

REM WORKING PAPER SERIES

**A DSGE Model to Evaluate the Macroeconomic Impacts of
Taxation**

José Alves

REM Working Paper 062-2018

November 2018

REM – Research in Economics and Mathematics

Rua Miguel Lúpi 20,
1249-078 Lisboa,
Portugal

ISSN 2184-108X

Any opinions expressed are those of the authors and not those of REM. Short, up to two paragraphs can be cited provided that full credit is given to the authors.



A DSGE Model to Evaluate the Macroeconomic Impacts of Taxation*

José Alves[†]

2018

Abstract

As recognized, taxation is not only an instrument for government to collect revenues from the economic agents but also an instrument of fiscal policy to influence the agents' behaviour. In this work, we develop a DSGE model to assess the macroeconomic impact of three tax items (taxes on individual income, on firms' income and on consumption) on the dynamics of both individual tax items and on the aggregate revenues as well. Moreover, we also intend to evaluate how macroeconomic aggregates behave in a presence of stochastic shocks in taxation.

Keywords: DSGE models; Tax effects; Fiscal Policy; Optimal taxation

JEL: D58; E62; H21; H30

*The opinions expressed herein are those of the author and do not necessarily reflect those of his employers. Any remaining errors are the author's sole responsibility.

[†]ISEG/UL - Universidade de Lisboa, Department of Economics; REM - Research in Economics and Mathematics, UECE - Research Unit on Complexity and Economics. UECE is supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal) Corresponding author: jose.borges.alves@phd.iseg.ulisboa.pt.

1 Introduction

As reflected in Polanyi (2001), the development of both modern economic markets and state cannot be understood separately. In addition to the capitalist strengthening process and the increasing of modern liberal state and its regulatory power on the modern economic markets activities, mainly in the XIX and XX centuries, there has been a certain consensus in the tax social contract. In fact, this social contract has been crucial for the state to support some transaction costs related with information asymmetries present in economic relations and in the promotion of these relations developments. However, the consensus is only apparent, since the existing tensions between the relevance of public services and the magnitude of expropriation on the economy through taxation have been intensely debated throughout society, and since there are different perceptions about the regulatory power of public institutions on economic markets across different social groups.

Accordingly, several economists have analysing the effects of taxation on the economic agents' behaviour in different perspectives, such as Atkinson and Stiglitz (1976), Brennan and Buchanan (1977), Brennan and Buchanan (2000) and Becker and Mulligan (2003).

In order to diminish the referred tension in the modern capitalist society, it is crucial to evaluate the impact of taxation on economic aggregates. This macro analysis can be useful to provide tax tools to guarantee an economic welfare along with the intensification of economic markets, while there is, at the same time, a public appropriation of both private income and wealth. Therefore, the application of the New Keynesian Stochastic Dynamic Models (DSGE) can shed light on the taxation effects on those aggregates. Yet, and despite of the criticisms due to an exacerbated adherence of fiscal and monetary authorities to this type of models relying on simplistic assumptions, there have been an effort to develop this kind of methodology by incorporating more realistic features (e.g., financial friction and agents' heterogeneous preferences). Moreover, using this methodology in a New Keynesian framework can give important insights economic relations across business cycles. In accordance to this, in this article we develop a New Keynesian DSGE model to assess the many effects of taxation in macroeconomic variables, namely in economic growth, capital stock accumulation and consumption and wages dynamics, among others. Additionally, we calibrate our model for the Portuguese economy and we simulate several macroeconomic shocks to retrieve conclusion on the macro taxation effects.

This article is organized as follows: section 2 reviews literature on some topics regarding the macroeconomic impact of taxation; section 3 describes our model. Section 4 details the parameters values employed to calibrate the model and, therefore, we reflect on the results obtained in our simulations. Lastly, in section 5 we summarize our conclusions.

2 The Literature Review

The existing literature on taxation is quite vast. There are literature regarding the effects of income taxation under homogeneous or heterogeneous preferences hypothesis among individuals. The assumption of homogeneous preferences is justified by a greater

simplicity in assessing fiscal impact derived from tax incidence policies. The studies as those conducted in Mirrlees (1971) and Kanbur and Tuomala (1994) assess the impact of taxation under homogeneous preferences features, and conclude that skills distribution and income-leisure preferences influence in not only tax design as well as inequality dynamics. However, it is recognized that individual preferences are not homogeneous at all. As referred by Kaplow (2008), individuals present many differences in utility values on the available goods and services that can be purchased. In addition, heterogeneous preferences arise from individuals' physical and psychological features, which are constrained by environment they face. In addition, Kaplow (2008), by considering heterogeneous preferences hypothesis, concludes that when preferences are observable, the income taxation optimality should be higher with higher utility values. On contrary, with unobservable preferences, the optimal income tax rates are equal as for homogeneous preferences case. In addition, Cremer et al. (2001) and Saez (2002) show that commodity taxes can play an important role when there is a presumption of a smooth optimal income tax incidence schedule. However, this result is contradicted in Golosov et al. (2013), since the authors find that, in a context where commodity taxes are assumed to be nonlinear functions of consumption and income, capital taxes present small welfare gains. Moreover, Boadway et al. (2002), by analysing an optimal tax scheme under a heterogeneous and quasilinear individual preferences for leisure reach to an important result: with a bigger proportion of hard-working people in the utilitarian welfare function, there are some income tax intervals favouring regressive redistribution. In addition, the important studies of Tarkiainen and Tuomala (2007), Blomquist and Christiansen (2008), Lockwood and Weinzierl (2015) analyse how heterogeneous preferences impact on tax design.

Besides the studies that analyse the microeconomic relations between preferences and taxation structure design, several other researches are resorting to DSGE models, which studies the overall behaviour of an economy, to assess several economic relations in an economy, by incorporating, at the same time, tax components. Some examples of this is the PESSOA model developed in Almeida et al. (2013) for the Portuguese economy, the RAMSES Swedish model created by Adolfson et al. (2008), AINO Finish economic model presented in and Kilponen and Ripatti (2006) and the FiMod model (Stähler and Thomas (2012)) jointly developed by the German and Spanish Central Banks and which has been used by the European System of Central Banks. Beyond the DSGE models, there is also important literature on specific topics of taxation and its implication on economic relations, such as the effects of taxation on income, inequalities and labour supply, on investment, on budget and government trajectory balances phenomena, and on other economic relations.

In what respects to the effects of taxation on income, wealth and inequalities, in Krusell and Smith (1998) seminar paper it is evidenced how heterogeneity in discounting behaviour influence the explanation of wealth inequality in a macroeconomic perspective. In Nishiyama and Smetters (2005), by assessing a replacement of progressive taxation through flat consumption taxes in an overlapping-generations model and where agents are considered to be heterogeneous in both working ability and individual wealth initial levels, the authors conclude that the output and wealth levels grow in the presence of an uninsurable wage shock. This result is corroborated by Coleman (2000) and Correia

(2010) which found a positive connection between consumption taxes and welfare dynamics. Regarding inequalities, García-Peñalosa and Turnovsky (2011) evidence changes in taxes seriously affect wealth and income distribution, highlighting the impact of tax redistribution. Moreover, in a previous study conducted in García-Peñalosa and Turnovsky (2007) which considers the initial condition of capital distribution, fiscal policy influences pre-tax income and welfare inequalities through the labour supply channel. In addition, it is found that labour and capital income taxes influence after-tax income parceling from a direct redistributive effect labour source of taxation and from an indirect impact through the capital income tax effect on labour supply. On the other hand, Heer and Trede (2003) assess the impact of a two-revenue neutral income tax reform for Germany. More specifically, the authors investigate the quantitative effects of a consumption tax implementation and a flat-rate income tax reforms in a general equilibrium model framework resorting to 1996 data. The results highlight both gains in efficiency and welfare through the application of these tax reforms when compared to the German taxation system. Although, it is important to stress that those results do not take into account any important features as the endogenization of labour productivity, and this paper do not assess possible welfare losses during the tax regime changes. Coleman (2000) also found a positive connection between consumption taxation and welfare. In fact, the author reports that welfare increases can be reached through a complete income tax replacement by a unique consumption tax. Finally, the studies of Bouza and Turnovsky (2012) and Lim and McNelis (2014) investigate the relation between inequalities and economic openness, incorporating the effects of taxation.

