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Abstract

This paper studies the impact of optimism on occupational choice using a general equilibrium framework. The model shows that optimism has four main qualitative effects: it leads to a misallocation of talent, drives up input prices, raises the number of entrepreneurs, and makes entrepreneurs worse off. We calibrate the model to match U.S. manufacturing data. This allows us to make quantitative predictions regarding the impact of optimism on occupational choice, input prices, the returns to entrepreneurship, and output. The calibration shows that optimism can explain the empirical puzzle of the low mean returns to entrepreneurship compared to average wages.

JEL Codes: D50; H21; J24; L26.

Keywords: General Equilibrium; Entrepreneurship; Optimism.

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1 Introduction

The seminal paper of occupational choice and firm size distribution of an economy is Lucas (1978). Individuals have heterogenous one-dimensional abilities as entrepreneurs and choose between entrepreneurship and paid employment. The most talented individuals become entrepreneurs and the less talented ones become workers. The ability differentials across entrepreneurs give rise to different spans of control (firm sizes). Two main predictions of Lucas’ model are that the mean returns to entrepreneurship are greater than average wages and that the return distributions of entrepreneurs and workers have non-overlapping supports.

These two predictions stand in contrast to empirical evidence on the returns to entrepreneurship. First, the returns to entrepreneurship are found, on average, not to be higher than wages. For example, Hamilton (2000) finds that after 10 years in business the median entrepreneurial earnings are 35 percent less than those on a paid job of the same duration. Similarly, Moskovitz and Vissing-Jorgensen (2002) find that the returns to entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio—the private equity puzzle. Second, the returns to entrepreneurship are found to be highly variable, more than wages, and more than the returns on public equity (Borjas and Bronars (1989), Hamilton (2000), and Moskovitz and Vissing-Jorgensen (2002)). Hence, the empirical return distributions of entrepreneurs and workers have overlapping supports.

Moreover, research on entrepreneurs’ traits and expectations casts serious doubts on the assumption that entrepreneurs are rational decision makers. Entrepreneurs are extremely optimistic about the future of their firms. Most businesses fail within a few years (Dunne et al. (1988)). However, entrepreneurs report the odds of their business ‘succeeding’ to be significantly higher than historically observed and substantially better than the odds of success for other similar businesses (Cooper et al. (1998)).

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1See Shane and Venkataraman (2000) and Astebro et al. (2014) for surveys on this topic.

2This result was obtained for the 1989-1998 period. However, Kartashova (2014) shows that the private equity puzzle does not extend to the 1989-2010 period.
Direct comparison of entrepreneur expectations to new venture outcomes shows that a representative sample of French entrepreneurs tend to overestimate employment expansion and sales growth (Landier and Thesmar (2009)). Nascent entrepreneurs overestimate the probability that their projects will result in operating ventures and, for those ventures that achieve operation, 62 percent overestimate future sales and 46 percent overestimate the number of employees in the first year of operation (Cassar (2010)). 48.8 percent of a sample of U.S. nascent entrepreneurs think that the likelihood of exit of their venture is zero in five years time (Hyytinen et al. (2014)). Individuals who switch into self-employment have an optimistic view of their future prior to switching into self-employment (Dawson et al. (2014)).

Empirical evidence on entrepreneurs’ traits and expectations also shows that entrepreneurs are more optimistic than employees (Busenitz and Barney (1997), Arabsheibani et al. (2000), Fraser and Greene (2006), and Koudstaal et al. (2015)). Entrepreneurs expect to live about 2 years longer than non-entrepreneurs after controlling for differences in smoking, race, and education-related mortality risk across groups (Puri and Robinson (2013)). In contrast, entrepreneurs’ risk attitudes are indistinguishable from those of wage earners (Wu and Knott (2006), Parker (2009), Holm et al. (2013), and Koudstaal et al. (2016)). Hence, the empirical puzzle of the low mean returns to entrepreneurship compared to average wages cannot be ex-

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3Busenitz and Barney (1997) find that entrepreneurs are more optimistic than managers. Arabsheibani et al. (2000) compare entrepreneurs’ and employees’ expectations of future prosperity to actual outcomes using a sample from the British Household Panel Survey (BHPS) during the years 1990-96. They find that entrepreneurs are 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. In contrast, for employees the ratio was only 2.9. Fraser and Greene (2006) find that self-employed Britons have higher income expectations than employees during the years 1984-99, but the difference diminishes with experience. Koudstaal et al. (2015) run a lab-in-the field experiment in the Netherlands and find that 58 percent of entrepreneurs can be classified as ‘very optimistic,’ i.e., have a score of 18 or more in the Revised Life Orientation Test, a commonly used measure of dispositional optimism. In contrast, only 32 percent of employees can be classified as ‘very optimistic.’
plained by assuming that entrepreneurs have different risk attitudes from those of employees.

In this paper we show that optimism can explain the empirical puzzle of the low mean returns to entrepreneurship compared to average wages. To do that we build up a fully specified general equilibrium model with labor, capital, and output markets. The main novelty is the assumption that the population is composed of optimists and realists. The occupational choices of optimists are driven by their biased expectations about ability and by input prices. Realists make their occupational choices like in Lucas (1978) but are affected by the behavior of optimists. Unlike the existing literature, the model allows us to make qualitative and quantitative predictions regarding the impact of optimism on occupational choices, input prices, the returns to entrepreneurship, and output.

Following Lucas (1978) we model a closed economy with a population of size \( N \) and a capital stock of \( K \) units of capital. Each individual is endowed with one unit of labor, with capital stock \( K/N \), and with a one-dimensional ability \( \theta \). Individuals are risk neutral and maximize their expected returns by choosing occupations. A firm in this economy is one entrepreneur together with the labor and capital under his control. The technology of the firm is as follows. Output is an increasing function of ability, labor, and capital. Ability is complementary to labor and capital. Decreasing returns to scale in labor and capital ensure that the competitive equilibrium exhibits a non-degenerate distribution of firm sizes.

We depart from Lucas (1978) by assuming that a fraction \( \lambda \in (0, 1] \) of individuals is optimistic about ability whereas the remaining fraction \( 1 - \lambda \) is realist. Realists know their ability is \( \theta \) whereas optimists think, mistakenly, that their true ability is \( \gamma \theta \), with \( \gamma > 1 \). Hence, realists who enter entrepreneurship know the true production function of their firms, whereas optimists believe their firms are more productive than they really are.

The competitive equilibrium is characterized by: (i) a cut-off ability level \( \hat{\theta}_R \) such that realists with ability less than \( \hat{\theta}_R \) become workers and those with ability greater
than \( \hat{\theta}_R \) become entrepreneurs, (ii) a cut-off ability level \( \hat{\theta}_O \) such that optimists with ability less than \( \hat{\theta}_O \) become workers and those with ability greater than \( \hat{\theta}_O \) become entrepreneurs, (iii) a market clearing wage that equates labor demand to supply, and (iv) a rental cost of capital that equates capital demand to supply.

We solve the competitive equilibrium assuming a generalized Cobb-Douglas production function and a uniform distribution of ability. We show that in equilibrium there is a misallocation of talent. The ablest people do not necessarily select into entrepreneurship: the lowest ability entrepreneurs are less talented at running a firm than the highest ability workers. This is an empirically attractive implication of the model since, in reality, the return distributions of entrepreneurs and workers have overlapping supports. The misallocation of talent that characterizes the competitive equilibrium can be corrected through a revenue-neutral tax-subsidy scheme that discourages low ability optimistic entrepreneurs and subsidies high ability workers. We also show that optimists are more likely to become entrepreneurs than realists and that entrepreneurs are more likely to be optimists than employees.

We discuss the robustness of the qualitative implications of our model to a number of extensions: if the return to entrepreneurship is stochastic rather than deterministic; if individuals have heterogeneous abilities both as workers and as entrepreneurs; if the occupational choice is extended to consider also firms run by owners without employees; if the ability of entrepreneurs is log-normally distributed; and if the intensity of optimistic beliefs is endogenous rather than exogenous.

We calibrate the model to match salient features of U.S. manufacturing data. The production function and the capital stock are calibrated following Atkeson and Kehoe (2005) and Adler (2016). The fraction of optimists \( \lambda \) and the intensity of optimism \( \gamma \) are calibrated using empirical evidence on the financial expectations of entrepreneurs and employees. The calibration shows that optimism can explain quantitatively the empirical puzzle of the low mean returns to entrepreneurship compared to average wages. This happens due to four reasons. First, optimism leads to the misallocation of talent since lower skill optimistic individuals crowd out higher skill
realistic individuals from entrepreneurship. Second, it raises input prices. Third, it
distorts the input choices of optimistic entrepreneurs. Fourth, it raises the number
of entrepreneurs.

The reminder of the paper proceeds as follows. Section 2 reviews related literature. Section 3 sets up the model. Section 4 characterizes the competitive equilibrium. Section 5 contains comparative statics results. Section 6 calibrates the model. Section 7 discusses the main assumptions of the model, extensions, and policy implications. Section 8 concludes the paper. All proofs can be found in the Appendix.

2 Related Literature

In this section we explain how our work contributes to the literature on occupational choice and entrepreneurship. We focus mostly on studies that, like ours, address the puzzle of the low mean returns to entrepreneurship compared to average wages. We distinguish three types of approaches: static general equilibrium models, dynamic general equilibrium models, and partial equilibrium models.

