - g$	0.637	[0.387, 0.765]	
common share, ten-year yield y^{10}	0.575	[0.507, 0.698]	
common share, current account balance	0.105	[0.061, 0.247]	
common share, gross capital formation	0.305	[0.148, 0.489]	
common share, change in gross debt	0.217	[0.070, 0.392]	
common share, primary balance	0.261	[0.089, 0.473]	
common share, change in financial assets	0.141	[0.078, 0.249]	
canonical correlation with the full-sample space, first	0.9998	[0.9973, 0.9999]	
canonical correlation with the full-sample space, second	0.9870	[0.9408, 0.9971]	
canonical correlation with the full-sample space, third	0.9709	[0.6404, 0.9842]	

Notes: identifying specification, 2000Q2–2019Q4, three factors. Panel A: the adopted fit exempts the first series from the penalty; the first row refits it with no exemption, the next five with the seventeen country blocks in a random order (seed and first country given), all at the adopted penalty, which each refit selects. Panel B: every series of a variable takes the order of differencing most countries take, a first difference for all seven variables; W and the Holm-adjusted block-bootstrap p are those of the coefficient-equality tests of Section 6.2. Panel C: the 79 quarters are resampled jointly in blocks of eight and the model refitted, penalty included; the full-sample space is held fixed. The common share is the mean over the seventeen countries of the share of a series' variance the common component explains.

therefore relates a quarterly movement in a rate to a quarterly movement in the annual rate of asset accumulation, and not to an asset purchase of a given size. Taken to the interest–growth differential of the accounting identity (2)–(3), whose i^{eff} is the effective rate on the debt already outstanding, the restriction is not supported, and it is not supported in any of the three specifications. In first differences the regime difference in the financial-asset coefficient is +0.032 with a standard error of 0.033 ($p = 0.340$), the low-stress coefficient is +0.008 (0.031, $p = 0.795$) and the high-stress coefficient +0.040 (0.033, $p = 0.238$), on 1,197 country-quarters for seventeen countries over 2001Q2–2019Q4; the two specifications in levels give +0.257 ($p = 0.566$) and +0.152 ($p = 0.116$). None of that is evidence that the difference is zero. These are failures to find a difference in designs whose power against one is not established, and the discussion in Section 6.2 of what the homogeneity rule could have detected applies in spirit here too: a test that cannot see an effect is silent about it, and the restriction is recorded as unsupported rather than as refuted. That null is not an artefact of where the grid happens to place the transition. Profiling the smoothness parameter over all forty-two of its grid values at which the transition is estimable, in each of the three specifications, the smallest p-value the regime difference attains anywhere on the pre-2020 sample is 0.112. The selected transition assigns 0.180 of the sample to the stressed regime. On $r - g$ there is therefore no significant difference between the regimes at the one, five or ten per cent level. Taken instead to the ten-year government bond yield, on the same country-quarters, the same seventeen countries and the same specification, the restriction is supported in its state dependence and not in its quiet-state level. The low-stress coefficient is −0.0091 (0.0030, $p = 0.007$), the high-stress coefficient +0.0928 (0.0329, $p = 0.012$) and the difference between them +0.102 (0.035, $p = 0.010$), so both regime coefficients are individually significant. Their signs need stating carefully, because only one of the two is what the model predicts. Equation (14) makes the premium's response to ΔFA equal to $\beta_{FA,PB}\,\delta_{GD}\,(1 - S_I)$ once debt accumulation and the primary balance are held fixed, as this regression holds them; with $\beta_{FA,PB} > 0$ that quantity is non-negative in every state and vanishes as debt stress does. The model therefore predicts a positive yield response under stress, which the high-stress coefficient delivers, and approximately none without it, which the low-stress coefficient does not: −0.0091 at $p = 0.007$ is significantly below the zero the model implies rather than a confirmation of it. The negative sign the model does predict is a sign for $r - g$, and it arises entirely from the growth channel Φ^{FA}, which subtracting g brings into the differential and which a market yield does not contain. What survives as predicted is the direction of the state dependence, the difference being positive, and that is the quantity this analysis reports; the quiet-state level is not, and is recorded here as unexplained by the model rather than as evidence for it. Three qualifications belong with that figure

and none of them is cosmetic. The smoothness parameter is estimated at 200, the top of its grid, so what the data select is a threshold in the limit rather than a smooth transition, and the value is a boundary rather than an interior estimate. The stressed regime covers 0.100 of the country-quarters, so the positive coefficient is identified on a tenth of the panel. And the dependent variable is not the i^{eff} of the accounting identity, which is the substitution that has to be argued rather than assumed: the positive sign Proposition 1 predicts comes from the premium channel of (14) and (15), a risk premium that bondholders price on new issuance, and an effective rate, being a weighted average of coupons contracted over the preceding decade, cannot carry one. The yield is the marginal cost of funds and is where that premium is priced; the effective rate is the average cost of funds and is what the identity (2)–(3) propagates. They are two objects answering to two equations, written i^{eff} and y^{10} in Section 3, and the result should be read as a statement about sovereign pricing and not about the arithmetic of the debt ratio. Further qualifications apply to it. The standard errors are computed conditional on the transition parameters chosen by the grid search, so they omit the sampling variability of that choice and are optimistic by an amount this design does not quantify; against that, at every one of the twenty-one grid points whose residual sum of squares falls inside the confidence set the difference lies between +0.101 and +0.105, is significant at five per cent and carries the same sign pattern as the headline estimate, a negative low-stress and a positive high-stress coefficient, so the conditioning is not what produces it. The standard errors are clustered by country, which allows for serial dependence within each country but not for any contemporaneous shock common to several countries at once. The transition switches on the lagged four-quarter change in the debt ratio, whereas the debt-stress scaler of Section 3 is a function of the debt level relative to a country threshold, so the prediction is being tested against debt accumulation rather than against the debt position the theory names, and the two need not order the same quarters as stressed. The two specifications in levels say as much, and one of them says more. The specification built on a reconstructed debt-level proxy returns +0.092 ($p = 0.410$) on the yield and simply does not reproduce the result; the specification that keeps debt accumulation as the transition variable but leaves the system in levels returns −0.232 ($p = 0.035$), significant at five per cent with the opposite sign, on a fit that places 0.979 of the sample in the stressed regime and identifies the other coefficient off the residual two per cent. We lead with the first-difference specification on the ground given above, that it is the one of the three whose maintained linear benchmark comes closest to a balanced regression, and a rejection against an unbalanced benchmark is a rejection against a null misspecified for a reason unrelated to the transition; but a result that reverses under a specification the paper itself estimated is weaker than one that does not, and it is offered on that basis rather than as a settled finding.