Since investment decisions are imperative to sustain the current and the future consumption of an economy, i.e., investment guarantees both current and long-term levels of prosperity, the understanding of how taxation decisively affects investment options is imperative for a suitability economic model that intends to forecast the dynamics of several economic episodes. Regarding the impact of taxation on investment, a good survey on this topic can be found in Hanlon and Heitzman (2010), where several tax roles on firms' decisions are analysed. Despite this, one main problem mentioned in the literature regarding taxation and investment dynamics is the uncertainty as reported in Hassett and Metcalf (1999). Therefore, a research conducted in Edmiston (2004) concludes for a negative connection between firms taxation volatility and investment growth, for 15 European Union, Japan and United States and for a 28 years timespan. In accordance to the previous results, El-Shazly (2009) also found that tax policy transparency and corporate tax reductions lead to capital accumulation. Lastly, a research conducted in Agliardi (2001), based in a real option approach, is also a valuable literature to assess linkages between tax structure and firms' decisions to invest. In what respects to aggregate investment, Salgado (2011) develops a DSGE model with heterogeneous firms to assess the cyclicity of investment rates distribution across firms with growth dynamics, and finds that the higher corporate taxes are, the lesser is the concentration between investment rates across firms' degree and the cyclical component of GDP is. Lastly, while Alesina et al. (2002) quantitatively evidence how several taxes impact firms' profits, in Afonso and Jalles (2015) it is found that taxation has not implication on both private and public investment decisions.

To conclude, and in what concerns to fiscal policy and its impact on economic performance through the taxation channel, we highlight the studies conducted in Romer and Romer (2010) and in Jaimovich and Rebelo (2017). It is found in Fernández-Villaverde (2010) that, in a context of financial friction, a reduction in taxes present lower impact than increases in public spending to promote growth. Moreover, in Mertens and Ravn (2011), which resort to a DSGE model to evaluate the impact of macroeconomic variables to tax shocks, unanticipated and anticipated ones, conclude that anticipated tax cuts have a detrimental impact on GDP until those cuts are implemented *de facto*. Under a New Keynesian DSGE model, Cloyne (2014) assesses the public expenditures of Keynesian multiplier, taking into account how government expenditures are financed and how consumers behave when they expect future tax increases. Consequently, this research found that household income taxation is detrimental for growth, consumption and salaries, while government expenditures financed by debt have positive effects on those variables. Finally, Afonso and Rault (2009) show that while countries as Spain and Portugal follow a spend-and-tax pattern, countries as Germany and United Kingdom are characterized by a tax-and-spend framework.

3 The Model

3.1 Households

In order to study the impact of taxation in the several economic aggregates, our model closely follows the methodology employed in Junior (2016). Therefore, our economy is closed and populated by two types of infinitely lived working households $\mathcal{H} \in \{\mathcal{A}, \mathcal{B}\}$ where \mathcal{A} -type represents Ricardian households and \mathcal{B} are non-Ricardian households, with no population growth, and $\omega_{\mathcal{A}}$ and $\omega_{\mathcal{B}}$ represent the proportion of Ricardian and non-Ricardian households, respectively.

On the other hand, and while \mathcal{B} -households do not have access to financial markets, \mathcal{A} -type households can buy bonds, accumulate physical capital, they rent services to firms. Moreover, we also assume there is internal habit formation in consumption and both households have homogeneous preferences.

Each household \mathcal{H} derives utility from consumption, $C_{\mathcal{H},t}$, and leisure (with labour $L_{\mathcal{H},t}$, in hours) as:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[\frac{(C_{\mathcal{H},t+k} - \phi_c C_{\mathcal{H},t+k-1})^{1-\sigma}}{1-\sigma} - \frac{L_{\mathcal{H},t+k}^{1+\varphi}}{1+\varphi} \right], \quad (1)$$

where \mathbb{E}_t is the expectations operator, β is the discount factor, σ represents the relative risk aversion, ϕ_c is the parameter associated with consumption habits' persistence, and φ is the marginal desutility of labour supply. In the following sections we detail the behaviour of each type of households.

3.1.1 Ricardian households

Each \mathcal{A} -type household maximizes its lifetime utility by choosing between consumption and leisure. Therefore, Ricardian households solve the following problem:

$$\max_{C_{\mathcal{A},t}, K_{t+1}^P, B_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \left[\frac{(C_{\mathcal{H},t} - \phi_c C_{\mathcal{H},t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{\mathcal{H},t}^{1+\varphi}}{1+\varphi} \right] \quad (2)$$

subject to the following budget constraint

$$P_t \cdot (1 + \tau_t^c) \cdot C_{\mathcal{A},t} + \frac{B_{t+1}}{R_t^B} = (1 - \tau_t^l) W_t \cdot L_{\mathcal{A},t} + (1 - \tau_t^k) R_t K_t^P + B_t + \omega_{\mathcal{A}} \cdot P_t \cdot TRANS_t, \quad (3)$$

and with the following law of motion of capital

$$K_{\mathcal{A},t+1}^P = (1 - \delta) K_t^P + I_t^P, \quad (4)$$

where P_t represents the price level, τ_t^c is the tax rate on goods consumed by households, $\frac{B_{t+1}}{R_t^B}$ is the actual value of bonds purchased by Ricardian households, τ_t^l and W_t are the tax on labour income and the wage rate, respectively, τ_t^k is the income tax levied from capital owned by this type of households, K_t^P , B_t represents the bonds issued by the government at period t , $TRANS_t$ reflects the lump-sum transfers to households (please note that the share of transfers is proportional to the size of each type of households) and, lastly, δ is the depreciation rate associate to the private capital.

Therefore, the Lagrangian applied to the mentioned problem is

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{(C_{\mathcal{A},t} - \phi_c C_{\mathcal{A},t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{\mathcal{A},t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{\mathcal{A},t} \left[P_t \cdot (1 + \tau_t^c) \cdot C_{\mathcal{A},t} + \frac{B_{t+1}}{R_t^B} \right. \right. \\ \left. \left. - (1 - \tau_t^l) W_t \cdot L_{\mathcal{A},t} - (1 - \tau_t^k) R_t K_t^P - B_t - \omega_{\mathcal{A}} \cdot P_t \cdot TRANS_t \right] \right. \\ \left. - Q_t [K_{t+1}^P - (1 - \delta) K_t^P - I_t^P] \right\} \quad (5)$$