2.1 Static General Equilibrium Models

Among static general equilibrium models, Manove (2000) is the closest to ours. Individuals have heterogeneous entrepreneurial abilities and choose to become entrepreneurs or workers. Entrepreneurs use internal resources (own savings and effort) and external resources (hired labor) to produce. Some individuals are optimists and others are realists. Optimists overestimate their ability whereas realists do not. Manove finds that optimistic entrepreneurs save too much, provide too much effort, and hire too many employees relative to realistic entrepreneurs. Manove also shows that optimistic entrepreneurs can survive by working and saving extra hard to compensate for the mistakes caused by their optimism.

Rigotti et al. (2011) study the role of optimism on technology choice. Individuals choose to be entrepreneurs or employees and between employing a traditional
technology or a new one which has ambiguous returns. A firm is an entrepreneur-employee pair operating a particular technology. Some individuals are optimists and others are pessimists. Rigotti et al. find that optimists are more likely to become entrepreneurs and that firms employing new technologies are run by optimistic entrepreneurs and employ optimistic employees.

We generalize Lucas (1978) by assuming that a fraction of individuals in the economy are optimists. This allows us to study the general equilibrium effects of optimism using Lucas’ powerful and rigorous analytical framework. Some of our qualitative predictions are in line with Manove (2000) but others are novel. Like Manove (2000), we find that optimists bid up the wage which makes workers better off and entrepreneurs worse off. Unlike Manove (2000) and Rigotti et al. (2011), we are able to characterize the impact of optimism on the capital market. We find that optimists bid up the rental cost of capital which further contributes to explain the low mean returns to entrepreneurship compared to average wages. In addition, we provide quantitative predictions about the impact of optimism on occupational choice, input prices, the returns to entrepreneurship, and output. As far as we know, we are the first to show that optimism can explain quantitatively the observed low mean returns to entrepreneurship compared to average wages.

2.2 Dynamic General Equilibrium Models

Dynamic general equilibrium models show that experimenting with entrepreneurship in order to learn about ability can explain qualitatively the observed low returns to entrepreneurship compared to average wages. For example, Vereshchagina and Hopenhayn (2009) consider a life-cycle model in which individuals can switch back

and forth between entrepreneurship and paid employment. Entrepreneurship is risky and paid employment provides a fixed outside option. Individuals face financing constraints and because of them they take more risk at low wealth levels than at high wealth levels. Vereshchagina and Hopenhayn show that the combination of occupational choice and financing constraints can lead entrepreneurs to display risk-taking behavior. Hence, entrepreneurs operate in a financial environment that leads them to engage in risky investment even in the absence of a return premium.

Campanale (2010) considers a life-cycle occupational and portfolio choice model with learning. The key assumption is that the quality of a business project is not precisely known upon entry and is learned over time. The model shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. Since individuals can switch back to paid-employment, they find it worthwhile experimenting with entrepreneurship to find out if the project is good even if initially the expected return is low. Campanale quantifies the amount of risk premia that would justify entry into entrepreneurship in this environment, and finds that it is still substantially larger than what we see in the data.

Poschke (2013) proposes a life-cycle model in which individuals differ in their efficiency as workers and in the productivity of the firms they start. Whereas efficiency as a worker is known, the productivity of entrepreneurial projects can only be found after implementing them. Poschke shows that the option to abandon bad projects attracts low-ability agents into entrepreneurship.

We show, using a static general equilibrium model, that optimism provides a compelling alternative explanation for the observed low returns to entrepreneurship compared to average wages. In addition, we show that optimism can explain this empirical puzzle not only qualitatively but also quantitatively.
2.3 Partial Equilibrium Models

Partial equilibrium models focus mostly on the impact of optimism on credit markets and financial intermediation rather than on the puzzle of the low mean returns to entrepreneurship compared to average wages. However, they also show how optimism can lower the returns to entrepreneurship.

In de Meza and Southey (1996) individuals choose between working in a safe occupation with a known return or undertaking a project with a risky return. Entrepreneurs must select the right mix of self-finance and debt-finance from risk neutral banks to develop their projects. All individuals have the same ability or probability of success of their projects. Banks and realistic entrepreneurs know a project’s true probability of success but optimistic entrepreneurs overestimate it. De Meza and Southey show that optimists select maximum internal finance and any form of external finance is a standard debt contract, that optimism can lead to excessive lending, and that only optimists become entrepreneurs. They also show that optimistic individuals who are denied loans and become workers may end up better off ex post than those who obtain loans and become entrepreneurs.

Manove and Padilla (1999) also study the role of optimism on investment and on the credit market. While de Meza and Southey (1996) assume that banks can distinguish between optimists and realists, Manove and Padilla assume that banks cannot differentiate optimists from realists. They find that, in the presence of optimists, perfectly competitive banks may be insufficiently conservative in their dealings with entrepreneurs, even if entrepreneurs themselves may practice self-restraint to signal realism. They also show that, in the presence of optimists, the use of collateral requirements by banks may reduce the efficiency of the credit market.

Coval and Thakor (2005) study the role of optimism and pessimism on financial intermediation. They consider a model where individuals do not have enough wealth to self-finance a project that can be either good or bad. Realists correctly assess a project’s probability of success, optimists overestimate it and pessimists underestimate it. Coval and Thakor show that realists form a financial intermediary that
raises funds from pessimists (who become investors in the intermediary) and lends
to optimists (who become entrepreneurs).

Some of our qualitative predictions are in line with those in de Meza and Southey
(1996). For example, the prediction that optimistic individuals are more likely to
become entrepreneurs, that entrepreneurs are more likely to be optimistic than work-
ers, and that optimism lowers the returns to entrepreneurship. However, given that
we use a general equilibrium approach, we are able to show that optimism raises
input prices which plays a critical role towards explaining the low mean returns to
entrepreneurship compared to average wages.

3 Set-up

The economy consists of a continuum of risk-neutral individuals. The population is
of size $N$ and the capital stock is of $K$ units of capital. Individuals derive utility
from consumption and can earn income either as workers or by running their own
firm. Each individual is endowed with 1 unit of labor, with capital stock $K/N$, and
with a one-dimensional ability $\theta$ drawn from the cumulative distribution function
$G(\theta)$ with support on $[0, \bar{\theta}]$, with $0 < \bar{\theta} < \infty$.

If an individual with ability $\theta$ becomes a worker he supplies his unit of labor on
the labor market, receives the competitive wage $w$ for his unit of labor, and receives
the competitive rental rate of capital for renting his capital $K/N$. Hence, a wage
worker ends up with an income

$$w + rK/N.$$  

If an individual with ability $\theta$ becomes an entrepreneur he can use without cost
a technology defined by the continuous production function

$$y = \theta f(l, k),$$

where $y$ is output, $l$ is labor, and $k$ is capital. Following Lucas (1978), $\theta$ enters into
the production function as the total factor productivity (TFP). Any individual can
run at most one firm. We assume that $f$ is twice continuously differentiable with $f_l > 0$, $f_k > 0$, $f_{ll} < 0$, $f_{kk} < 0$. This production function combines as inputs one entrepreneur, who is essential to operate the firm, $l$ homogeneous employees, and $k$ units of homogeneous capital. The production function exhibits decreasing returns to scale in the variable inputs, labor and capital, so that the competitive equilibrium exhibits a non-degenerate firm size distribution. This assumption implies that the size of firms is finite. This could be due for instance to limits in entrepreneurs’ span of control: as activity expands, it becomes more difficult to control, and the marginal product of the variable inputs diminishes.

Entrepreneurs hire labor at the competitive wage rate $w$ and rent capital at the competitive rental cost of capital $r$. Hence, an entrepreneur who employs $l$ workers and rents $k$ units of capital earns a profit of

$$\pi(\theta, w, r) = p\theta f(l, k) - wl + r(K/N - k).$$

From now on the price of output $p$ is normalized to be 1. Individuals can belong to one of two types: those with optimistic beliefs and realists. A fraction $\lambda \in (0, 1)$ of the population has optimistic beliefs about their ability as entrepreneurs and a fraction $1 - \lambda$ has realistic beliefs. The perceived profit of an optimistic entrepreneur with perception of ability $\gamma \theta$ who employs $l$ workers and rents $k$ units of capital is

$$\pi(\gamma \theta, w, r) = \begin{cases} 
\gamma \theta f(l, k) - wl + r(K/N - k) & \text{if } 0 \leq \theta < \frac{\bar{\theta}}{\gamma} \\
\bar{\theta} f(l, k) - wl + r(K/N - k) & \text{if } \frac{\bar{\theta}}{\gamma} \leq \theta \leq \bar{\theta}
\end{cases},$$

(1)

where $\gamma > 1$. The parameter $\gamma$ measures the strength or intensity of optimistic beliefs. Under this specification the perception of ability of individuals with ability above $\bar{\theta}/\gamma$ is set equal to the highest possible ability level $\theta = \bar{\theta}$. In other words, no individual thinks that he or she can be more productive than $\bar{\theta}$. This specification of optimistic beliefs is analytically convenient. The distributions of entrepreneurial abilities and types are assumed to be independent. Hence, realists and optimists are equally endowed in terms of their entrepreneurial abilities.
An individual who becomes an entrepreneur will choose to employ \( l(\gamma \theta; w, r) \) workers and \( k(\gamma \theta; w, r) \) units of capital where \( l(\gamma \theta; w, r) \) and \( k(\gamma \theta; w, r) \) are the values of \( l \) and \( k \) that solve the following problem

\[
\max_{l,k} \{ \gamma \theta f(l, k) - wl + r(K/N - k) \}.
\]

The first-order conditions to this problem are

\[
\gamma \theta f_l(l, k) = w. \tag{2}
\]

and

\[
\gamma \theta f_k(l, k) = r. \tag{3}
\]

It follows from (2), the assumption of decreasing returns to labor, \( f_{ll} < 0 \), and complementarity between ability and labor, i.e., \( f_{l\theta} > 0 \), that realistic entrepreneurs with a higher \( \theta \) hire more workers: \( \partial l(\gamma \theta, w, r)/\partial \theta = -\gamma f_{l\theta}/f_{ll} > 0 \). Similarly, it follows from (3), the assumption of decreasing returns to capital, \( f_{kk} < 0 \), and complementarity between ability and capital, i.e., \( f_{k\theta} > 0 \), that realistic entrepreneurs with a higher \( \theta \) hire more capital: \( \partial k(\gamma \theta, w, r)/\partial \theta = -\gamma f_{k\theta}/f_{kk} > 0 \). The same is true for optimistic entrepreneurs but only for those with \( \theta < \bar{\theta}/\gamma \), as we imposed an upper bound \( \bar{\theta} \) on the perceived ability of optimistic entrepreneurs. It also follows from (2) that an optimistic entrepreneur will demand more labor than a realist with the same ability. Similarly, it follows from (3) that an optimistic entrepreneur will demand more capital than a realist with the same ability.