Proposition 1 is in addition a partial statement, holding debt accumulation and the primary balance fixed, while the intervention that motivates it is a debt-financed purchase. The total effect of such a purchase would have to carry the associated debt change and its feedback through growth and borrowing costs; the coefficient reported here does not measure it. Finally, the quantity being decomposed is small to begin with: the linear fixed-effects benchmark explains 0.033 of the quarterly change in the yield, and the smooth-transition fit improves on it by 6.4 per cent of its residual sum of squares.

The honest reading is therefore split. That transmission is state-dependent is supported: linearity is rejected in the pooled panel at one per cent under each of the three specifications, on the third-order statistic — whose pooled bootstrap probabilities are anti-conservative by an amount this design does not quantify — and country by country, on the primary first-difference specification, in thirteen of the seventeen at five per cent, and the model's own restriction is supported in the pooled first-difference specification only, in its state dependence and not in its quiet-state level, on the ten-year yield. It is not supported on the interest–growth differential of the accounting identity, where no specification and no transition parameter brings the regime difference below $p = 0.10$, so the proposition is supported on its pricing half and unsupported on the half the paper's own identity is written in. *Which* state matters, and how sharply it separates, is not pinned down country by country, and this paper reports no per-country estimate of it. Establishing the mechanism at the country level would need either a longer sample or transition variables measured independently of the system, and we regard that as the natural next step rather than a settled result.

The asset variable of this paper is the four-quarter change in the stock of general-government financial assets as a ratio to GDP, and a change in a stock mixes transactions with revaluations and other changes in volume. For the fifteen EU countries of the panel the stock the paper uses equals the consolidated stock of Eurostat's quarterly financial accounts for general government, its ratio to that stock lying between 0.992 and 1.017 in every quarter, so the change can be split into the two. On the consolidated accounts transactions average between 0.12 per cent of GDP a year in Slovakia and 3.33 in Finland, and other flows between 0.06 in Italy and 2.80 in Finland. Outside Finland transactions account for between 0.40 and 0.86 of the variance of the four-quarter stock change, and correlate with the paper's asset change at between 0.47 and 0.90; in Finland, where other flows are nearly as large as transactions, they account for none of it (-0.04) and the correlation is -0.16. Table H5 sets out what the stock holds on the non-consolidated accounts, by instrument, at the end of 2019 and of 2025: equity and investment fund shares are the largest item in most countries. With the transactions measure in place of the paper's asset

change, on the same fifteen countries (S-D in Table 4), the targeted test of Section 6.4 does not reject ($p = 0.336$) for a difference of +0.166 between the debt states, whose 90 per cent interval, $[-0.016, +0.339]$, includes zero; because S-D changes the asset measure and the sample together, it does not separate the two.