Deriving the respective first-order conditions, we obtain:

$$\frac{\partial \mathcal{L}}{\partial C_{\mathcal{A},t}} = (C_{\mathcal{A},t} - \phi_c C_{\mathcal{A},t-1})^{-\sigma} - \lambda_{\mathcal{A},t} P_t (1 + \tau_t^c) - \phi_c \beta (\mathbb{E}_t C_{\mathcal{A},t+1} - \phi_c C_{\mathcal{A},t})^{-\sigma} \\ = 0, \quad (6)$$

that can be rearranged into equation 7:

$$\lambda_{\mathcal{A},t} = \frac{(C_{\mathcal{A},t} - \phi_c C_{\mathcal{A},t-1})^{-\sigma}}{P_t (1 + \tau_t^c)} - \phi_c \beta \frac{(\mathbb{E}_t C_{\mathcal{A},t+1} - \phi_c C_{\mathcal{A},t})^{-\sigma}}{P_t (1 + \tau_t^c)} \quad (7)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial K_{t+1}^P} &= \beta \mathbb{E}_t \{ \lambda_{\mathcal{A},t+1} \cdot (1 - \tau_{t+1}^k) \cdot R_{t+1} - Q_t + \beta \cdot Q_{t+1} (1 - \delta) \} \\ &= 0,\end{aligned}\tag{8}$$

and which can be written as

$$Q_t = \beta \mathbb{E}_t \{ (1 - \delta) Q_{t+1} + \lambda_{t+1} (1 - \tau_{t+1}^k) R_{t+1} \}\tag{9}$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= -\frac{\lambda_t}{R_t^B} + \beta \mathbb{E}_t \lambda_{t+1} \\ &= 0.\end{aligned}\tag{10}$$

From the previous equation, we can state that

$$\frac{\lambda_t}{R_t^B} = \beta \mathbb{E}_t \lambda_{t+1}\tag{11}$$

3.1.2 Non-Ricardian households

In what respects to non-Ricardian households, and as previously mentioned, this type of agents have no access to financial markets. Therefore, they only supply labour and maximize consumption in the current period. Therefore, their budget constraint is given in equation (12):

$$(1 + \tau_t^c) P_t \cdot C_{\mathcal{B},t} = (1 - \tau_t^l) W_t \cdot L_{\mathcal{B},t} + (1 + \omega_{\mathcal{B}}) P_t \cdot TRANS_t.\tag{12}$$

Therefore, the lagrangian for non-Ricardian households is given by:

$$\begin{aligned}\mathcal{L} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left\{ \left[\frac{(C_{\mathcal{B},t} - \phi_c C_{\mathcal{B},t-1})^{1-\sigma}}{1-\sigma} - \frac{L_{\mathcal{B},t}^{1+\varphi}}{1+\varphi} \right] \right. \\ \left. - \lambda_{\mathcal{B},t} [(1 + \tau_t^c) P_t \cdot C_{\mathcal{B},t} - (1 - \tau_t^l) W_t \cdot L_{\mathcal{B},t} - (1 + \omega_{\mathcal{B}}) P_t \cdot TRANS_t] \right\}\end{aligned}\tag{13}$$

Deriving the first-order conditions for these households we obtain

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_{\mathcal{B},t}} &= (C_{\mathcal{B},t} - \phi_c C_{\mathcal{B},t-1})^{1-\sigma} - \lambda_{\mathcal{B},t} (1 + \tau_t^c) P_t - \phi_c \beta (\mathbb{E}_t C_{\mathcal{B},t+1} - \phi_c C_{\mathcal{B},t})^{-\sigma} \\ &= 0\end{aligned}\tag{14}$$

which can be rewritten as

$$\lambda_{\mathcal{B},t} = \frac{(C_{\mathcal{B},t} - \phi_c C_{\mathcal{B},t-1})^{-\sigma}}{P_t (1 + \tau_t^c)} - \phi_c \beta \frac{(\mathbb{E}_t C_{\mathcal{B},t+1} - \phi_c C_{\mathcal{B},t})^{-\sigma}}{P_t (1 + \tau_t^c)}.\tag{15}$$

3.2 Wage's dynamics

In our model we do not admit differences between the salaries among households, i.e, both Ricardian and non-Ricardian households earn the same wage rates. Households supply differentiated labour in a monopolistic structure market competition, and they *sell* their labour to a representative firm. Therefore, the differentiated types of labour supply are aggregated into one labour type input, L , that is

$$L_t = \left(\int_0^1 L_{j,t}^{\frac{\psi_W-1}{\psi_W}} dj \right)^{\frac{\psi_W}{\psi_W-1}}, \quad (16)$$

being ψ_W the elasticity of substitution among the different jobs, $L_{j,t}$ is the amount of supplied labour of the mentioned different jobs supplied by household j , and, consequently, each work that supplies labour j -type earns the wage $W_{j,t}$.

Moreover, and given that the aggregating labour firms' problem is to maximizes their profits, the following expression translates this problem:

$$\max_{L_{j,t}} W_t L_t - \int_0^1 W_{j,t} L_{j,t} dj, \quad (17)$$

which can be rewritten by replacing the L_t 16 as:

$$\max_{L_{j,t}} W_t \left(\int_0^1 L_{j,t}^{\frac{\psi_W-1}{\psi_W}} dj \right)^{\frac{\psi_W}{\psi_W-1}} - W_{j,t} \int_0^1 L_{j,t} dj. \quad (18)$$

Consequently we can derive the F.O.C. for the above problem:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_{j,t}} &= 0 \\ \Leftrightarrow W_t \left(\int_0^1 L_{j,t}^{\frac{\psi_W-1}{\psi_W}} dj \right)^{\frac{1}{\psi_W-1}} L_{j,t}^{-\frac{1}{\psi_W}} - W_{j,t} &= 0. \end{aligned} \quad (19)$$

Since equation 16 can be stated as

$$L_t^{\frac{1}{\psi_W}} = \left(\int_0^1 L_{j,t}^{\frac{\psi_W-1}{\psi_W}} dj \right)^{\frac{1}{\psi_W-1}}, \quad (20)$$

substituting the obtained F.O.C in equation 19, we reach to the equation for differentiated labour j 's demand:

$$L_{j,t} = L_t \left(\frac{W_t}{W_{j,t}} \right)^{\psi_W} \quad (21)$$

By substituting equation 21 in equation 16, and after some algebraic manipulation, we get to the aggregated wage level equation:

$$W_t = \left(\int_0^1 W_{j,t}^{1-\psi_W} dj \right)^{\frac{1}{1-\psi_W}}. \quad (22)$$

In addition, we assume the existence of wage stickiness, i.e., in every period there is a fraction of households, $1 - \theta_W$, that optimizes their wages, by choosing $W_{j,t}^*$. On the other hand, the other portion of workers, θ_W , are characterized by following a wage stickiness rule, which maintains the same wage rate of the present period equal to the previous one ($W_{j,t} = W_{j,t-1}$). Furthermore, by trying to optimize the wage rate, households face a probability of θ_W^N which translates into wages remaining equal for N periods. Therefore, the wage definition problem for \mathcal{H} household, and considering the taxes on labour income, is given by:

$$\begin{aligned} \max_{W_{j,t}^*} \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \cdot \theta_W)^i & \left\{ -\frac{1}{1+\varphi} \left[L_{\mathcal{H},t+i} \left(\frac{W_{t+i}}{W_{j,t}^*} \right)^{\psi_W} \right]^{1+\varphi} \right. \\ & \left. + \lambda_{\mathcal{H},t+i} \left[W_{j,t}^* L_{\mathcal{H},t+i} \left(\frac{W_{t+i}}{W_{j,t}^*} \right)^{\psi_W} (1 - \tau_{t+i}^l) \right] \right\}. \end{aligned} \quad (23)$$