A realist with ability \( \theta \) chooses to become a worker at wage \( w \) and rental cost of capital \( r \) when

\[
\theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \leq w. \tag{4}
\]

He selects to be an entrepreneur if

\[
\theta f(l(\theta, w, r), k(\theta, w, r)) - wl(\theta, w, r) - rk(\theta, w, r) \geq w, \tag{5}
\]
and he is indifferent if the equality holds in (4) and (5). An optimist with perception of ability $\gamma \theta$ chooses to become a worker at wage $w$ and rental cost of capital is $r$ when
\[
\gamma \theta f(l(\gamma \theta, w, r), k(\gamma \theta, w, r)) - wl(\gamma \theta, w, r) - rk(\gamma \theta, w, r) \leq w.
\] (6)
He selects to be an entrepreneur if
\[
\gamma \theta f(l(\gamma \theta, w, r), k(\gamma \theta, w, r)) - wl(\gamma \theta, w, r) - rk(\gamma \theta, w, r) \geq w,
\] (7)
and he is indifferent if the equality holds in (6) and (7).

Since there are only three markets—output, labor, and capital—by Walras’ Law, general equilibrium is realized when the labor and capital markets clear. At the equilibrium wage, the labor demanded by individuals who choose to become entrepreneurs equals that supplied by individuals who choose to become workers. At the equilibrium rental cost of capital, the capital demanded by individuals who choose to become entrepreneurs equals the exogenous capital stock of the economy, $K$. Formally, an equilibrium is (i) a partition $\{[0, \hat{\theta}_R], [\hat{\theta}_R, \bar{\theta}]\}$ of $[0, \bar{\theta}]$ where for all $\theta \in [0, \hat{\theta}_R]$ (4) holds and for all $\theta \in [\hat{\theta}_R, \bar{\theta}]$ (5) holds, (ii) a partition $\{[0, \hat{\theta}_O], [\hat{\theta}_O, \bar{\theta}]\}$ of $[0, \bar{\theta}]$ where for all $\theta \in [0, \hat{\theta}_O]$ (6) holds and for all $\theta \in [\hat{\theta}_O, \bar{\theta}]$ (7) holds, (iii) a wage $w$ for which labor demand equals labor supply
\[
N \left[(1 - \lambda) \int_{\hat{\theta}_R}^{\theta} l(\theta, w, r)dG(\theta) + \lambda \int_{\hat{\theta}_O}^{\theta} l(\gamma \theta, w, r)dG(\theta)\right] = N \left[(1 - \lambda) \int_{0}^{\theta} dG(\theta) + \lambda \int_{0}^{\theta} dG(\theta)\right],
\] (8)
and (iv) a rental cost of capital $r$ for which capital demand equals the exogenous capital supply
\[
N \left[(1 - \lambda) \int_{\hat{\theta}_R}^{\theta} k(\theta, w, r)dG(\theta) + \lambda \int_{\hat{\theta}_O}^{\theta} k(\gamma \theta, w, r)dG(\theta)\right] = K.
\] (9)

5The term $rK/N$ cancels out because an agent receives the rental price of his $K/N$ unit of capital both when he decides to be a worker and an entrepreneur.
In equilibrium, realists with ability below \( \hat{\theta}_R \) become workers whereas those with ability above \( \hat{\theta}_R \) become entrepreneurs. Similarly, optimists with below \( \hat{\theta}_O \) become workers whereas those with ability above \( \hat{\theta}_O \) become entrepreneurs. We refer to a realist with ability \( \hat{\theta}_R \) as the marginal realistic entrepreneur. We refer to an optimist with ability \( \hat{\theta}_O \) as the marginal optimistic entrepreneur.

## 4 Competitive Equilibrium

In this section we determine the competitive equilibrium under a generalized Cobb-Douglas production function and a uniform distribution of ability with support on \([0,1]\).\(^6\) The production function given by

\[
y = \theta f(l, k) = \theta l^\alpha k^\beta, \text{ with } \alpha + \beta \equiv \eta \in (0,1).
\]

Hence, the variable inputs, labor and capital, are combined under a generalized Cobb-Douglas production function with decreasing returns to scale.\(^7\) The profit of a realistic entrepreneur with ability \( \theta \) is

\[
\pi(\theta, w, r) = \theta l^\alpha k^\beta - wl + r(K/N - k).
\]

The perceived profit of an optimistic entrepreneur with perception of ability \( \gamma \theta \) is

\[
\pi(\gamma \theta, w, r) = \begin{cases} 
\gamma \theta l^\alpha k^\beta - wl + r(K/N - k) & \text{if } 0 \leq \theta < \frac{1}{\gamma} \\
l^\alpha k^\beta - wl + r(K/N - k) & \text{if } \frac{1}{\gamma} \leq \theta \leq 1 
\end{cases}
\]

where \( \gamma > 1 \). The assumptions: (i) individuals are risk neutral, (ii) ability \( \theta \) belongs to \([0,1]\), and (iii) \( \theta \) is the total factor productivity, imply that \( \theta \) can be interpreted

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\(^6\)In Section 7.1 we discuss how to relax a number of assumptions of the model, including how to deal with distributions of ability that are not uniform, and how to allow individuals to have heterogeneous abilities both as entrepreneurs and as workers.

\(^7\)This is a standard assumption in models with heterogeneous ability. See, for example, Lucas (1978), Evans and Jovanovic (1989), Murphy et al. (1991), de Meza and Southey (1996), and Poschke (2013).
as the probability of success of the firm (the project either succeeds with probability \( \theta \) or fails with probability \( 1 - \theta \), in which case output is zero).

An entrepreneur with perception of ability \( \gamma \theta \in [\theta, 1] \) chooses to employ \( l \) workers and \( k \) units of capital where \( l \) and \( k \) are the solution to

\[
\max_{l,k} \left[ \gamma \theta l^\alpha k^\beta - wl + r(K/N - k) \right].
\]

The first-order conditions are

\[
\alpha \gamma \theta l^{\alpha - 1} k^\beta = w,
\]

and

\[
\beta \gamma \theta l^\alpha k^{\beta - 1} = r.
\]

Solving for \( l \) and \( k \) we obtain the input demands:

\[
l(\gamma \theta, w, r) = (\gamma \theta)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\beta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\gamma}{1-\eta}}, \tag{11}
\]

and

\[
k(\gamma \theta, w, r) = (\gamma \theta)^{\frac{1}{1-\eta}} \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1-\alpha}{1-\eta}}, \tag{12}
\]

respectively. The input demands determine the size of the firm given the ability of the entrepreneur, the wage, the rental cost of capital, and the entrepreneur’s optimism.

We see from (11) and (12) that entrepreneurs’ demands for labor and capital are greater among those with higher ability \( \theta \). That is, more talented entrepreneurs run larger firms than less talented entrepreneurs, irrespective of whether firm size is defined in terms of labor or capital. We also see from (11) and (12) that optimists

\footnote{In other words, under this specification entrepreneurial optimism coincides with overestimation of ability. The strongest cross-national covariate of an individual’s entrepreneurial propensity is whether the person believes herself to have the sufficient skills and knowledge to start a business (Koellinger et al. (2007)). The probability of becoming an entrepreneur increases with a person’s confidence in his/her ability to perform entrepreneurship related tasks (Cassar and Friedman (2009)). Entrepreneurs are more overconfident about their abilities than non-entrepreneurs: 59 percent of entrepreneurs, 56 percent of the managers, and 52 percent of the employees overestimate their performance on a cognitive ability test (Koudstaal et al. (2015)).}
(\gamma > 1) run larger firms than realists (\gamma = 1) of the same ability. Substituting (11) and (12) into (10) and setting \gamma = 1 we obtain the profit of a realistic entrepreneur:

$$\pi(\theta, w, r) = \theta^{1-\eta}(1-\eta) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} + rK/N. \quad (13)$$

We see from (13) that the assumption of decreasing returns to scale, i.e., \eta \in (0, 1), implies that the profits of realistic entrepreneurs are an increasing and convex function of \theta. The perceived profit of an optimistic entrepreneur is:

$$\pi(\gamma \theta, w, r) = \begin{cases} (\gamma \theta)^{1-\eta}(1-\eta) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} + rK/N \text{ if } 0 \leq \theta < \frac{1}{\gamma} \\ (1-\eta) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} + rK/N \text{ if } \frac{1}{\gamma} \leq \theta \leq 1 \end{cases}. \quad (14)$$

We see from (14) that the assumption of decreasing returns to scale also implies that the perceived profits of optimistic entrepreneurs with perception of ability \gamma \theta and ability \theta < 1/\gamma are an increasing and convex function of \theta and of \gamma.