Table H5: Composition of general government financial assets, 2019Q4 and 2025Q4

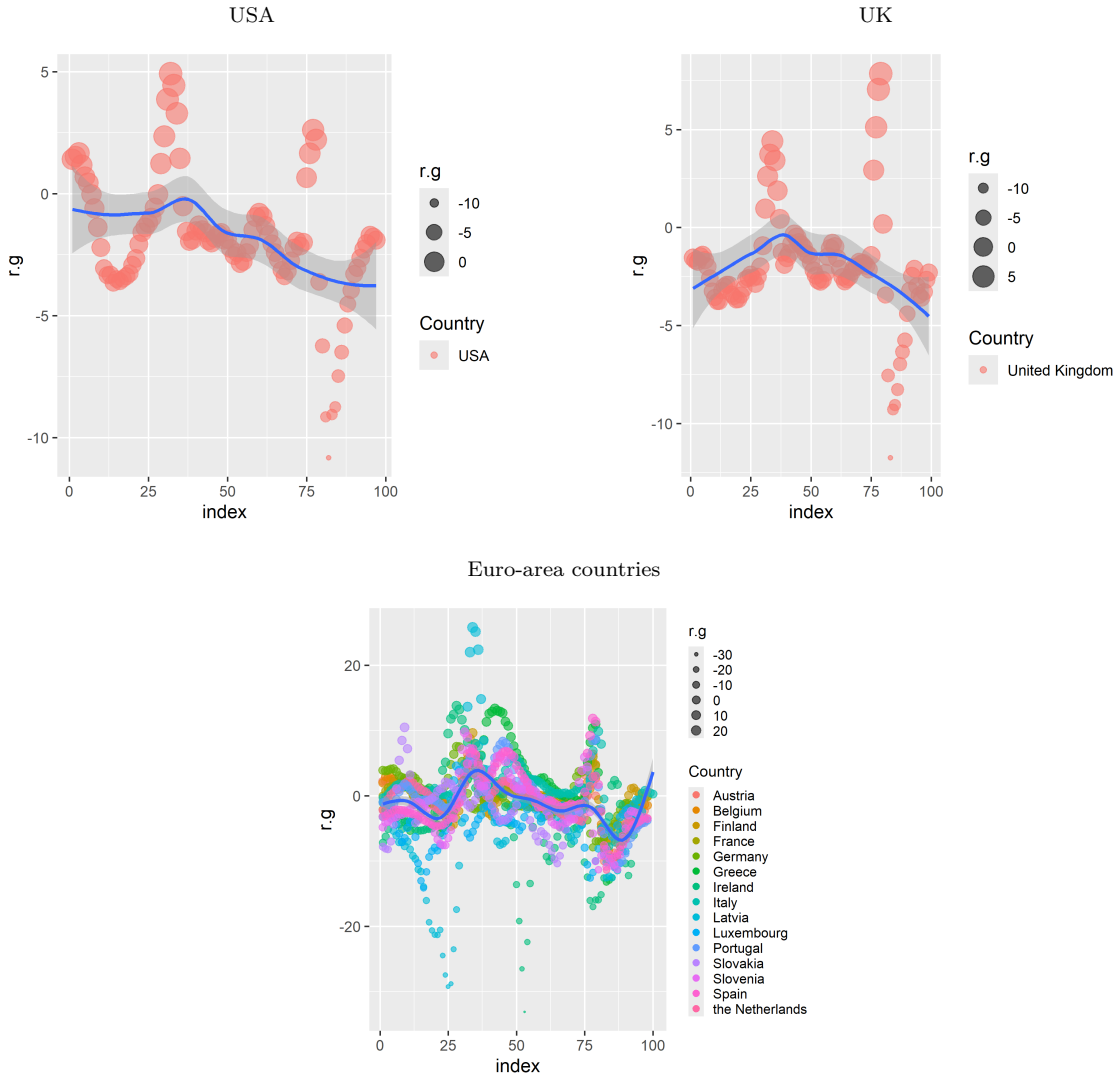
	2019Q4					2025Q4				
Country	F2	F3	F4	F5	Other	F2	F3	F4	F5	Other
Austria	10.6	9.0	25.6	39.9	14.8	7.5	6.9	30.0	39.8	15.8
Belgium	7.8	11.2	26.2	39.6	15.2	10.7	9.6	29.7	35.0	15.0
Finland	4.7	10.9	11.3	67.3	5.7	5.5	6.8	13.1	69.4	5.2
France	12.2	4.0	8.4	49.3	26.1	11.6	2.8	10.4	46.7	28.5
Germany	25.3	10.5	16.7	43.7	3.7	17.9	7.7	17.3	49.7	7.3
Greece	31.3	10.6	24.1	22.2	11.8	19.7	6.7	38.4	18.8	16.4
Ireland	23.6	6.9	9.0	36.3	24.2	16.8	38.4	11.3	20.8	12.7
Italy	14.5	6.8	24.2	32.3	22.2	13.6	15.7	20.0	32.2	18.6
Latvia	23.7	0.4	12.4	43.6	19.9	27.4	1.8	11.7	40.4	18.8
Luxembourg	20.8	18.6	2.9	44.4	13.3	16.3	18.3	2.5	47.5	15.4
Netherlands	15.5	2.7	24.8	30.5	26.5	24.4	1.8	21.0	25.6	27.2
Portugal	21.3	13.4	20.1	37.2	8.0	19.4	16.8	10.8	46.7	6.4
Slovakia	33.2	0.1	33.0	25.5	8.3	35.4	0.2	35.2	19.1	10.2
Slovenia	25.2	1.5	7.5	52.3	13.4	26.0	2.2	6.0	49.7	16.1
Spain	13.0	3.9	41.9	27.3	13.8	19.0	4.8	41.7	17.7	16.8

Notes: per cent of the total stock of financial assets (F) of general government (S13), non-consolidated, Eurostat gov_10q_ggfa (dataset updated 21 July 2026). F2 currency and deposits; F3 debt securities; F4 loans; F5 equity and investment fund shares; Other = 100−(F2+F3+F4+F5): F6 insurance, pension and standardised guarantee schemes, F7 financial derivatives, F8 other accounts receivable, and F1 monetary gold and SDRs if any. The United Kingdom and the United States are not covered by this EU database.

Appendix I Additional figures

Figure I1: Government interest rates–economic growth $(r - g)$ by within-country observation index