The first-order condition for this problem is given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial W_{j,t}^*} &= 0 \\ \Leftrightarrow 0 & \\ &= \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta_W)^i \left\{ \psi_W \cdot L_{\mathcal{H},j,t+i}^{\varphi} \frac{1}{W_{j,t}^*} + (1 - \psi_W) \lambda_{\mathcal{H},t+i} (1 - \tau_{t+i}^l) \right\}, \end{aligned} \quad (24)$$

that can be rewritten as:

$$W_{j,t}^* = \frac{\psi_W}{\psi_W - 1} \mathbb{E}_t \sum_{i=0}^{\infty} (\beta \theta_W)^i \left[\frac{L_{\mathcal{H},j,t+i}^{\varphi}}{\lambda_{\mathcal{H},t+i}} (1 - \tau_{t+i}^l) \right]. \quad (25)$$

Consequently, and taken into account the wage stickiness hypothesis, the wage rate level at period t is given as:

$$W_t = \left[\theta_W W_{t-1}^{1-\psi_W} + (1 - \theta_W) W_t^{*1-\psi_W} \right]^{\frac{1}{1-\psi_W}}. \quad (26)$$

3.3 Firms

In the economy, there are two types of firms. On the one hand, there are firms that sell intermediate goods. On the other hand, a representative retail firm sells the final goods purchased from the first group of firms. While the retail firm, a representative of several firms, acts in a perfect market competition framework, monopolistic competition

features characterize the market structure of intermediate firms. Moreover, given the firms' characteristics and the purpose of our model, we then detail the dynamics within intermediate firms.

For the intermediate firms we admit the non-existence of fixed costs, implying that the average total costs are equal to the variable ones. Moreover, in the production function of these firms, we include as inputs not only labour and private capital, but also public capital. Therefore, the production function is given by:

$$Y_{j,t} = A_t K_{j,t}^{P,\alpha_1} L_{j,t}^{\alpha_2} K_{j,t}^{G,\alpha_3}, \quad (27)$$

where α_1 , α_2 and α_3 represent the elasticity of private capital ($K_{j,t}^P$), of labour ($L_{j,t}$) and of public capital ($K_{j,t}^G$), respectively, regarding the production process, and we admit the constant returns to scale hypothesis, $\alpha_1 + \alpha_2 + \alpha_3 = 1$. Moreover, productivity A_t follows an AR(1) process as

$$\log A_t = (1 - \rho_A) \log A_{ss} + \rho_A \log A_{t-1} + \varepsilon_t, \quad (28)$$

being A_{ss} the steady state level of productivity, $|\rho_A| < 1$ the autoregressive parameter, and ε_t the error term following a normal distribution with standard-deviation σ_A , i.e., $\varepsilon_t \sim N(0, \sigma_A^2)$. In addition, these firms solve their problem by minimizing the production costs subject to the production function expressed in equation 27. Consequently, we can state the Lagrangian for this problem as:

$$\mathcal{L} = W_t L_{j,t} + R_t K_{j,t} + \mu_{j,t} \left(Y_{j,t} - A_t K_{j,t}^{P,\alpha_1} L_{j,t}^{\alpha_2} K_{j,t}^{G,\alpha_3} \right) \quad (29)$$

From the Lagrangian equation, and given that $\mu_{j,t} = MC_{j,t}$, $MC_{j,t}$ the marginal cost, equivalent to $MC_{j,t} = \frac{1}{A_t K_{j,t}^{G,\alpha_3}} \left(\frac{W_t}{\alpha_2} \right)^{\alpha_2} \left(\frac{R_t}{\alpha_1} \right)^{\alpha_1}$, we can derive the first order conditions for these firms' problems and, consequently, find the firm's demand for each input:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial L_{j,t}} &= 0 \\ &\Leftrightarrow \frac{\alpha_2}{A_t K_{j,t}^G} \left(\frac{W_t}{\alpha_2} \right)^{\alpha_2} \left(\frac{R_t}{\alpha_1} \right)^{\alpha_1} \frac{Y_{j,t}}{W_t} \end{aligned} \quad (30)$$

and,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_{j,t}^P} &= 0 \\ &\Leftrightarrow \frac{\alpha_1}{A_t K_{j,t}^G} \left(\frac{W_t}{\alpha_2} \right)^{\alpha_2} \left(\frac{R_t}{\alpha_1} \right)^{\alpha_1} \frac{Y_{j,t}}{R_t} \end{aligned} \quad (31)$$

In what respects to the price associated with the sell of these intermediate goods, and likewise the wage stickiness, the intermediate firms also face sticky prices. Therefore, and similar to what happens with households' wages, we can state the aggregate price level as:

$$P_t = \left[\theta P_{t-1}^{1-\psi} + (1-\theta) P_t^{*,1-\psi} \right]^{\frac{1}{1-\psi}}, \quad (32)$$

where θ is the probability associated with the firm maintaining the same price, P_t , of the present period in the next one, ψ is the elasticity of substitution degree among the intermediate firms' goods and, lastly, P_t^* represents the optimal price seek by firms in every periods.

3.4 Fiscal and Monetary Policies

The government is responsible for collecting taxes, T_t , using these revenues to finance not only the acquisition of goods and services from the economy, G_t , but also to invest, I_t^G , and transfer a share of revenues to households, $TRANS_t$, as mentioned before. In addition, and since tax revenues could be insufficient to finance all types of public spending, the government can also issue debt, B_t . Furthermore, the public capital dynamics is given by:

$$K_{t+1}^G = (1 - \delta_G) K_t^G + I_t^G, \quad (33)$$

with δ_G being the depreciation rate associated to public capital. Therefore, the budget constraint for the fiscal policy is given by:

$$\frac{B_{t+1}}{R_t^B} - B_t + T_t = P_t \cdot G_t + P_t \cdot I_t^G + P_t \cdot TRANS_t, \quad (34)$$

where $\frac{B_{t+1}}{R_t^B} - B_t$ represents the present value of government debt variation in period t , and T_t is the total nominal tax revenues as:

$$T_t = \tau_t^c \cdot C_t + \tau_t^l \cdot W_t L_t + \tau_t^k (R_t - \delta) K_t^P, \quad (35)$$

being τ_t^c , τ_t^l and τ_t^k the tax rates levied on consumption, on labour and on net capital's income, respectively. These tax rates types and government spending variables follow the subsequent policy rule:

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Z_{t-1}}{Z_{ss}} \right)^{\gamma_Z} \left(\frac{B_t}{Y_{t-1} P_{t-1}} \right)^{(1-\gamma_Z)\phi_Z} S_t^Z, \quad (36)$$

where $Z_t = \{\tau_t^c, \tau_t^l, \tau_t^k, G_t, I_t^G, TRANS_t\}$, γ_Z is the persistence associate to each variable included in variables set Z , and S_t^Z is the shock associated to each fiscal policy variable, following an AR(1) process, as detailed below:

$$\log S_t^Z = (1 - \rho_Z) \log S_{ss}^Z + \rho_Z \log S_{t-1}^Z + \varepsilon_t^Z. \quad (37)$$

In what respects to the monetary policy setup, the central bank is only concerned with price stability and economic growth trajectories. In fact, there are several interactions between fiscal and monetary policies with important impacts on other economic aggregates, as demonstrated in Corsetti and Pesenti (2005), Kirsanova et al. (2016) Afonso and Balhote