A realist with ability \hat{\theta}_R is indifferent between being an entrepreneur and a worker when

$$\hat{\theta}_R^{1-\eta}(1-\eta) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w. \quad (15)$$

An optimist with perception of ability \gamma \hat{\theta}_O and ability \hat{\theta}_O < 1/\gamma is indifferent between being an entrepreneur and a worker when

$$(\gamma \hat{\theta}_O)^{1-\eta}(1-\eta) \left( \frac{\alpha}{w} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{\beta}{1-\eta}} = w. \quad (16)$$

In equilibrium, labor demand must equal labor supply. The assumption that \theta is uniformly distributed on [0, 1] and (1) imply that (8) becomes:

$$(1-\lambda) \int_{\hat{\theta}_R}^{1} l(\theta, w, r)d\theta + \lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} l(\gamma \theta, w, r)d\theta + \int_{1/\gamma}^{1} l(1, w, r)d\theta \right] = (1-\lambda)\hat{\theta}_R + \lambda\hat{\theta}_O. \quad (17)$$

In equilibrium, capital demand must equal capital supply. The assumption that \theta is uniformly distributed on [0, 1] and (1) imply that (9) becomes:

$$(1-\lambda) \int_{\hat{\theta}_R}^{1} k(\theta, w, r)d\theta + \lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} k(\gamma \theta, w, r)d\theta + \int_{1/\gamma}^{1} k(1, w, r)d\theta \right] = K/N. \quad (18)$$
Equations (15), (16), (17), and (18) form a system of four equations and four unknowns \((\hat{\theta}_R, \hat{\theta}_O, w, r)\) which defines a unique competitive equilibrium. Since the profits of realistic entrepreneurs are an increasing and convex function of \(\theta\) it follows from (15) that there exists an unique ability cut-off between realistic entrepreneurs and realistic workers, i.e., \(\hat{\theta}_R\) is unique. Similarly, since the perceived profits of optimists are an increasing and convex function of \(\gamma\theta\) it follows from (16) that there exists an unique ability cut-off between optimistic entrepreneurs and optimistic workers, i.e., \(\hat{\theta}_O\) is unique. Solving (15) and (16) for the unique cut-offs \(\hat{\theta}_R\) and \(\hat{\theta}_O\) and substituting these into (17) and (18) we obtain the unique equilibrium vector of input prices \((w^*, r^*)\). Finally, from \((\hat{\theta}_R, \hat{\theta}_O, w^*, r^*)\) we obtain the equilibrium output level \(Y^*\). Hence, the existence and uniqueness of the equilibrium, a standard result in the Lucas “span-of-control” model, is not affected by the presence of optimists. Our first result characterizes the equilibrium.

**Proposition 1:** If the technology is \(f(l, k, \theta) = \theta^\alpha k^\beta\), ability \(\theta\) is uniformly distributed on \([0,1]\), and \(\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)\), then there exists a unique competitive equilibrium where the marginal realistic entrepreneur has ability

\[
\hat{\theta}_R = \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{1 - \eta}{2 - \eta}}, \tag{19}
\]

the marginal optimistic entrepreneur has ability

\[
\hat{\theta}_O = \frac{1}{\gamma} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{1 - \eta}{2 - \eta}}, \tag{20}
\]

the wage is

\[
w^* = \frac{\alpha^\eta (1 - \eta)^{1 - \eta} (K/N)^\beta}{\left(1 - \lambda + \frac{\lambda}{\gamma}\right)^\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{(1 - \eta)(1 - \beta)}{2 - \eta}}, \tag{21}
\]
the rental cost of capital is
\[ r^* = \frac{\beta (1 - \eta)^{1 - \eta}}{\alpha^{1 - \eta}(K/N)^{1 - \beta}} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{1 - \beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)} \right]^{\frac{(1 - \eta)(2 - \beta)}{2 - \eta}}, \tag{22} \]

the number of workers is
\[ L^* = N \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)} \right]^{\frac{1 - \eta}{2 - \eta}}, \tag{23} \]

and the output level is
\[ Y^* = N \left( \frac{\alpha}{w^*} \right)^{\frac{\alpha}{2 - \eta}} \left( \frac{\beta}{r^*} \right)^{\frac{\beta}{2 - \eta}} \left[ \frac{1 - \lambda + \frac{\lambda}{2 - \eta}}{2 - \eta} \left( 1 - \eta - \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) + \frac{\lambda - \frac{\lambda}{\gamma}}{2} \right]. \tag{24} \]

Proposition 1 shows us that the existence of optimists leads to a misallocation of talent. In a competitive equilibrium without optimists (where \( \lambda = 0 \) or \( \gamma = 1 \)) the marginal entrepreneur has ability
\[ \hat{\theta} = \left( \frac{\alpha}{2 - \beta} \right)^{\frac{1 - \eta}{2 - \eta}}, \tag{25} \]
which implies that individuals with ability \([0, \hat{\theta}]\) become workers and individuals with ability \([\hat{\theta}, 1]\) become entrepreneurs. Hence, in the competitive equilibrium without optimists the ablest people become entrepreneurs.

In a competitive equilibrium with optimists, realists with ability \([0, \hat{\theta}_R]\) and optimists with ability \([0, \hat{\theta}_O]\) become workers whereas realists with ability \([\hat{\theta}_R, 1]\) and optimists with ability \([\hat{\theta}_O, 1]\) become entrepreneurs. It follows from (19), (20), and \( \gamma > 1 \) that:
\[ \hat{\theta}_O < \hat{\theta}_R. \tag{26} \]

Hence, in the competitive equilibrium with optimists, the ablest people do not necessarily become entrepreneurs. Moreover, the lowest ability entrepreneur (an optimist
with ability $\hat{\theta}_O$) is less talented at running a firm than the highest ability worker (a realist with ability $\hat{\theta}_R$). This is an empirically attractive implication of the model since, in reality, the income distributions of workers and entrepreneurs have overlapping supports.

Proposition 1 implies that optimists are more likely to become entrepreneurs than realists. In other words, the probability an optimist becomes an entrepreneur is greater than the probability a realist becomes an entrepreneur. To see this note that the probability an optimist becomes an entrepreneur is

$$
Pr(E|O) = \frac{Pr(E \cap O)}{Pr(O)} = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda} = 1 - \hat{\theta}_O,
$$

and the probability a realist becomes an entrepreneur is

$$
Pr(E|R) = \frac{Pr(E \cap R)}{Pr(R)} = \frac{(1 - \lambda)(1 - \hat{\theta}_R)}{1 - \lambda} = 1 - \hat{\theta}_R.
$$

It follows from (26), (27), and (28) that $Pr(E|O) > Pr(E|R)$. This result is in line with empirical evidence that shows optimistic individuals are more likely to become entrepreneurs. For example, Puri and Robinson (2007) find that optimism is an important determinant of self-employment after controlling for a range of family, demographic, and wealth characteristics.

Proposition 1 also implies that entrepreneurs are more likely to be optimists than workers. In other words, the probability an entrepreneur is an optimist is greater than the probability a worker is an optimist. To see this note that the probability an entrepreneur is an optimist is

$$
Pr(O|E) = \frac{Pr(O \cap E)}{Pr(E)} = \frac{\lambda(1 - \hat{\theta}_O)}{\lambda(1 - \hat{\theta}_O) + (1 - \lambda)(1 - \hat{\theta}_R)},
$$

and the probability a worker is an optimist is

$$
Pr(O|L) = \frac{Pr(O \cap L)}{Pr(L)} = \frac{\lambda\hat{\theta}_O}{\lambda\hat{\theta}_O + (1 - \lambda)\hat{\theta}_R}.
$$
It follows from (26), (29), and (30) that \( \Pr(O|E) > \Pr(O|L) \). This result is in line with the empirical evidence in Arabsheibani et al. (2000), Fraser and Greene (2006), Puri and Robinson (2013), and Koudstaal et al. (2015).

To close this section we discuss the assumption \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \). This inequality tells us that the competitive equilibrium is well defined as long as the overall optimism in the economy—the product of the fraction of optimists \( \lambda \) by \((1 - 1/\gamma)\), a term that is increasing in the intensity of optimistic beliefs—is not too high.\(^9\)

## 5 Comparative Statics

In this section we perform comparative statics on equilibrium outcomes. There are two parameters which can be used to perform this analysis: the fraction of optimists \( \lambda \) and the intensity of optimism \( \gamma \). We focus on comparative statics with respect to \( \lambda \). At the end of this section we discuss briefly the comparative statics with respect to \( \gamma \).

**Proposition 2:** If the technology is \( f(l,k,\theta) = \theta l^\alpha k^\beta \), ability \( \theta \) is uniformly distributed on \([0,1]\), and \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \), then an increase in the fraction of optimists: (i) raises the market clearing wage, i.e., \( \partial w^*/\partial \lambda > 0 \), (ii) raises the rental cost of capital, i.e., \( \partial r^*/\partial \lambda > 0 \), and (iii) raises the number of entrepreneurs, i.e., \( \partial E^*/\partial \lambda > 0 \).

Part (i) shows that an increase in the fraction of optimists raises the market clearing wage. The intuition behind this result as follows. Wage effects can occur through two channels: through firm’s derived demand for labor and through labor-supply de-

\(^9\)When \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \) is violated there exists a unique competitive equilibrium where optimists who select to become entrepreneurs hold the highest possible perception of ability, i.e., \( \gamma \theta = 1 \). In addition, a positive mass of individuals with \( \gamma \theta = 1 \) choose to be workers since their entrepreneurial ability \( \theta \) is not high enough to make entrepreneurship more attractive than working as an employee.
cisions of individuals, who must choose to be either workers or entrepreneurs. The fact that optimists overestimate their ability implies that, for given input prices, the demand for labor of an optimist is higher than the demand for labor of a realist of the same ability. This leads to an expansion of labor demand. An optimist is, for given input prices, more attracted to entrepreneurship than a realist of the same ability. This leads to a contraction of labor supply. The expansion of labor demand and contraction of labor supply raise the market clearing wage.