$r - g$ time series

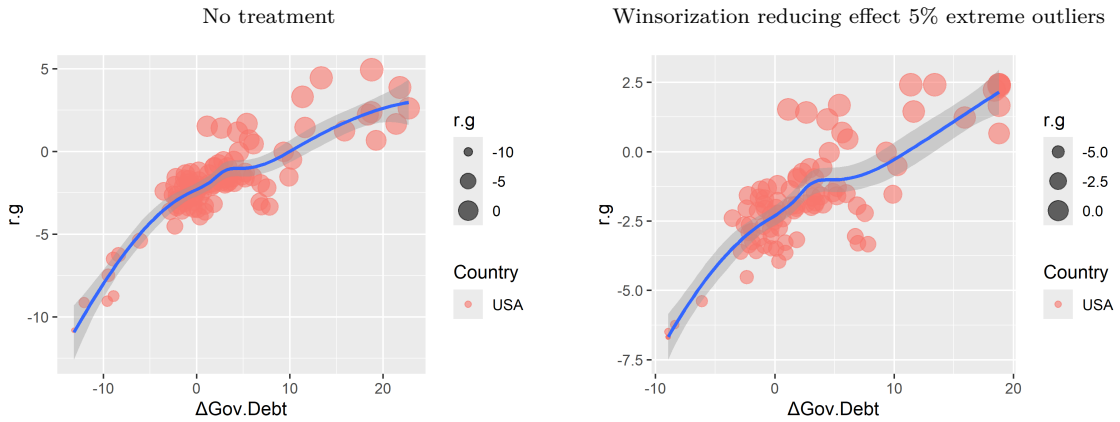


Notes: quarterly observations of $r - g$, in percentage points per year; sources and definitions as in Table B1. Each country is drawn over the quarters in which both $r - g$ and the change in government debt plotted in Figures I2 and I3 (Appendix I) are observed, the two series being filtered together so that all nine panels rest on one sample: a span running from 2001Q1 to 2026Q1 at its widest, 97 quarters for the United States and 99 for the United Kingdom, and not the transformed sample from which the estimates in the rest of the paper are drawn. The horizontal axis is a within-country observation index and not a calendar date, so the first observation of every country sits at the origin — in the euro panel Italy's 2001Q1 and Slovakia's 2007Q4 coincide there — and the fifteen members are not aligned in time. That panel pools them into a single unweighted scatter of 1,423 country-quarters coloured by country, so its smoothed line is not a euro-area time series; the line is the default `geom_smooth` fit, a LOESS with span 0.75 in the single-country panels and a penalised cubic regression spline with shrinkage in the euro panel.

Figure I2: Government interest rates and economic growth ($r - g$): $r - g$ against the change in government debt

United States and United Kingdom

USA

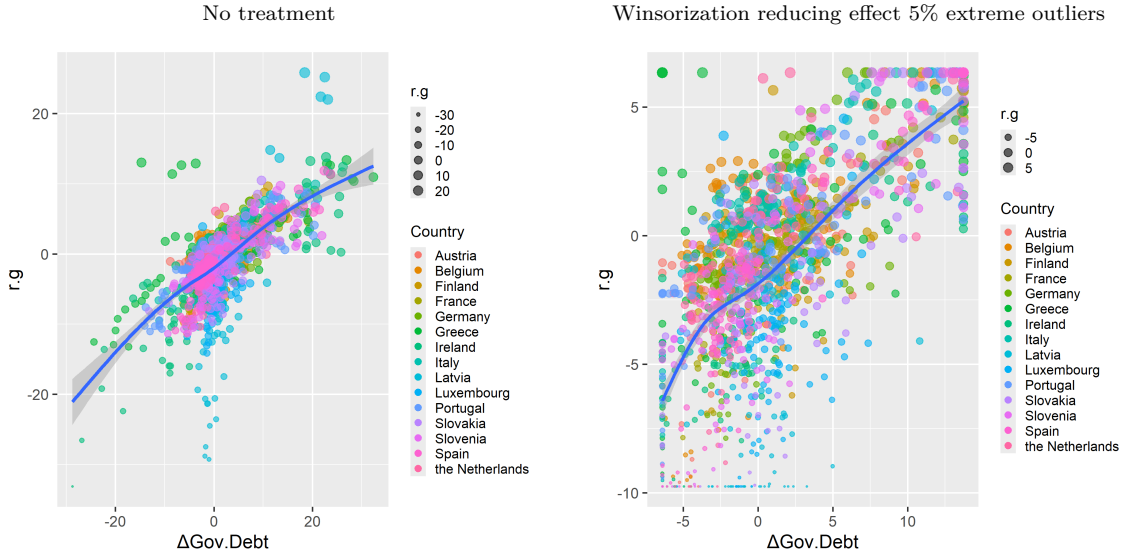


UK

Notes: $r - g$ in percentage points per year on the vertical axis against the increase in government debt, in percentage points of GDP, on the horizontal, the latter the four-quarter change in the ratio of gross debt to GDP defined in Table B1; both series as reported and without the stationarity transformations of Section 5; sample and sources as in Figure I1 in Section 4. The winsorization clips both variables at their 5th and 95th percentiles, computed on the sample actually plotted. The line is the default `geom_smooth` fit, a LOESS with span 0.75 at these sample sizes.

Figure I3: Government interest rates and economic growth ($r - g$): euro-area countries

Euro-area countries



Notes: axes, treatment and sources as in Figure I2, with the fifteen euro-area members pooled into a single scatter of 1,423 country-quarters coloured by country. The winsorization cut-offs are computed on that pooled cloud rather than country by country, and the smoothed line is a penalised cubic regression spline with shrinkage rather than a LOESS, the panel holding at least one thousand observations.

Appendix J The nested forecast comparison

Section 6.1 asks whether government financial assets carry information about the interest–growth differential that gross debt does not. This appendix sets out the nested forecast comparison that examines that question for $r - g$ a year ahead. Its design, sample, test and decision rule, and the sentence to be reported for each possible outcome, were fixed in a pre-registration dated 23 September 2026 and hashed before the registered comparison was run. They were not fixed blind. Exploratory scripts run on 19 September, while the design was being chosen, had forecast $r - g$ and the ten-year yield a year ahead from debt-and-macro baselines, and, in specifications other than the one registered (with the outcome measured against the cross-country mean, ridge regression, and the asset side entered as the ratio of financial assets to GDP and its four-quarter change), with the asset side added. Adding it seldom improved those forecasts: the change in out-of-sample R^2 was negative in most specifications, as low as -0.08, and no gain exceeded 0.011. After the plan was hashed, the scripts that implement the comparison were debugged on copies of the data in which the interest rates and $r - g$ were replaced by random noise. In those trial runs the outcome of the secondary analysis S14, the effective interest rate less the growth of annual nominal GDP, kept the actual growth series, so the S14 statistics, which relate assets to a partly actual series, were computed and printed after the design was fixed and before the scripts were hashed. The

scripts that compute the statistics each ran once on the actual data, after they were hashed, in a fixed order, with the size-and-power simulation run and logged before the primary comparison; the script that writes this appendix from their output was run a second time, to add the S1 power curves and the S19 country means that the pre-registration requires and to correct its wording, and no number it had reported changed. The exercise is pseudo-out-of-sample on final-vintage data that earlier stages of this project had already examined in other specifications.