(2014), among others. Since our focus is only in the taxation analysis we set the following Taylor rule:

$$\frac{R_t^B}{R_{ss}^B} = \left(\frac{R_{t-1}^B}{R_{ss}^B} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi_{ss}} \right)^{\gamma_\pi} \left(\frac{Y_t}{Y_{ss}} \right)^{\gamma_Y} \right]^{(1-\gamma_R)} S_t^m, \quad (38)$$

where γ_π and γ_Y are the sensitiveness degree of interest rate to inflation and to the aggregate product, respectively, γ_R is a smoothing parameter and, lastly, S_t^m represents the monetary shock, which also follows an AR(1) process as expressed below:

$$\log S_t^m = (1 - \rho_m) \log S_{ss}^m + \rho_m \log S_{t-1}^m + \varepsilon_{t,m} \quad (39)$$

To sum up, and taking into consideration the model described before, the equilibrium conditions of our economy are:

- **Equilibrium condition**

$$Y_t = C_t + I_t^P + I_t^G + G_t \quad (40)$$

- **Aggregate consumption**

$$C_t = \omega_A C_{A,t} + \omega_B C_{B,t} \quad (41)$$

- **Aggregate labour**

$$L_t = \omega_A L_{A,t} + \omega_B L_{B,t} \quad (42)$$

4 Calibration and empirical analysis

In order to simulate our model it is necessary to calibrate the parameters associated to it. Consequently, and taking into account not only the calibrated values used in the literature, we present in table 1 the values used to calibrate our variables. Moreover, and trying to analyse the Portuguese economy, the basis to calibrate our model was the PESSOA model, presented in Almeida et al. (2013). In addition, we also assume that each ε_t follows a normal distribution, i.e, $\varepsilon_t \sim N(0, 1)$. Therefore, and as described in the following paragraphs, we were able to simulate our model and to analyse the empirical results obtained.

First, in section 4.1, we analyse the impact of each tax shock not only on the aggregate tax revenues, but also in the total revenues of each tax item. Subsequently, in section 4.2 we analyse the taxation impact over the main economic aggregates detailed in our model.

Table 1: Parameter values used for the model's calibration.

Parameter	Value	Parameter	Value
-----------	-------	-----------	-------

β	0.996	θ_W	0.800
σ	2.000	ψ_W	21.000
φ	1.500	τ^c	0.129
ϕ_c	0.700	τ^l	0.430
α_1	0.300	τ^k	0.298
α_2	0.600	ω_R	0.730
α_3	0.100	γ_Y	0.160
δ_{IP}	0.025	γ_π	2.430
θ	0.800	δ_{IG}	0.025
ψ	8.000	γ_R	0.790

4.1 The impact of tax shocks in taxation

In what respects to the impact of tax items' shocks, not only in the overall tax revenues but also in each tax revenues component (revenues from consumption, labour and capital), we can state, from the obtained results present in figure 1, that a shock on taxes over consumption and capital presents similar and immediate negative effects in the overall tax revenues, although with a different magnitude. In detail, a shock in tax consumption seems to reduce 4% in the overall taxation in the period that occurs the shock. This negative effect loses its magnitude by half after 10 periods, and it has no impact on taxation only 20 periods after the shock's occurrence. This also happens with taxation on capital shocks. However, the negative impact of capital tax shocks is much less detrimental - almost residual - for the overall revenues, than what it can be observed for the overall taxes regarding a shock on consumption taxes. On the contrary, and despite of the initial negative impact of labour tax rates' relaxation on aggregate tax revenues, the overall effect of a shock regarding this tax item evidence a positive impact, which increase the aggregate tax revenues in the long-run.

When we analyse the individual impact of tax rates shocks in the respective tax items revenues, we can conclude for the overall negative impact for all the tax items. In particular, the labour tax revenues is the tax item most affected by an increase of its tax rate, followed by tax revenues from capital and, lastly, from tax on consumption.

4.2 The behaviour of economic aggregates to tax shocks

In this section, we provide a reflection on the results obtained for the impact of tax rates shocks on the economic aggregates detailed in section 3.

After running our model, and through figure 2, we can verify that if fiscal authorities decide to increase tax rates on consumption, it will lead to a positive impact on the aggregate economic activity, on private investment, and on aggregate consumption, and it will lead to a higher positive impact for the non-Ricardian households' consumption.

Additionally, a consumption tax rate increase evidence to pressure up the government

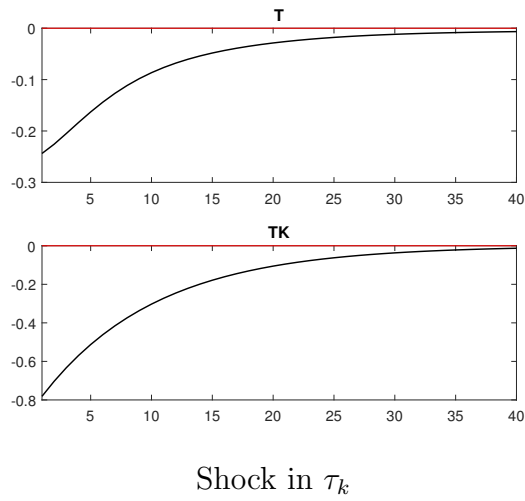
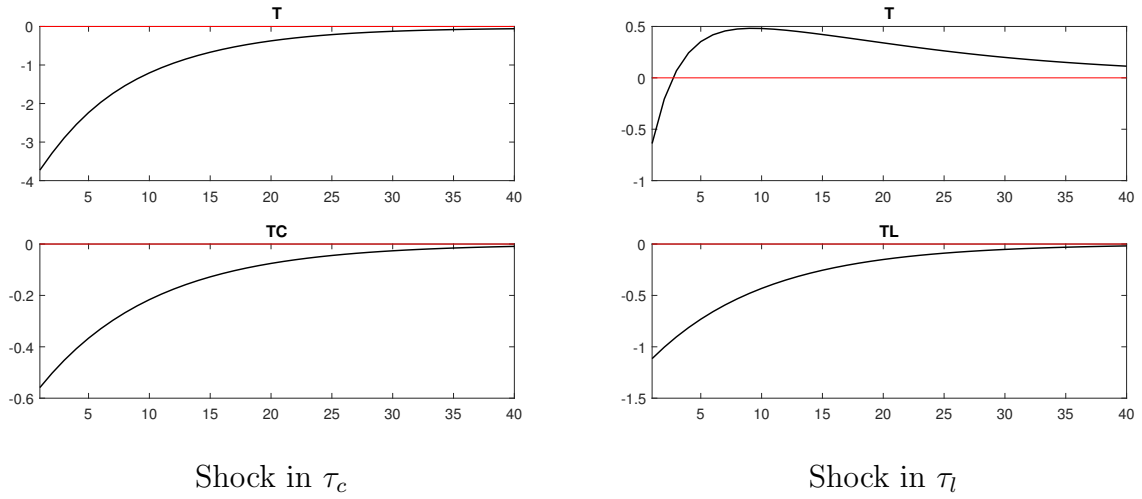
debt, while also stimulates the government transfers to households. On the other hand, an increase in tax rates has a temporary and reduce impact not only in inflation rate but also in the wage rate growth.

Moreover, a consumption tax relaxation seems to positively affect on labour supply and, on the opposite side, to reduce non-Ricardian household's labour supply. In fact, this pattern is found for shocks in all tax items (see figures 3 and 4). In addition, we can state that the dynamics found through a shock in labour tax rate on the labour supply for both Ricardian and non-Ricardian households translate into a substitution and an income effects, respectively.