Part (ii) shows that an increase in the fraction of optimists raises the rental cost of capital. The fact that optimists overestimate their ability implies that, for given input prices, the demand for capital of an optimist is higher than the demand for capital of a realist of the same ability. This leads to an expansion of capital demand. Since the supply of capital is fixed the expansion of capital demand raises the rental cost of capital.

Finally, part (iii) shows that an increase in the fraction of optimists raises the number of entrepreneurs. We have that, on the one hand, an increase in the fraction of optimists lowers the number of realistic entrepreneurs, and, on the other hand, it raises the number of optimistic entrepreneurs. Hence, at first sight, an increase in \( \lambda \) has an ambiguous effect on the number of entrepreneurs. However because optimists are more likely to become entrepreneurs than realists, as we showed in the previous section, the second effect always dominates the first and therefore an increase in \( \lambda \) raises the number of entrepreneurs.

To close this section we briefly discuss the comparative statics with respect to the intensity of optimism. An increase in \( \gamma \) raises the ability of the marginal realistic entrepreneur \( \hat{\theta}_R \) and lowers the ability of the marginal optimistic entrepreneur \( \hat{\theta}_O \). In addition, an increase in \( \gamma \) raises the wage, the rental cost of capital, and the number of entrepreneurs.
6 Calibration

This section calibrates the model to illustrate quantitatively the general equilibrium effects of optimism. The calibration parameterizes the economy to match salient features of U.S. manufacturing data and is summarized in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>0.850</td>
<td>decreasing returns to scale</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.612</td>
<td>labor's average income share</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.238</td>
<td>capital's average income share</td>
</tr>
<tr>
<td>( K )</td>
<td>0.906</td>
<td>capital stock</td>
</tr>
<tr>
<td>( N )</td>
<td>1</td>
<td>population</td>
</tr>
</tbody>
</table>

Following Atkeson and Kehoe (2005) we set \( \eta \) to 0.85. Following Adler (2016), given \( \eta \) equal to 0.85, a value of 0.612 for \( \alpha \) matches labor’s average income share (including managerial compensation) in U.S. manufacturing between 1998 and 2005. Again, following Atkeson and Kehoe (2005) and Adler (2016) we assume a capital-output ratio \( K/Y \) of 1.46 which together with a value for \( Y \) of 0.62032 in the model without optimists (\( \lambda = 0 \)) implies a capital stock \( K \) of 0.906.

We are left with the behavioral parameters \( \lambda \) and \( \gamma \) to calibrate. Recall that \( \lambda \) represents the fraction of optimists and \( \gamma \) the intensity of optimistic beliefs. The ideal data to calibrate \( \lambda \) and \( \gamma \) would consists of representative samples of entrepreneurs and employees with measures of optimism that compare expectations to realized financial outcomes in the U.S. manufacturing sector. We are unaware of such data so we take the following approach. First, we review the evidence of empirical studies.
on optimism of entrepreneurs and employees in the U.S. and elsewhere. Second, we use the empirical evidence to obtain a lower bound for the fraction of optimists in the U.S. \( \lambda \) and a lower bound for the share of optimistic entrepreneurs in the U.S., \( E^*_O/E^* \). Third, using the lower bounds for \( \lambda \) and \( E^*_O/E^* \) together with the values for \( \alpha \) and \( \beta \) we calibrate \( \gamma \) to satisfy the equilibrium condition

\[
\frac{\lambda \left(1 - \hat{\theta}_R/\gamma \right)}{\lambda(1 - \hat{\theta}_R/\gamma) + (1 - \lambda)(1 - \hat{\theta}_R)} = \frac{E^*_O}{E^*}.
\]

(31)

Finally, the calibration must satisfy \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \).

The empirical evidence shows that: (i) the fraction of optimists in the U.K. is 31 percent, (ii) the fraction of optimistic entrepreneurs in the U.S. varies from 33 to 48.8 to 62 percent, and (iii) the fraction of optimistic entrepreneurs is quite high in the U.S., U.K., Finland, and Netherlands.10 Based on these studies we set the lower bound for the fraction of optimists in the U.S. at \( \lambda = 0.31 \) and we set the lower bound for the fraction of optimistic entrepreneurs in the U.S. at \( E^*_O/E^* = (33 + 48.8 + 62)/3 = 0.48 \). Setting \( \alpha = 0.612, \beta = 0.238, \lambda = 0.31 \) and \( E^*_O/E^* = 0.48 \) in (31) and solving for \( \gamma \) we obtain \( \gamma = 1.275 \). The calibration satisfies \( \lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta) \) since \( 0.31 - 0.31/1.275 = 0.06863 < 0.19685 = (1 - 0.85)/(1 - 0.238) \).

Table II summarizes the results of the calibration. The first column in Table II lists the variables. The second and the third columns report the competitive

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10Cooper et al. (1988) find that 33 percent of a sample of 2994 U.S. nascent entrepreneurs perceive their chances of success as 10 out of 10 or “absolutely certain.” Arabsheibani et al. (2000) report results from a sample of 2909 entrepreneurs and 20056 employees from the British Household Panel Study. They find that 31 percent of individuals are optimists, 34.754 percent of entrepreneurs are optimists, and 30.46 percent of employees are optimists. Cassar (2010) reports that 62 percent of a sample of 386 U.S. nascent entrepreneurs from the Panel Study of Entrepreneurial Dynamics (PSED) who achieve operation overestimate projected first-year sales. Hyytinen et al. (2014) report that 48.8 percent of a sample of 487 U.S. nascent entrepreneurs from the PSED think that the likelihood of exit of their venture is zero in five years time. They also report that 34.5 percent of a sample of 891 Finnish nascent entrepreneurs think that the likelihood of exit of their new venture is zero in three years’ time. Koudstaal et al. (2015) find that 58 percent of entrepreneurs, 54 percent of the managers, and 32 percent of the employees can be classified as very optimistic.
equilibrium without and with optimists, respectively. The fourth column reports the percent change in the variables common to both models.

<table>
<thead>
<tr>
<th>Table II Calibration</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>Lucas (1978) and $\gamma = 1.275$</td>
</tr>
<tr>
<td>Output ($Y^*$)</td>
</tr>
<tr>
<td>Wage ($w^*$)</td>
</tr>
<tr>
<td>Rental cost of capital ($r^*$)</td>
</tr>
<tr>
<td>Mean returns to entrepreneurship</td>
</tr>
<tr>
<td>Mean returns of realistic entrep.</td>
</tr>
<tr>
<td>Mean returns of optimistic entrep.</td>
</tr>
<tr>
<td>Fraction of workers ($L^*$)</td>
</tr>
<tr>
<td>Ability marginal realistic entrep. ($\hat{\theta}_R$)</td>
</tr>
<tr>
<td>Ability marginal optimistic entrep. ($\hat{\theta}_O$)</td>
</tr>
<tr>
<td>Fraction of entrep. ($E^*$)</td>
</tr>
<tr>
<td>Fraction of entrep optimists ($E^<em>_O/E^</em>$)</td>
</tr>
</tbody>
</table>

The calibration tells us that optimism leads to a 3.93 percent decline in output. This result is expected since we know from Lucas (1978) that, in the absence of distortions, the competitive equilibrium maximizes output. The calibration also shows that optimism leads to an overuse of scarce resources in equilibrium and bids up input prices: the wage increases by 5.94 percent and the rental rate of capital by 4.90 percent. Furthermore, optimism leads to 57.05 percent decline in the mean returns to entrepreneurship. The mean returns to entrepreneurship without optimism ($\lambda = 0$) is equal to

$$\frac{\pi^*_0}{E^*_0} = \frac{Y^*_0 - w^*_0L^*_0 - r^*_0K}{N(1 - \hat{\theta}_0)} = 0.71506,$$

whereas the mean returns to entrepreneurship with optimism ($\lambda = 0.31$ and $\gamma = 1.275$)
\[ \pi^* = \frac{Y^* - w^* L^* - r^* K}{N \lambda (1 - \hat{\theta}_O) + N (1 - \lambda) (1 - \hat{\theta}_R)} = 0.30713. \]

The sharp decline in the mean returns to entrepreneurship happens due to four reasons. First, optimism leads to a misallocation of talent since lower skill optimistic individuals crowd out higher skill realistic individuals from entrepreneurship.\(^{11}\) Second, optimism raises input prices. Third, optimism distorts the input choices of optimistic entrepreneurs. Fourth, optimism raises the number of entrepreneurs.\(^{12}\)

The calibration also tells us that optimistic entrepreneurs earn less than realists. This is consistent with empirical evidence that shows that optimism is on average bad for performance (Landier and Thesmar (2009)), and that entrepreneurs’ level of optimism has, on average, a negative relationship with the performance of their new ventures (Hmieleski and Baron (2009)).\(^{13}\)

In sum, the calibration shows that optimism can explain quantitatively the empirical puzzle of the low mean returns to entrepreneurship compared to average wages.

7 Assumptions, Extensions and Implications

In this section, we discuss the main assumptions of the model and some extensions. Then we discuss policy implications of the analysis.

\(^{11}\)The misallocation of talent affects 8.11 percent of the population since \(N \lambda (\hat{\theta}_0 - \hat{\theta}_O) + N (1 - \lambda) (\hat{\theta}_R - \hat{\theta}_0) = 0.0811.\)

\(^{12}\)Optimism leads to a 1.20 percent decline in the fraction of workers (from 87 to 86 percent) and to a 8.11 percent increase in the fraction of entrepreneurs (from 13 to 14 percent).