The variable forecast is $r-g$ four quarters after the predictor date s, in percentage points per year, where r is the effective interest rate, four quarters of interest over gross debt four quarters earlier, and g the year-on-year growth of quarterly nominal GDP. The debt-only model R regresses it on seventeen country intercepts and on the following, all dated s: $r-g$, g, the primary balance, the gross debt ratio d and its square, and the four-quarter change in the debt stock over current annual GDP, f_D, and its square, 24 parameters in all. Model U adds exactly four terms: the ratio of total government financial assets to GDP, a, its square, the four-quarter change in the asset stock over current annual GDP, f_A, and its square, 28 parameters in all. Debt and assets enter in the same form, so the two models differ only in whether the asset side is known. Stock changes are used rather than differences of ratios because a difference of ratios would also carry the growth of nominal GDP through its denominator. The null hypothesis is that the four asset coefficients are zero, which for nested models is equality of the population mean squared errors; under the alternative U has the lower population error, so the test is one-sided.

The sample is the 1,154 country-quarter pairs with predictor dates from 2001Q2 to 2018Q4 that are complete for U, all inside the identifying window 2000Q2–2019Q4, and R uses the same pairs. Forecasts are made at the 36 quarterly origins from 2010Q1 to 2018Q4, for targets from 2011Q1 to 2019Q4, which gives a balanced evaluation panel of 612 country-quarters for each model. At each origin both models are re-estimated by pooled least squares, with country intercepts and common slopes, on every pair whose target is dated at or before the origin, an expanding window that grows from 491 pairs at 2010Q1 to 1,086 at 2018Q4, and the forecast uses the predictors dated at the origin. Nothing is selected, tuned or shrunk, and a rank-deficient fit would have stopped the run.

The statistic is the adjusted difference in squared forecast errors of Clark and West (2007), $f = e_R^2 - [e_U^2 - (\hat{y}_R - \hat{y}_U)^2]$, which removes from the comparison the noise that estimating four further coefficients adds to the forecasts of U. It is averaged across the seventeen countries within each origin before any variance is formed, which makes the inference robust to arbitrary dependence across countries at a date, and then over the origins. Its long-run variance is the equal-weighted cosine estimator of Lazarus et al. (2018) with $B = 4$ cosine terms,

which absorbs the overlap of the four-quarter targets, and the one-sided p-value is read from Student-t with four degrees of freedom; the null is rejected if and only if $t_{CW} > 2.1318$. That reference is a fixed-b approximation rather than an exact distribution for recursively estimated nested models, so the size of the test at this sample is measured by simulation (S1).

Because the Clark–West mean $\bar{F}$ is, to first order, twice the variance of the asset signal net of estimation noise, $S = \sqrt{\max(0, \bar{F}/2)}$ is the root-mean-square size of that signal in percentage points of $r - g$. Weighting each cell by $(d/100)^2$ expresses it, to first order, in points of GDP of the contribution of $r - g$ to the next year's change in the debt ratio, S_{dd}, with a 90 per cent interval from the same variance estimator. The threshold is 0.5 points of GDP, the minimum average annual reduction that the debt sustainability safeguard of Regulation (EU) 2024/1263 requires while debt is between sixty and ninety per cent of GDP. It is set against S_{dd} only when the test rejects; when the test does not reject, as here, S_{dd} is reported with no threshold, and 0.5 is set instead against the smallest information that the test detects with probability 0.80 (S1 below). The threshold only words the reported sentence and never enters the test decision. On the evaluation cells the median debt ratio is 91.4 per cent of GDP, and 20.3, 27.1 and 52.6 per cent of the cells lie at or below sixty, between sixty and ninety, and above ninety per cent.

Over the 612 evaluation cells the root mean squared errors are 5.08 percentage points for R and 5.08 for U, so the out-of-sample R^2 of U against R is 0.1 per cent; in debt-dynamics units, to first order, they are 5.75 and 5.63 points of GDP. The Clark–West mean is $\bar{F} = 0.393$, with $t_{CW} = 0.30$ and one-sided $p = 0.389$, and the 90 per cent interval for the difference in mean squared errors, R minus U, runs from -2.786 to 2.853. Net of estimation noise, $S = 0.44$ percentage points and $S_{dd} = 1.01$ points of GDP (90 per cent interval 0.00 to 2.10). The mean squared error of R is 1.21 times that of the no-change forecast, which projects $r - g$ at its value at the origin; R forecasts the sign of $r - g$ correctly in 67.0 per cent of the cells and U in 66.2 per cent; and the Mincer–Zarnowitz intercept and slope are -0.40 and 0.31 for R and -0.41 and 0.31 for U. These figures are reported with the test and are not tested; Table J2 gives the comparison country by country.