In what concerns to the impact of labour tax rates increases, we also assist to a positive impact in the economic aggregate. However, this positive impact appears to last longer than what we observe for the consumption tax rates' increases. In perspective, and comparing to the results highlighted in figures 2 to 4, an upward release of labour tax rates evidences to be the most positive one for the economy. In contrast to the consumption tax rates, private investment seems to react negatively to labour taxes. On the other hand, and although we find a similar positive impact of an increase in tax rate from labour on aggregate consumption - in this case the positive magnitude of labour taxes is higher than the other two tax items - it seems that Ricardian households' consumption evidences a reduction, not only in the short but also in the long-run, contrarily to the other tax items' impacts. Furthermore, while gross inflation rate tends to slightly increase in the first periods, there is a negative effect on that variable. This dynamics is identical for the wage growth rate, although the effect of this tax item on wages quickly fades. Lastly, government debt seems to be pressured up as the previous tax item's case, although with a lesser negative magnitude.

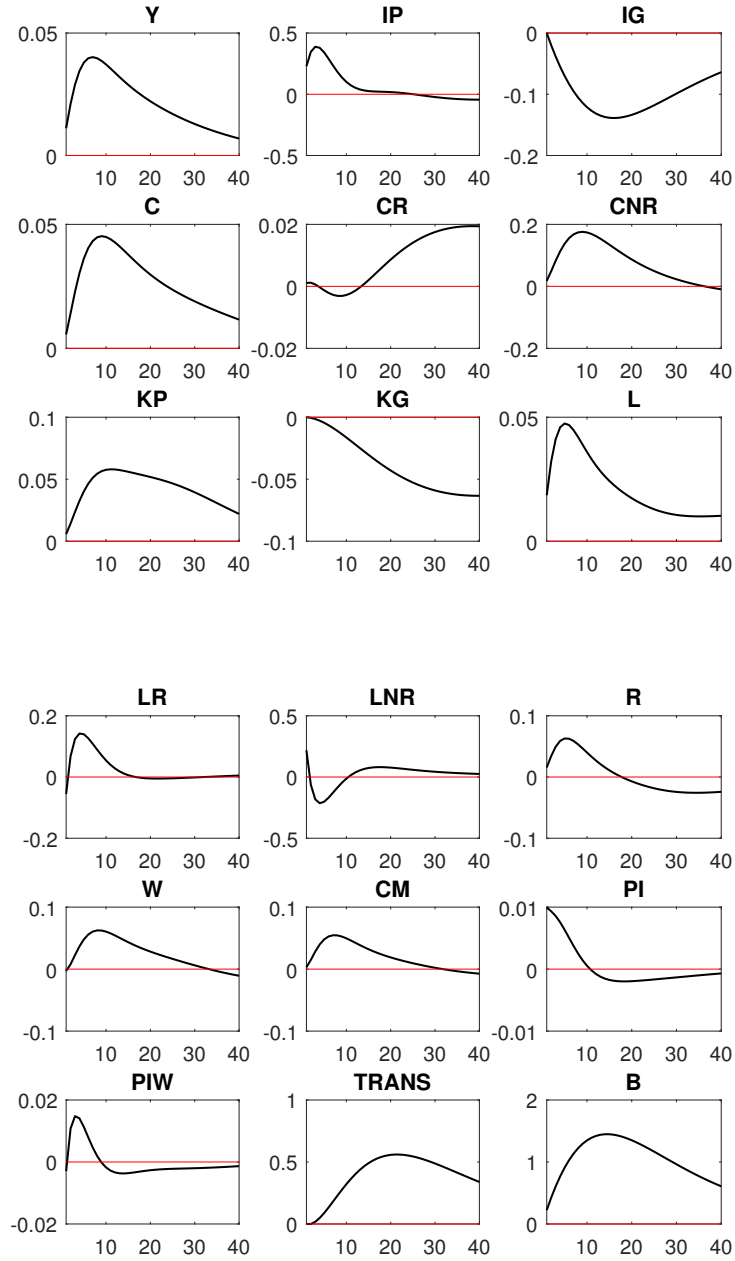
By analysing the effects of a shock in private capital taxation (4), we can conclude that it has little impact on aggregate economic activity, consumption and labour supply. Moreover, it seems that a relaxation on this tax rate has a positive and contemporaneous impact on private investment. This can be explained by the necessity of private capital owners to invest more to face not only the capital stock depreciation but also this upward movement on tax rates with higher capital's profitability, through more private capital stock, as also demonstrated in figure 4. In addition, while a shock in this tax rates show a reduction in labour supply from Ricardian households, the non-Ricardian ones increase their labour supply. Furthermore, it is observable that if the fiscal policy intends to increase this tax item, there is an overall positive effect in wage rate and its growth, in public transfers to households, and an increase effect on government debt. Finally, the results show that inflation is less sensitive to shocks for this tax source than it is in the other previously discussed shocks.

Figure 1: The impact of tax items' shocks on aggregate tax revenues and tax items revenues, in percentage.



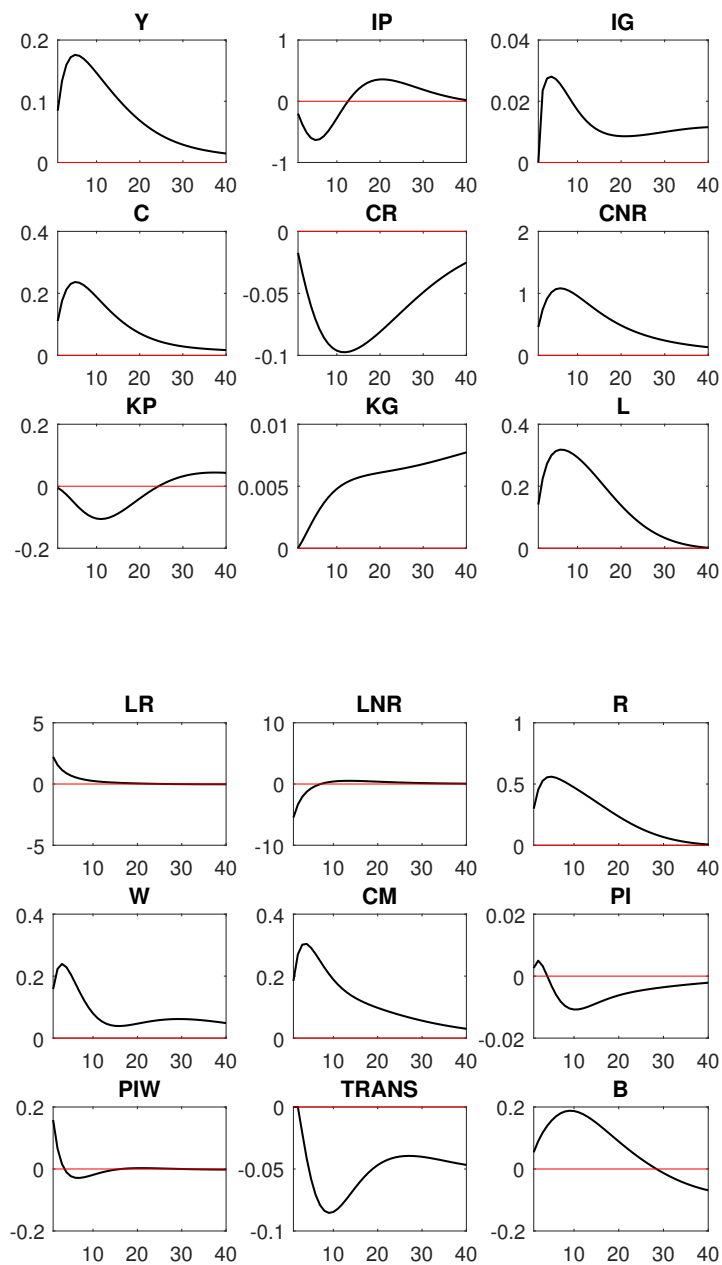
Notes: T represents the overall aggregate tax revenues, while TC , TL and TK represents the aggregate tax revenues from consumption, labour and capital, respectively.