\(^{13}\)Dawson et al. (2015) examine how entrepreneurs’ forecasts predict entrepreneurship performance using the BHPS during the years 1991-2008 and find that optimists, on average, earn less than pessimists.
### 7.1 Assumptions and Extensions

We assume that the returns from entrepreneurship are deterministic. It is possible to extend the model by including a random component $\varepsilon$ in entrepreneurial revenues. For example, letting $y = \theta f(l, k) + \varepsilon$, where $\varepsilon$ has mean 0 and variance $0 < \sigma^2 < \infty$. Since individuals are risk neutral all results are left unchanged as long as there is no optimism about the realization of $\varepsilon$. If individuals are not only optimistic about $\theta$ but also about $\varepsilon$, then entrepreneurship would be more attractive relative to paid employment. In this case the main qualitative effects of optimism would still hold but its quantitative effects would be larger.

We assume individuals have different abilities to run a firm and the same productivity (or ability) as workers. This implies that different entrepreneurs obtain different amounts of profit but that workers receive the same wage. This is a natural simplification since the empirical evidence shows that the returns to entrepreneurship are much more variable than wages (Borjas and Bronars (1989), Hamilton (2000)). Still, the model could be extended by letting individuals have different abilities in both occupations. Following Jovanovic (1994), we could let the returns to paid employment be equal to $w\psi(\theta)$ where $\psi(\theta)$ is the wage-working ability of an individual with ability $\theta$. If $\psi$ is a strictly increasing function (good entrepreneurs are also good workers), then optimists would overestimate the returns to entrepreneurship as well as the returns to paid employment. Since these two effects would partially cancel out, the main qualitative effects of optimism would still hold but its quantitative

---

14 Jovanovic generalizes Lucas (1978) by allowing for heterogeneous working abilities, i.e., the labor income of a worker is given by $wy$ where $y$ represents working ability. Working ability $y$ is correlated with entrepreneurial ability $\theta$ if $y = \psi(\theta)$. Jovanovic shows that when $\psi$ is either (i) strictly decreasing or (ii) strictly increasing and not very steep at high levels of $\theta$, then the best potential entrepreneurs are drawn into entrepreneurship. In contrast, when $\psi$ is strictly increasing and very steep at high levels of $\theta$, then the best potential entrepreneurs end up as wage workers.

15 The empirical evidence supports the assumption that entrepreneurial and wage-working abilities are positively correlated, i.e., $\psi$ is a strictly increasing function. See Murphy et al. (1991), Javonovic (1994), and Braguinsky et al. (2011).
effects would be smaller.\textsuperscript{16}

In our model an entrepreneur hires workers and rents capital to produce output. However, the empirical evidence shows that many firms have no employed workers, i.e., the owners of these firms are self-employed without employees (see Braguinsky et al. (2011) and Salas-Fumas et al. (2014)). The model could also be extended to incorporate this third type of occupational choice. This could be done by assuming that the returns of firms without employed workers are given by $B + \theta$, where $B > 0$ represents a non-pecuniary benefit like the utility derived from “being your own boss” (see Hurst and Pugsley (2011) and Åstebro et al. (2014)). In this case, realists with ability $\theta$ such that $w > \max\{B + \theta, \pi(\theta, w, r)\}$ would become workers, those with ability $\theta$ such that $B + \theta > \max\{w, \pi(\theta, w, r)\}$ would open a firm without employed workers, and those with ability $\theta$ such that $\pi(\theta, w, r) > \max\{B + \theta, w\}$ would become entrepreneurs. Similarly, optimists with perception of ability $\gamma \theta$ such that $w > \max\{B + \gamma \theta, \pi(\gamma \theta, w, r)\}$ would become workers, those with perception of ability $\gamma \theta$ such that $B + \gamma \theta > \max\{w, \pi(\gamma \theta, w, r)\}$ would open a firm without employed workers, and those with perception of ability $\gamma \theta$ such that $\pi(\gamma \theta, w, r) > \max\{B + \gamma \theta, w\}$ would become entrepreneurs.

We assume ability is uniformly distributed. However, empirical evidence shows that firm size might be better described by a lognormal distribution (Cabral and Mata (2003)). A right-skewed distribution of ability, like the lognormal, interacts with the optimistic beliefs of individuals. Under our simple specification for optimistic beliefs, individuals with intermediate ability levels are the ones who overestimate their abilities the most. If the number of these individuals is small, then the qualitative effects of optimism would still hold but its quantitative effects would be smaller.

We also assume that the intensity of optimistic beliefs is exogenous. We extended the model by endogeneizing the intensity of optimistic beliefs using the optimal ex-

\textsuperscript{16}We are assuming here that $\psi$ is strictly increasing and not very steep at high levels of $\theta$. In this case the most talented individuals become entrepreneurs. In contrast, when $\psi$ is strictly increasing and very steep at high level of $\theta$, the most talented individuals become workers.
pectations model of Brunnermeier and Parker (2005). We found that the qualitative and quantitative impact of optimism on labor, capital and output markets would be similar as the ones obtained under exogenous optimistic beliefs. The main novelty of this extension would be that the intensity of optimistic beliefs would become a function of tastes and technology.

We focus on differences in ability and optimism as the main determinants which explain who becomes an entrepreneur and who works as an employee. There are of course many other factors which could influence this choice. For example, entrepreneurial effort (and the disutility of exerting it), access to funds needed to create a firm, risk aversion, and learning about ability. We do not model entrepreneurial effort and therefore we rule out any positive effects of optimism on entrepreneurial effort like the ones found in Manove (2000). If ability and effort are complements, then optimistic entrepreneurs would provide more effort than realistic ones. In this case the impact of optimism on the returns to entrepreneurship and on output would be ambiguous. We assume individuals are risk neutral so we cannot discuss the role that risk aversion together with optimism might play in the decision to become an entrepreneur or a worker. In addition, our model is static so we rule out the possibility that optimists learn their true abilities over time. We believe these are interesting avenues for future research.

7.2 Policy Implications

Are there any policy implications one can take away from this model? Given their mistaken beliefs, individuals in this economy are not maximizing their utility and so the economy is in a second-best situation. If the goal of a policymaker is to move the economy back to the first-best, then it is possible to do so with a revenue-neutral tax-subsidy scheme. The scheme consists of a lump-sum tax to (optimistic) entrepreneurs with profits below the market clearing wage and a lump-sum subsidy to workers. The tax revenues come only from low ability optimistic entrepreneurs and induces them to stay in the labor force. The tax revenues are redistributed to
workers as a lump-sum subsidy which further induces low ability optimists to stay in the labor force. The full characterization of this tax-subsidy scheme is available upon request.

8 Conclusion

We present a fully specified general equilibrium model of occupational choice where a fraction of individuals are optimists about ability. We find that optimism has four qualitative effects: it leads to a misallocation of talent, drives up input prices, raises the number of entrepreneurs, and makes entrepreneurs worse off.

We calibrate the model to match salient features of U.S. manufacturing data. We find that optimism may significantly change the distribution of income by lowering the mean returns to entrepreneurship and driving up the wage. Overall, the calibration shows that optimism can explain quantitatively the empirical puzzle of the low mean returns to entrepreneurship compared to average wages.

9 Appendix

Proof of Proposition 1: Let $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$. The first step to determine the competitive equilibrium is to find out the labor market equilibrium condition. The labor demand from realistic entrepreneurs is

$$L^D_R = N(1 - \lambda) \int_{\theta_R}^{1} l(\theta, w, r)d\theta$$

$$= N(1 - \lambda) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{\frac{\eta}{1-\eta}} \int_{\theta_R}^{1} \theta^{\frac{1}{1-\eta}}d\theta$$

$$= N(1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R^{\frac{2}{\gamma}} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{\frac{\eta}{1-\eta}}. \quad (32)$$

Note that for $L^D_R$ to be well defined it must be that $\hat{\theta}_R < 1$. Recall that $\hat{\theta}_O$ is the ability threshold that determines the marginal optimistic entrepreneur. If $\hat{\theta}_O < 1/\gamma$, ...
then labor demand from optimistic entrepreneurs is the sum of the demand for labor coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with \( \theta \in (\hat{\theta}_O, 1/\gamma) \), to the demand for labor coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with \( \theta \in (1/\gamma, 1) \):

\[
L^D_O = N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} l(\gamma \theta, w, r) d\theta + \int_{1/\gamma}^1 l(1, w, r) d\theta \right]
\]

\[
= N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} (\gamma \theta)^{1-\eta} d\theta + \int_{1/\gamma}^1 d\theta \right] \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
= N\lambda \left( \gamma^{1-\eta} \int_{\hat{\theta}_O}^{1/\gamma} \theta^{1-\eta} d\theta + 1 - \frac{1}{\gamma} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
= N\lambda \left( \gamma^{1-\eta} \frac{1 - \eta}{2 - \eta} \left[ \frac{\theta^{2-\eta}}{2-\eta} \right]^{1/\gamma}_{\hat{\theta}_O} + 1 - \frac{1}{\gamma} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
= N\lambda \left[ \gamma^{1-\eta} \frac{2 - \eta - 1/\gamma}{2 - \eta} \left( 1 - \frac{\hat{\theta}_O^{2-\eta}}{1-\eta} \right) + 1 - \frac{1}{\gamma} \right] \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
= N\lambda \left[ \frac{2 - \eta - 1/\gamma}{2 - \eta} \left( 1 - \frac{\hat{\theta}_O^{2-\eta}}{1-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}.
\]