The simulation S1 measures the size and power of the test at this sample. Model R's in-sample fitted values plus a circular block bootstrap of whole seventeen-country cross-sections of U's in-sample residuals, in blocks of eight quarters drawn from the 45 quarters 2007Q4–2018Q4 on which every country is observed, form a simulated null under which the four asset coefficients are zero; 1,000 draws are made, with the same draws for every design point. An asset signal is then added in three directions, the asset level, the asset flow, and the square of the asset flow with every linear component removed, explaining a share ρ of the debt-only model's residual variance

from 0 to 0.40, and the whole pipeline is rerun on every draw. Under the null the test rejects in 2.4 per cent of samples at its nominal five. The rejection rate first reaches 0.80 at $\rho = 0.075$ for the level, $\rho = 0.200$ for the flow and $\rho = 0.200$ for the squared flow, which corresponds to asset information that moves the projected contribution of $r - g$ to the next year's change in the debt ratio by 1.34, 3.05 and 4.38 points of GDP in root-mean-square terms, to first order. The minimum detectable information in the least favourable form is therefore 4.38 points of GDP, more than the 0.5-point threshold, and the power status is UNDERPOWERED. Table J3 gives the rejection rate at every point of the grid for each form, together with the reduction in the root mean squared error of the debt-only forecast that knowing the simulated signal exactly would give at ρ^*, the smallest grid value at which the rate reaches 0.80: 3.8 per cent for the level, 10.6 per cent for the flow and 10.6 per cent for the squared flow.

Table J1 reports the nineteen registered secondary analyses whatever they show. Each changes one element of the primary and is otherwise identical to it: S2 forecasts eight quarters ahead; S3 imposes a one-quarter release lag on the predictors; S4 replaces the stock-change flows by the paper's own ratio differences; S5 adds every second-order interaction of d, f_D, a and f_A, which spans any quadratic in net debt; S6 replaces the quadratics by penalised regression splines whose knots are placed within each training window; S7 and S8 replace total assets by their liquid part, and by their liquid and illiquid parts, from the non-consolidated Eurostat instrument breakdown, for the fifteen countries other than the United Kingdom and the United States; S9 drops the country intercepts; S10 gives each country its own linear asset slopes; S11 uses a rolling window of 32 predictor dates; S12 drops one country at a time; S13 evaluates only the origins from 2013Q1; S14 replaces $r - g$ by the effective rate less the growth of annual nominal GDP, with that growth as the control; S15 forecasts the ten-year yield; S16 extends the origins to 2024Q4, pandemic quarters included, as a companion that is never identifying; S17 adds the ten-year yield, the current account and capital formation to both models; S18 forecasts the absolute one-year change in $r - g$; and S19 describes the two forecasts on the country-quarters where a gross and a net debt anchor disagree. None of them can change the classification of the primary result: S1, S12, S13 and S17 enter the reported text only through qualifiers written before the comparison was run.

Appendix K The second forecast comparison (NF2)

NF2 asks the question of the comparison of Appendix J on the consistent interest–growth differential of Section 4: whether a specified asset measure improves the

Table J1: The nested forecast comparison: primary and registered secondary analyses

Analysis	Pairs/cells	P/B	c.v.	t	p	R^2_{OOS}	ΔRMSPE	S_{dd}
Primary	1,154/612	36/4	2.1318	0.30	0.389	0.1	−0.003	1.01
S1 Size and power[b]	1,154/612	36/4	2.1318	size 2.4%; MDE_{dd} 4.38				
S2 Horizon $h = 8$	1,086/544	32/4	2.1318	−1.21	0.854	−4.1	0.193	0.00
S3 One-quarter release lag	1,137/612	36/4	2.1318	−1.17	0.847	−5.1	0.131	0.00
S4 Ratio-difference flows	1,145/612	36/4	2.1318	−0.05	0.518	−1.6	0.038	0.39
S5 Full quadratic	1,154/612	36/4	2.1318	0.86	0.220	10.8	−0.416	4.75
S6 Penalised splines	1,154/612	36/4	2.1318	0.93	0.202	11.8	−0.401	3.04
S7 Liquid assets	1,015/540	36/4	2.1318	0.70	0.263	−5.2	0.149	0.61
S8 Asset composition	1,015/540	36/4	2.1318	0.51	0.318	−6.5	0.186	1.72
S9 No country intercepts	1,154/612	36/4	2.1318	0.60	0.290	0.0	0.000	1.06
S10 Country asset slopes	1,154/612	36/4	2.1318	−0.06	0.522	−156.4	3.056	0.00
S11 Rolling window	1,154/612	36/4	2.1318	−0.31	0.615	−4.0	0.101	0.00
S12 Leave one country out[a]	1,083/576	36/4	2.1318	−1.27	0.863	−3.1	0.073	0.00
S13 Origins from 2013Q1	1,154/408	24/3	2.3534	−1.93	0.926	−5.7	0.121	0.00
S14 Timing-consistent $r - g$	1,154/612	36/4	2.1318	0.18	0.433	−7.0	0.135	0.66
S15 Ten-year yield outcome	1,154/612	36/4	2.1318	1.21	0.147	4.5	−0.057	0.76
S16 Full sample to 2024Q4	1,557/1,015	60/6	1.9432	0.08	0.471	−0.5	0.016	0.55
S17 Macro-financial controls	1,127/612	36/4	2.1318	−0.48	0.673	−2.4	0.077	0.66
S18 Dispersion outcome	1,154/612	36/4	2.1318	−0.54	0.692	−1.6	0.042	0.43
S19 Anchor disagreement	–/86	–	–	descriptive, no test; see notes				