Figure 2: Consumption tax shocks' impacts on economic aggregates, in percentage.



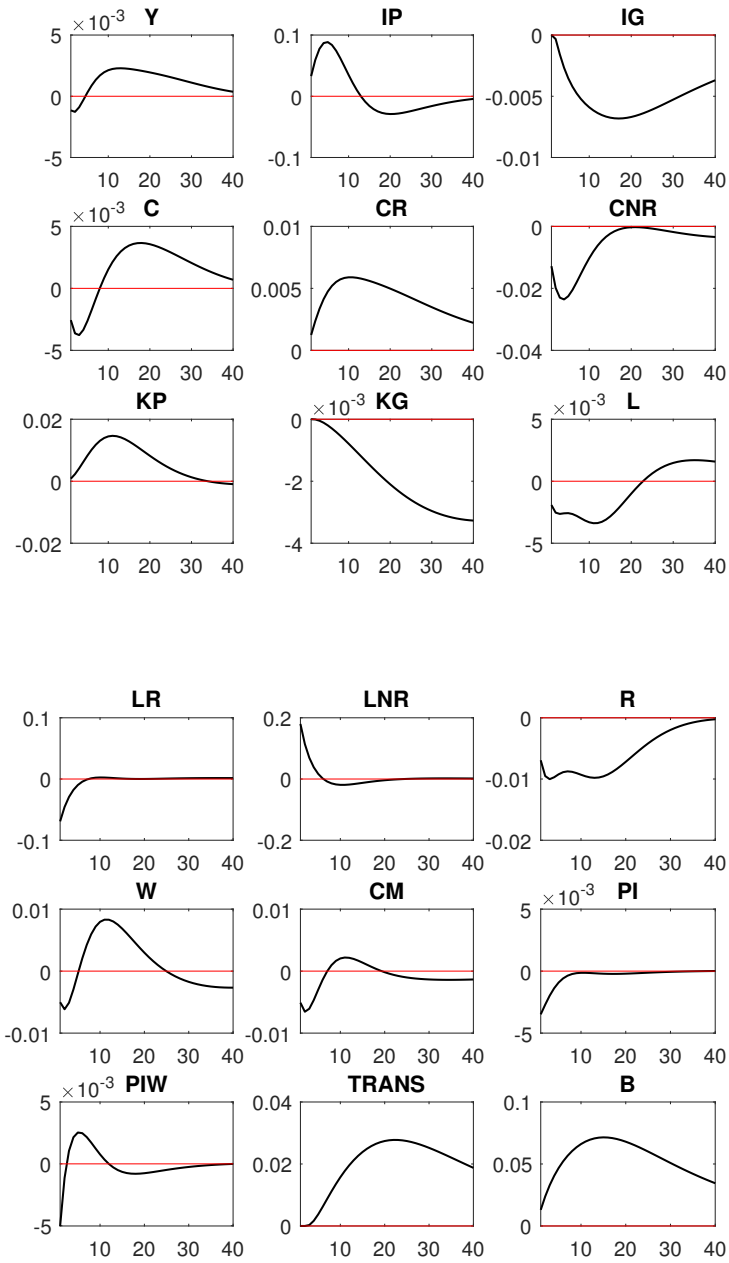
Notes: Y represents the aggregate economic output, IP and IG represent the private and public investment, respectively, C is the aggregate consumption, CR is the Ricardian households' consumption and CNR is the consumption from non-Ricardian agents. KP and KG are the capital stock of private and public sectors, respectively, L , LR and LNR are the aggregate labour supply and the labour supply of each type of households - Ricardian and non-Ricardian - respectively. Moreover, R is the interest rate, W translates the wage rate, CM is the marginal cost, and PI and PIW are the respective inflation and wage growth rates. Lastly, $TRANS$ is the government transfers to households and B is the government debt.

Figure 3: Labour tax shocks' impacts on economic aggregates, in percentage.



Notes: Y represents the aggregate economic output, IP and IG represent the private and public investment, respectively, C is the aggregate consumption, CR is the Ricardian households' consumption and CNR is the consumption from non-Ricardian agents. KP and KG are the capital stock of private and public sectors, respectively, L , LR and LNR are the aggregate labour supply and the labour supply of each type of households - Ricardian and non-Ricardian - respectively. Moreover, R is the interest rate, W translates the wage rate, CM is the marginal cost, and PI and PIW are the respective inflation and wage growth rates. Lastly, $TRANS$ is the government transfers to households and B is the government debt.

Figure 4: Private capital tax shocks' impacts on economic aggregates, in percentage.



Notes: Y represents the aggregate economic output, IP and IG represent the private and public investment, respectively, C is the aggregate consumption, CR is the Ricardian households' consumption and CNR is the consumption from non-Ricardian agents. KP and KG are the capital stock of private and public sectors, respectively, L , LR and LNR are the aggregate labour supply and the labour supply of each type of households - Ricardian and non-Ricardian - respectively. Moreover, R is the interest rate, W translates the wage rate, CM is the marginal cost, and PI and PIW are the respective inflation and wage growth rates. Lastly, $TRANS$ is the government transfers to households and B is the government debt.

5 Concluding Remarks

The subjects around the relevance and the adherence of DSGE models by public authorities, including both fiscal and monetary authorities, have been highly debated. These intense discussions arise from the absence of criticisms in the use of this methodology. However, and as many economists pointed out and despite all the possible criticisms that can arise, the use of DSGE models can provide important insights of a certain policy effects on the several economic aggregates, not only in the short but also in the long-run.

Therefore, and being aware of both positive and negative aspects that can emerge from DSGE models, we develop a simple DSGE model to analyse the impact of taxation on an economy, like the Portuguese economy, not only in an aggregate perspective - through the study of the overall revenues impact -, but also by analysing the individual impacts of each tax items that could be part of *our* tax system. Our model include some realistic features, such as Ricardian and non-Ricardian households, interactions between fiscal and monetary policies, among others. The assumptions and the characteristics employed in our model allow us to assess the impact of taxation.

Lastly, our results reflect that a shock in the overall taxation affects positively the economic output and consumption. Moreover, we conclude for different effects in labour supply for each type of households, derived from a shock in labour tax rates. Additionally, the wage rates seem to react positively to all tax items' rates shocks, as also the government debt. In fact, and as the government debt dynamics respond endogenously to tax revenues and, at the same time, the overall tax shocks tends to decrease the overall tax revenues, the outstanding public debt ends up increasing.

To conclude, and despite the fact that we did not include some hypothesis that would bring to our results a more realistic context, as the inclusion of heterogeneous preferences, or financial frictions, the overall analysis detailed in this Gap allows to elucidate, in general, how taxation affects the several economic relations. Moreover, our results are important in the sense that they can shed light to public authorities in the design of efficient policies for a better allocation of resources in an economy.

References

- Adolfson, M., Laséen, S., Lindé, J., and Villani, M. (2008). Evaluating an estimated new Keynesian small open economy model. *Journal of Economic Dynamics and Control*, 32(8):2690 – 2721.
- Afonso, A. and Ballhote, R. (2014). Interactions between Monetary Policy and Fiscal Policy. Working Papers Department of Economics 2014/13, ISEG - Lisbon School of Economics and Management, Department of Economics, University of Lisbon.
- Afonso, A. and Jalles, J. T. (2015). How Does Fiscal Policy Affect Investment? Evidence from a Large Panel. *International Journal of Finance & Economics*, 20(4):310–327.