Note that for \( L^D_O \) to be well defined it must be that \( \hat{\theta}_O < 1/\gamma \). From (32) and (33), labor demand is equal to

\[
L^D = L^D_R + L^D_O
\]

\[
= N(1 - \lambda) \frac{2 - \eta}{2 - \eta} \left( 1 - \frac{\hat{\theta}_R^{2-\eta}}{1-\eta} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
+ N\lambda \frac{2 - \eta - 1/\gamma}{2 - \eta} \left( 1 - \frac{\hat{\theta}_O^{2-\eta}}{1-\eta} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]

\[
= N \frac{2 - \eta}{2 - \eta} \left[ (1 - \lambda) \left( 1 - \frac{\hat{\theta}_R^{2-\eta}}{1-\eta} \right) + \lambda \left( 1 - \frac{\hat{\theta}_O^{2-\eta}}{1-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta}
\]
Since each worker provides a unit of labor, labor supply is

\[
L^S = N \left[ (1 - \lambda) L^S_R + \lambda L^S_O \right]
\]

\[
= N \left[ (1 - \lambda) \int_{\theta_R}^{\hat{\theta}_R} d\theta + \lambda \int_{\theta_O}^{\hat{\theta}_O} d\theta \right]
\]

\[
= N \left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right]. \quad (34)
\]

In equilibrium, labor demand must equal labor supply:

\[
\frac{1 - \eta}{2 - \eta} \left[ (1 - \lambda) \left( 1 - \hat{\theta}^{2 - \eta}_R \right) + \lambda \left( \frac{2 - \eta - 1/\gamma}{1 - \eta} - \gamma^{1 - \eta} \hat{\theta}^{2 - \eta}_O \right) \right] \left( \frac{\alpha}{w} \right)^{1 - \eta} \left( \frac{\beta}{r} \right)^{\frac{1 - \eta}{\gamma}}
\]

\[
= (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O, \quad (35)
\]

The second step to determine the competitive equilibrium is to find out the capital market equilibrium condition. The capital demand from realistic entrepreneurs is

\[
K^D_R = N(1 - \lambda) \int_{\theta_R}^{1} k(\theta, w, r) d\theta
\]

\[
= N(1 - \lambda) \left( \frac{\alpha}{w} \right)^{\frac{1}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{1 - \eta}{\gamma}} \int_{\theta_R}^{1} \theta^{\frac{1}{1 - \eta}} d\theta
\]

\[
= N(1 - \lambda) \frac{1 - \eta}{2 - \eta} \left( 1 - \hat{\theta}_R^{2 - \eta} \right) \left( \frac{\alpha}{w} \right)^{\frac{1}{1 - \eta}} \left( \frac{\beta}{r} \right)^{\frac{1 - \eta}{\gamma}}. \quad (36)
\]

Note that for \( K^D_R \) to be well defined it must be that \( \hat{\theta}_R < 1 \). Recall that \( \hat{\theta}_O \) is the ability threshold that determines the marginal optimistic entrepreneur. If \( \hat{\theta}_O < 1/\gamma \), then capital demand from optimistic entrepreneurs is the sum of the demand for capital coming from the mass of entrepreneurs with heterogeneous optimistic expectations, i.e., those with \( \theta \in (\hat{\theta}_O, 1/\gamma) \), to the demand for capital coming from the mass of entrepreneurs with homogeneous optimistic expectations, i.e., those with
\( \theta \in (1/\gamma, 1): \)

\[
K_D^O = N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} k(\theta \gamma, w, r) d\theta + \int_{1/\gamma}^{1} k(1, w, r) d\theta \right]
\]

\[
= N\lambda \left[ \int_{\hat{\theta}_O}^{1/\gamma} \frac{1}{\gamma} \frac{1}{1-\gamma} d\theta + \int_{1/\gamma}^{1} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}} d\theta \right]
\]

\[
= N\lambda \frac{1-\eta}{2-\eta} \left[ \frac{2-\eta-1/\gamma}{1-\eta} - \frac{\gamma}{1-\eta} \hat{\theta}_O^{2-\eta} \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}}.
\]

(37)

Note that for \( K_D^O \) to be well defined it must be that \( \hat{\theta}_O < 1/\gamma \). From (36) and (37), capital demand is equal to

\[
K^D = K_D^R + K_D^O
\]

\[
= N(1-\lambda) \frac{1-\eta}{2-\eta} \left( 1 - \hat{\theta}_R^{2-\eta} \right) \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}}
\]

\[
+ N\lambda \frac{1-\eta}{2-\eta} \left[ \frac{2-\eta-1/\gamma}{1-\eta} - \frac{\gamma}{1-\eta} \hat{\theta}_O^{2-\eta} \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}}
\]

\[
= N \frac{1-\eta}{2-\eta} \left[ (1-\lambda) \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda \left( \frac{2-\eta-1/\gamma}{1-\eta} - \frac{\gamma}{1-\eta} \hat{\theta}_O^{2-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}}
\]

In equilibrium, capital demand must equal the exogenous capital supply:

\[
\frac{1-\eta}{2-\eta} \left[ (1-\lambda) \left( 1 - \hat{\theta}_R^{2-\eta} \right) + \lambda \left( \frac{2-\eta-1/\gamma}{1-\eta} - \frac{\gamma}{1-\eta} \hat{\theta}_O^{2-\eta} \right) \right] \left( \frac{\alpha}{w} \right)^{\frac{1-\eta}{1-\eta}} \left( \frac{\beta}{r} \right)^{\frac{1}{1-\eta}}
\]

\[
= K/N.
\]

(38)

The third step to determine the competitive equilibrium is to find out the ability level of the marginal realistic entrepreneur \( \hat{\theta}_R \) and of the marginal optimistic entrepreneur \( \hat{\theta}_O \). A realist with ability \( \hat{\theta}_R \) is indifferent between being an entrepreneur and a worker when

\[
\hat{\theta}_R \left[ l(\hat{\theta}_R, w, r) \right]^\alpha \left[ k(\hat{\theta}_R, w, r) \right]^{\beta} - w(\hat{\theta}_R, w, r) + r \left[ K/N - k(\hat{\theta}_R, w, r) \right] = w + rK/N,
\]
or
\[ \hat{\theta}_R \left[ \hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \right]^\alpha \left[ \hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \right]^\beta \]

or
\[ -w\hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} - r\hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} = w, \]

or
\[ \hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \left[ 1 - w \left( \frac{\alpha}{w} \right) - r \left( \frac{\beta}{r} \right) \right] = w, \]

or
\[ \hat{\theta}_R^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} (1-\eta) = w, \]

or
\[ \hat{\theta}_R^{1-\eta} \alpha^{1-\eta} \beta^{1-\eta} (1-\eta) = w^{1-\eta} r^{1-\eta}, \]

or
\[ \alpha^\alpha \beta^\beta \beta^\beta (1-\eta)^1 \hat{\theta}_R = w^{1-\eta} r^\beta. \]  \hspace{1cm} (39)

An optimist with perception of ability \( \theta^* = \gamma \hat{\theta}_O \) and ability \( \hat{\theta}_O \) is indifferent between being an entrepreneur and a worker when

\[ \gamma \hat{\theta}_O \left[ l(\gamma \hat{\theta}_O, w, r) \right]^\alpha \left[ k(\gamma \hat{\theta}_O, w, r) \right]^\beta - w l(\gamma \hat{\theta}_O, w, r) r \left[ K/N - k(\gamma \hat{\theta}_O, w, r) \right] = w + r K/N, \]

or
\[ \gamma \hat{\theta}_O \left[ (\gamma \hat{\theta}_O)^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \right]^\alpha \left[ (\gamma \hat{\theta}_O)^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} \right]^\beta = w, \]

or
\[ (\gamma \hat{\theta}_O)^{1-\eta} \left( \frac{\alpha}{w} \right)^{1-\eta} \left( \frac{\beta}{r} \right)^{1-\eta} = w, \]

or
\[ \alpha^\alpha \beta^\beta (1-\eta)^{1-\eta} \hat{\theta}_O = w^{1-\eta} r^\beta. \]  \hspace{1cm} (40)
It follows from (39) and (40) that
\[ \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \hat{\theta}_O = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta} \hat{\theta}_R, \]
or
\[ \hat{\theta}_O = \frac{1}{\gamma} \hat{\theta}_R. \] (41)

Substituting (39) and (41) into (35) we obtain
\[ (1 - \lambda) \left( 1 - \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R} \right) + \lambda \left[ \frac{2 - \eta}{1-\eta} \left( 1 - \frac{1}{\gamma} \right) + \frac{1}{\gamma} - \gamma \frac{1}{\gamma} \gamma^{\frac{2-\eta}{1-\eta}} \right] \]
\[ = \frac{2 - \eta}{\alpha} \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R} \left[ (1 - \lambda) \hat{\theta}_R + \frac{1}{\gamma} \hat{\theta}_R \right], \]
or
\[ (1 - \lambda + \frac{\lambda}{\gamma}) \left( 1 - \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R} \right) + \lambda \frac{2 - \eta}{1-\eta} \left( 1 - \frac{1}{\gamma} \right) = \frac{2 - \eta}{\alpha} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R}, \]
or
\[ \lambda \frac{2 - \eta}{1-\eta} \left( 1 - \frac{1}{\gamma} \right) + \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) = \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \frac{2 - \beta}{\alpha} \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R} \]
or
\[ \hat{\theta}^{\frac{2-\eta}{1-\eta}}_{\gamma R} = \frac{\lambda^{\frac{2-\eta}{1-\eta}} \left( 1 - \frac{1}{\gamma} \right) + \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)}{\left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{2-\beta}{\alpha}}} \]
\[ = \frac{\alpha}{2 - \beta} \left( 1 + \frac{2 - \eta}{1-\eta} \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) \]