Notes: each row compares model R with model U by the Clark–West statistic averaged across countries within each origin, with the equal-weighted cosine variance estimator and B cosine terms; c.v. is the one-sided five per cent critical value of Student-t with B degrees of freedom, from which p is also read. Pairs are the country-quarter pairs of the analysis and cells the evaluated forecasts. $R^2_{OOS} = 100(1 - \text{MSPE}_U/\text{MSPE}_R)$, in per cent; ΔRMSPE is the root mean squared error of U less that of R, in percentage points; S_{dd} is the noise-netted size of the asset information in points of GDP of the next year's change in the debt ratio, to first order, and for S15 and S18, whose outcome is not $r - g$, it is only a mechanical conversion with $d/100$. [b] Rejection rate under the simulated null at the nominal five per cent, and the minimum detectable information, in points of GDP, in the least favourable of the three simulated forms; power status UNDERPOWERED. S7 and S8 cover the fifteen countries other than the United Kingdom and the United States, with liquid assets $(F2 + F3)/F$ and illiquid assets $(F4 + F5)/F$ of the Eurostat non-consolidated breakdown. [a] The rerun with the lowest t, without Greece; 17 of the 17 reruns have $t \leq 2.1318$ (without Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Luxembourg, the Netherlands, Portugal, Slovakia, Slovenia, Spain, the United Kingdom and the United States); Table J2 gives each. S13 uses the primary forecasts. S16 is a companion that includes the pandemic quarters and is never identifying; its panel is unbalanced at the end and each origin averages over the countries present. S19 describes, with no test and no interval, the evaluation country-quarters on which a sixty-per-cent gross anchor and the count-matched net anchor ($d - a < 27.40$, the pre-2020 threshold) disagree, giving for each set, overall and by country, the means of $(\hat{y}_R - \hat{y}_U)\, d/100$, $e_R\, d/100$ and $e_U\, d/100$, in points of GDP and in that order. On the 58 that the gross anchor flags and the net anchor clears they are 0.20, −0.20 and 0.01: Finland (27 cells) 0.35, −0.23 and 0.12; Slovenia (25) 0.10, 0.06 and 0.16; Germany (5) 0.01, −1.19 and −1.18; the Netherlands (1) −0.25, −0.68 and −0.94. On the 28 that the net anchor flags and the gross anchor clears they are −0.07, −0.05 and −0.12: Slovakia (27 cells) −0.07, 0.03 and −0.04; the Netherlands (1) −0.07, −2.18 and −2.25.

Table J2: The nested forecast comparison by country

Country	MSPE_R	MSPE_U	$\text{MSPE}_U/\text{MSPE}_R$	$\bar{f}_i$	S12 t without it
Austria	4.38	4.58	1.047	−0.10	−0.01
Belgium	3.08	2.61	0.847	0.62	0.28
Germany	8.02	8.34	1.040	−0.24	0.42
Spain	24.02	23.43	0.975	0.62	0.20
Finland	6.29	11.33	1.801	−3.69	−0.71
France	4.69	5.13	1.094	−0.39	0.04
United Kingdom	4.74	5.06	1.067	−0.24	0.32
Greece	99.46	87.46	0.879	15.25	−1.27
Ireland	175.80	179.58	1.022	−3.61	−0.09
Italy	4.63	4.79	1.035	−0.13	0.22
Luxembourg	16.70	18.63	1.116	−1.83	0.64
Latvia	32.46	32.01	0.986	0.64	1.05
Netherlands	8.83	9.55	1.082	−0.66	0.33
Portugal	14.51	14.22	0.980	0.39	0.09
Slovakia	8.05	8.29	1.030	−0.11	0.07
Slovenia	17.32	17.56	1.014	−0.09	−0.14
United States	6.26	6.11	0.976	0.24	0.31

Notes: MSPE in squared percentage points over the 36 origins 2010Q1–2018Q4; $\bar{f}_i$ is the country's mean of the Clark–West adjusted difference f, whose average over countries and origins is $\bar{F} = 0.393$. The last column is the S12 statistic when that country is left out of both estimation and evaluation (P = 36, B = 4, c.v. 2.1318). These country rows are reported with the test and are not tested.

Table J3: The nested forecast comparison: size and power (S1)

ρ	Asset level	Asset flow	Squared asset flow
0.000	0.024	0.024	0.024
0.010	0.187	0.144	0.124
0.020	0.341	0.238	0.216
0.030	0.484	0.322	0.299
0.050	0.692	0.455	0.419
0.075	0.814	0.566	0.527
0.100	0.895	0.656	0.614
0.150	0.962	0.767	0.740
0.200	0.989	0.835	0.824
0.300	1.000	0.909	0.922
0.400	1.000	0.950	0.966
ρ^*	0.075	0.200	0.200
Oracle RMSPE reduction (%)	3.8	10.6	10.6
$\text{MDE}_{dd,k}$ (points of GDP)	1.34	3.05	4.38

Notes: the share of 1,000 simulated samples in which the primary test rejects, $t_{CW} > 2.1318$, with the same draws at every design point; ρ is the share of the debt-only model's residual variance that the simulated asset signal explains, so the row $\rho = 0$ gives the size of the test. The squared asset flow has every linear component removed. ρ^* is the smallest grid value at which the rejection rate reaches 0.80; the oracle RMSPE reduction, $100(1 - \sqrt{1 - \rho^*})$, is the per cent reduction in the root mean squared error of the debt-only forecast that knowing the simulated signal exactly would give at ρ^*; and $\text{MDE}_{dd,k}$ is the root-mean-square size of that signal over the 612 evaluation cells in points of GDP of the contribution of $r - g$ to the next year's change in the debt ratio, to first order. These rates are reported with the test and are not tested.

one-year-ahead forecast of $r - g$ beyond a credible benchmark. Its plan was written and hashed before any NF2 forecast was computed (SHA-256 bdc9712bd91595c6 6126f2a862e54813 6787665bf203dd0f 7425551b71aa90f6) and was not deposited with a registry, so the comparison is described as pre-specified. The outcome is $r - g$ four quarters ahead, in percentage points per year. At each forecast origin every fiscal and output predictor is dated one quarter earlier, a release lag, and the models are trained only on targets already observed at that date. Three benchmarks are compared: the no-change forecast (B0), a pooled first-order autoregression with country effects (B1), and the debt-and-macro model of Appendix J re-specified on the consistent measure (B2). Each of B1 and B2 is extended by the asset block — the financial-asset stock, its four-quarter change and their squares, as ratios to GDP — into U1 and U2. All models are pooled least squares with country intercepts, fitted on an expanding window.