- Afonso, A. and Rault, C. (2009). Spend-and-tax: a panel data investigation for the EU. *Economics Bulletin*, 29(4):2542–2548.
- Agliardi, E. (2001). Taxation and Investment Decisions: A Real Options Approach. *Australian Economic Papers*, 40(1):44–55.
- Alesina, A., Ardagna, S., Perotti, R., and Schiantarelli, F. (2002). Fiscal Policy, Profits, and Investment. *The American Economic Review*, 92(3):571–589.
- Almeida, V., Castro, G. L., Félix, R. M., Júlio, P., and Maria, J. R. (2013). Inside PESSOA - A Detailed Description of the Model. Working Papers w201316, Banco de Portugal, Economics and Research Department.
- Atkinson, A. and Stiglitz, J. (1976). The design of tax structure: Direct versus indirect taxation. *Journal of Public Economics*, 6(1):55 – 75.
- Becker, G. S. and Mulligan, C. B. (2003). Deadweight Costs and the Size of Government. *Journal of Law and Economics*, 46(2):293–340.
- Blomquist, S. and Christiansen, V. (2008). Taxation and Heterogeneous Preferences. *FinanzArchiv / Public Finance Analysis*, 64(2):218–244.
- Boadway, R., Marchand, M., Pestieau, P., and Del Mar Racionero, M. (2002). Optimal Redistribution with Heterogeneous Preferences for Leisure. *Journal of Public Economic Theory*, 4(4):475–498.
- Bouza, S. and Turnovsky, S. J. (2012). The distributional consequences of foreign transfers: do they reduce or exacerbate inequality? *Oxford Economic Papers*, 64(4):702–735.
- Brennan, G. and Buchanan, J. (2000). *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, volume 9 of *Collected Works of James M. Buchanan*. Liberty Fund, Indianapolis, United States.
- Brennan, G. and Buchanan, J. M. (1977). Towards a tax constitution for Leviathan. *Journal of Public Economics*, 8(3):255 – 273.
- Cloyne, J. (2014). Government spending shocks, wealth effects and distortionary taxes. Technical Report 1413, Centre for Macroeconomics (CFM).
- Coleman, W. J. (2000). Welfare and optimum dynamic taxation of consumption and income. *Journal of Public Economics*, 76(1):1 – 39.
- Correia, I. (2010). Consumption Taxes and Redistribution. *The American Economic Review*, 100(4):1673–1694.
- Corsetti, G. and Pesenti, P. (2005). International dimensions of optimal monetary policy. *Journal of Monetary Economics*, 52(2):281 – 305.

- Cremer, H., Pestieau, P., and Rochet, J.-C. (2001). Direct Versus Indirect Taxation: The Design of the Tax Structure Revisited. *International Economic Review*, 42(3):781–800.
- Edmiston, K. D. (2004). Tax Uncertainty and Investment: A Cross-Country Empirical Examination. *Economic Inquiry*, 42(3):425–440.
- El-Shazly, A. (2009). Investment Under Tax Policy Uncertainty: A Neoclassical Approach. *Public Finance Review*, 37(6):732–749.
- Fernández-Villaverde, J. (2010). Fiscal Policy in a Model with Financial Frictions. *American Economic Review*, 100(2):35–40.
- García-Peñalosa, C. and Turnovsky, S. J. (2007). Growth, Income Inequality, and Fiscal Policy: What Are the Relevant Trade-offs? *Journal of Money, Credit and Banking*, 39(2-3):369–394.
- García-Peñalosa, C. and Turnovsky, S. J. (2011). Taxation and Income Distribution Dynamics in a Neoclassical Growth Model. *Journal of Money, Credit and Banking*, 43(8):1543–1577.
- Golosov, M., Troshkin, M., Tsyvinski, A., and Weinzierl, M. (2013). Preference heterogeneity and optimal capital income taxation. *Journal of Public Economics*, 97:160 – 175.
- Hanlon, M. and Heitzman, S. (2010). A review of tax research. *Journal of Accounting and Economics*, 50(2):127 – 178.
- Hassett, K. A. and Metcalf, G. E. (1999). Investment with Uncertain Tax Policy: Does Random Tax Policy Discourage Investment. *The Economic Journal*, 109(457):372–393.
- Heer, B. and Trede, M. (2003). Efficiency and distribution effects of a revenue-neutral income tax reform. *Journal of Macroeconomics*, 25(1):87 – 107.
- Jaimovich, N. and Rebelo, S. (2017). Nonlinear effects of taxation on growth. *Journal of Political Economy*, 125(1):265–291.
- Junior, C. J. C. (2016). *Understanding DSGE Models: Theory and Applications*. Vernon Series in Economic Methodology. Vernon Press, Malaga, Spain.
- Kanbur, R. and Tuomala, M. (1994). Inherent Inequality and the Optimal Graduation of Marginal Tax Rates. *The Scandinavian Journal of Economics*, 96(2):275–282.
- Kaplow, L. (2008). Optimal Policy with Heterogeneous Preferences. *The B.E. Journal of Economic Analysis & Policy*, 8(1):1–30.
- Kilponen, J. and Ripatti, A. (2006). Labour and product market competition in a small open economy – Simulation results using a DGE model of the Finnish economy. Research Discussion Papers 5/2006, Bank of Finland.

- Kirsanova, T., Leith, C., and Wren-Lewis, S. (2016). Should central banks target consumer prices or the exchange rate?*. *The Economic Journal*, 116(512):F208–F231.
- Krusell, P. and Smith, A. A. J. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896.
- Lim, G. C. and McNelis, P. D. (2014). Income Inequality, Trade and Financial Openness. Melbourne Institute Working Paper Series wp2014n07, Melbourne Institute of Applied Economic and Social Research, The University of Melbourne.
- Lockwood, B. B. and Weinzierl, M. (2015). *De Gustibus non est Taxandum*: Heterogeneity in preferences and optimal redistribution. *Journal of Public Economics*, 124:74 – 80.
- Mertens, K. and Ravn, M. O. (2011). Understanding the aggregate effects of anticipated and unanticipated tax policy shocks. *Review of Economic Dynamics*, 14(1):27 – 54.
- Mirrlees, J. A. (1971). An Exploration in the Theory of Optimum Income Taxation. *The Review of Economic Studies*, 38(2):175–208.
- Nishiyama, S. and Smetters, K. (2005). Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks. *Journal of Political Economy*, 113(5):1088–1115.
- Polanyi, K. (2001). *The Great Transformation: The Political and Economic Origins of Our Time*. Beacon Press, Massachusetts, United States.
- Romer, C. D. and Romer, D. H. (2010). The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks. *The American Economic Review*, 100(3):763–801.
- Saez, E. (2002). The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes. *Journal of Public Economics*, 83(2):217 – 230.
- Salgado, S. (2011). Investment Dynamics in a DSGE Model with Heterogeneous Firms and Corporate Taxation. Working Papers Central Bank of Chile 638, Central Bank of Chile.
- Stähler, N. and Thomas, C. (2012). FiMod — A DSGE model for fiscal policy simulations. *Economic Modelling*, 29(2):239 – 261.
- Tarkiainen, R. and Tuomala, M. (2007). On optimal income taxation with heterogeneous work preferences. *International Journal of Economic Theory*, 3(1):35–46.