Hence, the ability of the marginal realistic entrepreneur is
\[ \hat{\theta}_R = \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)} \right]^{\frac{1-\eta}{2-\eta}}. \] (42)
From (41) and (42) the ability of the marginal optimistic entrepreneur is
\[
\hat{\theta}_O = \frac{1}{\gamma} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{1 - \eta}{1 - \beta}}.
\]

From (35) and (38) we have
\[
\left[ (1 - \lambda)\hat{\theta}_R + \lambda \hat{\theta}_O \right] \left( \frac{w}{\alpha} \right)^{\frac{1 - \beta}{1 - \eta}} \left( \frac{r}{\beta} \right)^{\frac{\beta}{1 - \eta}} = (K/N) \left( \frac{w}{\alpha} \right)^{\frac{\alpha}{1 - \eta}} \left( \frac{r}{\beta} \right)^{\frac{1 - \alpha}{1 - \eta}},
\]
or
\[
\alpha r K/N = \beta w \left[ (1 - \lambda)\hat{\theta}_R + \lambda \hat{\theta}_O \right],
\]
or
\[
r = \frac{\beta w}{\alpha K/N} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R. \tag{43}
\]

Substituting (43) into (39) we obtain
\[
\alpha^\beta (1 - \eta)^{1 - \eta} \hat{\theta}_R = w^{1 - \beta} \left( \frac{\beta}{\alpha} \right)^{\beta} w^\beta \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\beta} \hat{\theta}_R^\beta (K/N)^{-\beta}.
\]

Solving this equality with respect to \( w \) we obtain the equilibrium wage:
\[
w^* = \frac{\alpha (1 - \eta)^{1 - \eta} (K/N)^\beta \hat{\theta}_R^{1 - \beta}}{(1 - \lambda + \frac{\lambda}{\gamma})^{\beta}} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right)} \right]^{\frac{(1 - \eta)(1 - \beta)}{1 - \eta}}.
\]
The equilibrium rental cost of capital is equal to
\[
    r^* = \frac{\beta w^*}{\alpha K/N} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R
\]
\[
    = \frac{\beta \alpha^\eta (1-\eta)^{1-\eta} (K/N)^{\beta \hat{\theta}_R^{1-\beta}}}{\alpha (K/N) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^\beta} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R
\]
\[
    = \frac{\beta (1-\eta)^{1-\eta} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{1-\beta} \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{1-\eta}} \right]}{\alpha^{1-\eta} (K/N)^{1-\beta} (1-\eta)^{\frac{1-\eta}{2-\eta} \alpha}} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{\alpha}{2-\eta}} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{2-\eta}}.
\]

The equilibrium labor force is equal to
\[
    L^* = N \left[ (1 - \lambda) \hat{\theta}_R + \lambda \hat{\theta}_O \right] = N \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \hat{\theta}_R
\]
\[
    = N \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \left[ \frac{\alpha}{2 - \beta} \frac{1 - \eta + \lambda - \frac{\lambda}{\gamma}}{(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{1-\eta}} \right].
\]

The equilibrium output level is
\[
    Y^* = (1 - \lambda) N \int_{\hat{\theta}_R}^{1} \theta [l(\theta, w^*, r^*)]^{\alpha} [k(\theta, w^*, r^*)]^{\beta} d\theta
\]
\[
    + \lambda N \int_{\hat{\theta}_O}^{\frac{1}{\gamma}} \theta [l(\gamma \theta, w^*, r^*)]^{\alpha} [k(\gamma \theta, w^*, r^*)]^{\beta} d\theta
\]
\[
    + \lambda N \int_{\frac{1}{\gamma}}^{1} \theta [l(1, w^*, r^*)]^{\alpha} [k(1, w^*, r^*)]^{\beta} d\theta.
\]
This can be simplified to

\[ Y^* = N \left( \frac{\alpha}{w^*} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r^*} \right)^{\frac{\beta}{1-\eta}} \left[ (1 - \lambda) \int_{\hat{\theta}_L}^{1} \theta^{1-\eta} d\theta + \lambda \gamma^{\frac{\eta}{1-\eta}} \int_{\hat{\theta}_O}^{1/\gamma} \theta^{1-\eta} d\theta + \lambda \int_{1/\gamma}^{1} \theta d\theta \right] \]

\[ = N \left( \frac{\alpha}{w^*} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r^*} \right)^{\frac{\beta}{1-\eta}} \left\{ \frac{1 - \eta}{2 - \eta} \left[ (1 - \lambda) \left[ \theta^{\frac{2-\eta}{1-\eta}} \right]_{\hat{\theta}_R} + \lambda \gamma^{\frac{\eta}{1-\eta}} \left[ \theta^{\frac{2-\eta}{1-\eta}} \right]_{\hat{\theta}_O} + \frac{\lambda}{2} \left[ \theta^2 \right]_{1/\gamma} \right] \right\} \]

\[ = N \left( \frac{\alpha}{w^*} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{\beta}{r^*} \right)^{\frac{\beta}{1-\eta}} \left[ \frac{1 - \eta}{2 - \eta} \left( 1 - \frac{\lambda + \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) \left( 1 - \eta - \frac{\alpha}{2 - \beta} \frac{1 - \eta + \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) + \frac{\lambda - \frac{\lambda}{\gamma^2}}{2} \right] \]

For the equilibrium to be well defined we need to make sure that \( \hat{\theta}_R \) is less than 1, i.e.,

\[ \left[ \frac{\alpha}{2 - \beta} \left( 1 + \frac{2 - \eta}{1 - \eta} \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} \right) \right]^{\frac{\alpha}{2 - \eta}} < 1 \]

or

\[ \frac{2 - \eta}{1 - \eta} \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} < \frac{2 - \beta}{\alpha} - 1 \]

or

\[ \frac{\lambda - \frac{\lambda}{\gamma}}{1 - \lambda + \frac{\lambda}{\gamma}} < \frac{1 - \eta}{\alpha} \]

or

\[ \alpha \left( \lambda - \frac{\lambda}{\gamma} \right) < (1 - \alpha - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \]

or

\[ \alpha \left( \lambda - \frac{\lambda}{\gamma} \right) + \alpha \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) < (1 - \beta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) \]

or

\[ \lambda - \frac{\lambda}{\gamma} < \frac{1 - \alpha - \beta}{1 - \beta}, \]

which is true by assumption. \( Q.E.D. \)
Proof of Proposition 2: Let $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$. It follows directly from (19) and (20) that
\[
\frac{\partial \hat{\theta}_R}{\partial \lambda} > 0 \text{ and } \frac{\partial \hat{\theta}_O}{\partial \lambda} > 0.
\]
(i) It follows directly from (21) that
\[
\frac{\partial w^*}{\partial \lambda} > 0.
\]
(ii) The rental cost of capital is given by (22). To show that $\frac{\partial r^*}{\partial \lambda} > 0$ we need to show:
\[
\frac{\partial}{\partial \lambda} \left[ \left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\frac{\alpha}{2-\eta}} \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right)^{\frac{(1-\eta)(2-\beta)}{2-\eta}} \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right)^{-1} \right] > 0.
\]
We have that
\[
\frac{\alpha}{2-\eta} \left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\frac{\alpha}{2-\eta}-1} \left(-1 + \frac{1}{\gamma}\right) \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right)^{\frac{(1-\eta)(2-\beta)}{2-\eta}} + \frac{(1-\eta)(2-\beta)}{2-\eta} \left(1 - \lambda + \frac{\lambda}{\gamma}\right)^{\frac{\alpha}{2-\eta}} \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right)^{-1} \left(1 - \frac{1}{\gamma}\right) > 0
\]
or
\[
(1 - \eta)(2 - \beta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right) > \alpha \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right).
\]
Since $\lambda - \lambda/\gamma < (1 - \alpha - \beta)/(1 - \beta)$ is equivalent to $\alpha < (1 - \beta)(1 - \lambda + \lambda/\gamma)$ the above inequality is satisfied if
\[
(1 - \eta)(2 - \beta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right) > (1 - \beta) \left(1 - \lambda + \frac{\lambda}{\gamma}\right) \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right),
\]
or
\[
(1 - \eta)(2 - \beta) > (1 - \beta) \left(1 - \eta + \lambda - \frac{\lambda}{\gamma}\right),
\]
or
\[
(1 - \alpha - \beta) > (1 - \beta) \left(\lambda - \frac{\lambda}{\gamma}\right),
\]
or
\[
\lambda - \frac{\lambda}{\gamma} < \frac{1 - \beta}{1 - \alpha - \beta};
\]
38
which is true.

(iii) The number of workers is given by (23). To show that \( \partial L^*/\partial \lambda < 0 \) we need to show:

\[
\frac{\partial}{\partial \lambda} \left[ \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{1}{1-\eta}} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{1-\eta}} \right] < 0.
\]

We have that

\[
\frac{1}{2-\eta} \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{1}{1-\eta}} \left( -1 + \frac{1}{\gamma} \right) \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{1-\eta}} \\
+ \left( 1 - \lambda + \frac{\lambda}{\gamma} \right)^{\frac{1}{1-\eta}} \frac{1-\eta}{2-\eta} \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right)^{\frac{1-\eta}{1-\eta}} \left( 1 - \frac{1}{\gamma} \right) < 0
\]

or

\[
(1 - \eta) \left( 1 - \lambda + \frac{\lambda}{\gamma} \right) < \left( 1 - \eta + \lambda - \frac{\lambda}{\gamma} \right),
\]

which is true. \( Q.E.D. \)

10 References


Manove, M. 2000. Entrepreneurs, Optimism and the Competitive Edge, Boston University and CEMFI.