The primary test sets U against the stronger of B1 and B2, the stronger being the one with the lower mean squared prediction error on the origins 2005Q1–2009Q4, which are never used for evaluation; B1 was selected (Table K1). The test is the one-sided Clark–West statistic with the variance of the comparison of Appendix J, read against Student-t with four degrees of freedom, on forecasts made each quarter from 2010Q1 to 2018Q4 for the seventeen countries, 612 cells. Its economic margin is the size of the asset information in the projected contribution of $r - g$ to the next year's change in the debt ratio, S_{dd}, net of estimation noise and to first order, against the 0.5 points of GDP of the debt sustainability safeguard of Regulation (EU) 2024/1263 (Table K3). Rules fixed in the plan class the result as a gain established, gains of policy size excluded, or unresolved; it is unresolved. Five secondary analyses and a temporal validation on the origins 2022Q1–2025Q1 were fixed with it and are reported whatever they show (Tables K4 and K5); none of them changes the class.

Table K1: NF2: the primary nested comparison

Quantity	Value
Primary benchmark (selected on 2005Q1–2009Q4)	B1
MSPE of B1 / B2 on the selection origins	56.20 / 75.96
Comparison	U1 vs B1
Clark–West t	−0.28
One-sided p (Student-t, 4 degrees of freedom)	0.604
Five per cent critical value	2.1318
MSPE of B1 / U1	13.05 / 15.38
Change in MSPE from adding the asset block	17.9 per cent higher
Origins / cells / EWC bandwidth	36 / 612 / 4
Size under a simulated null (1,000 replications)	0.038 [0.027, 0.052]
Class	unresolved

Note. Pseudo-out-of-sample, final-vintage data, fiscal and output data lagged one quarter; pre-specified. The size is the rejection rate at five per cent when the asset block carries no information by construction, with an exact 95 per cent binomial interval; it is a diagnostic and does not enter the class.

Table K2: NF2: mean squared prediction error of every model relative to the no-change forecast

Model	Specification	MSPE	Relative to B0
B0	No change: the differential at $\tau - 1$	19.90	1.000
B1	Pooled AR(1) with country effects	13.05	0.656
B2	Debt-and-macro model	17.90	0.899
U1	B1 plus the asset block	15.38	0.773
U2	B2 plus the asset block	18.24	0.916

Note. Interest–growth differential four quarters ahead (pp per year), forecasts made each quarter from 2010Q1 to 2018Q4 for the seventeen countries (612 cells), predictors lagged one quarter. The asset block is the asset stock, its four-quarter change, and their squares.

Table K3: NF2: size of the asset information in next year's debt dynamics

Quantity	Points of GDP
S_{dd} (net of estimation noise, to first order)	0.00
90 per cent interval, block bootstrap over origins	[0.00, 0.79]
90 per cent interval, t-based (not used for the class)	[0.00, 1.21]
Margin (Regulation (EU) 2024/1263, Art. 7)	0.50
Class	unresolved

Note. Bootstrap: 999 draws of circular blocks of 4 consecutive origins, all countries of an origin together, forecasts held fixed; percentile interval. The t-based interval is the one used in the comparison of 23 September 2026 and is printed for comparison only.

Table K4: NF2: secondary analyses

Analysis	Comparison	t	p	MSPE ratio	S_{dd} [90% interval]
NS1: against no change (Diebold–Mariano, two-sided)	U1 vs B0	0.83	0.454	0.773	–
NS2: the other nested pair	U2 vs B2	0.63	0.281	1.019	0.81 [0.00, 1.47]
NS3: transactions in place of the stock change (15 countries)	U1 vs B1	1.82	0.071	1.094	1.17 [0.38, 1.72]
NS4: ten-year yield outcome	U1 vs B1	1.32	0.128	0.957	–
NS5: conventional $r - g$ outcome	U1 vs B1	−0.35	0.627	1.056	0.00 [0.00, 0.77]

Note. Each analysis was run once and is reported whatever it shows; none changes the class of the primary comparison. Clark–West tests are one-sided; the MSPE ratio is that of the forecast with the asset block to its benchmark (to the no-change forecast for NS1). S_{dd} is not reported for the yield and for the non-nested comparison. NS3 uses consolidated transactions in financial assets (Eurostat).

Table K5: NF2: temporal validation, forecast origins 2022Q1–2025Q1

Quantity	Value
MSPE relative to B0	B1 0.617, B2 0.572, U1 0.574, U2 0.594
Comparison	U1 vs B1
Clark–West t	0.67
One-sided p (Student-t, 2 degrees of freedom)	0.286
S_{dd} and its 90 per cent bootstrap interval	1.48 [0.70, 2.02]
Origins / cells	13 / 200

Note. Reported separately and never part of the classification; no class is computed. The models are re-estimated on all data available at each origin, 2020–21 included. Countries per origin: 17 from 2022Q1 to 2023Q4, 16 from 2024Q1 to 2024Q3, 15 in 2024Q4 and 1 (the United Kingdom) in 2025Q1; the origin averages weight the countries present. Five per cent critical value of t with 2 degrees of freedom: 2.920.