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Regional development of education as a "coordination game"

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Abstract

In this paper, we try to assess the ability of educationally backward countries, such as Portugal, to catch-up with more developed nations within the EU. For that purpose, we use a framework composed by a symmetric coordination n person game that is played by a set of candidates to attend a post-compulsory educational degree, such as university. Higher education has a positive payoff only if a "critical mass" (indeed the unanimity) of students with a low socioeconomic background decide to attend the university. Two strict Nash equilibria exist in this game: either all players decide to attend the university or none does it in equilibrium. By using the "risk dominance" approach to the selection of a unique Nash equilibrium that was suggested by HARSANYI and SELTEN (1988), we are able to recognize the factors that make either strict Nash equilibrium the likely solution. In spite of the progress they have achieved in schooling, structurally lagging countries such as Portugal seem to be hindered in education development by the fact that, in a large majority of households, income is low and parents lack post-compulsory education. While low household income makes the relative cost of university education high even if tuition fees are modest, a small share of highly educated parents makes the achievement of a "critical mass" of students who attend the university more difficult and thus renders the benefits of college education riskier and less safe.

Keywords: Higher Education; Regional Development; Coordination Games; Risk Dominance.

JEL classification: C72, I20, O12, R11.

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1. Introduction

From a geographical perspective, two main issues concerning the relationship between education and economic growth arise (REIS, 1993). Firstly, the investment in education means the accumulation of a specific kind of capital (namely *human capital*) so that it is assumed to help economic development. Secondly, the differential degree of success in the development of education across territories (namely "regions within a country, or "countries" within a regional integration area) is influenced by economic factors. In this paper, we deal with the latter class of issues.

In the literature, the determinants of participation and attainment level in post-compulsory education are usually treated as individual or family features, which are mostly related with socioeconomic family background. For the university, SPIESS and WROLICH (2010) single out factors such as the education level of parents, an emphasis being put on the education level of the father, and the per capita income of the household of origin.

Nevertheless, we can argue that it is likely that, in addition to an individual impact, economic factors influence educational performance at the neighborhood or local area level. Indeed, participation in higher education is positively related with the wage premium of skilled labor in comparison to unskilled. Such premium results from the fact that the workings of the labor market are regionally specific, given the existence of significant restrictions to labor mobility.

Even more important is the fact that the impact of distance to the nearest college upon the youngster's decision to enroll in higher education interacts with the candidate's socioeconomic background. Clearly, such impact appears to be stronger for students coming from relatively less favored families for different post-compulsory education levels (DICKERSON and MCINTOSH, 2013; FRENETTE, 2006).

It is also evident that per capita incomes and education levels are strongly spatially correlated. Based upon empirical evidence, BENABOU (1993) contends that the effort cost for a candidate living in a given neighborhood to enter the university decreases both with the share of highly educated people and the average income in the local area. He gives two kinds of explanations for this local area effect. Firstly, there is a "fiscal externality" which is present if colleges are to be financed from local resources and they provide complementary inputs to individual effort. Secondly, we have a set of "peer group effects" of different kinds (see BENABOU, 1993, p. 625), which arise from the fact that human capital is indeed a "social capital", so that individuals who participate in an educational process in fact "learn with each other" within a group of neighbors (LUCAS, 1988).

In this paper, we assess the economic factors that either help or hinder the development of post-compulsory education at the regional level through the framework of a coordination game, an idea that was put by WYDICK (2008). Since in many countries post-compulsory education is nowadays coincident with higher or "tertiary" education, the main emphasis will lie on the expansion of the university, although some reference will be made to secondary education.

2. Modelling the success in education development at the regional level through a coordination game

2.1. The rules of the coordination ("Stag Hunt") game

We assume that a country owns m regions, that are indexed by $i = 1, 2, \ldots, m$. We may also interpret this setting as referring to countries within a regional integration area, such as the European Union. Henceforth, the subscript i will be dropped since we presuppose that the factors determining educational success are internal to the region concerned.

We assume that the region contains n families, each family being composed by an adult and a youngster who has the proper age (say 18 years old) to enroll in the university. There are two types of families

1. $\overline{n} < n$ families where the adult has attended university with success.
2. $n - \overline{n}$ families where the adult does not hold a university degree.

The youngsters belonging to the set of $\overline{n}$ families with university background are predetermined to participate in higher education. The remaining $n - \overline{n}$ youngsters play a symmetric coordination game (the so called *Stag Hunt* game), where each player has two pure strategies at his disposal, namely

α , which consists in refraining from enrolling in the university, thus deciding to take a job immediately. This strategy leads to a certain payoff x.

β , which consists in entering the university and completing a degree. This strategy entails a certain cost c and one of two possible rewards,

- $y > 0$, if **all** less favored $n - \overline{n}$ youngsters decide to enter the university.

- 0, otherwise.

This is an **unanimity** game, where the investment in higher education has a positive return only if all potential students decide indeed to attend the university (see, for instance, VAN DAMME, 2002). We can rationalize the unanimity requirement as follows.

We presuppose that the region has the shape of a circumference with perimeter L, where both the families with and without university background (in numbers $\overline{n}$ and $n - \overline{n}$, respectively) are distributed uniformly. The regional government builds and keeps a network of q regularly spaced colleges. The cost of each college is entirely fixed (a structural cost) and given by g, so that it does not vary with the number of students enrolled in it. The regional government collects a tuition fee of f monetary units from each enrolled student. The revenue yielded by such fees is the only available funding to support the university in the region.

We presuppose that k regularly spaced families decide to send their youngsters to college. This set contains the families with a college background that send certainly their children to the university so that the inequality $\overline{n} < k < n$ holds. Then, the requirement of economic feasibility of higher education, expressed by the equality between revenues and expenditures, can be written as follows.

$$kf = qg \tag{1}$$

Furthermore, with q regularly spaced colleges, the maximal distance that an university student has to commute is approximated by d given by

$$d = \frac{L}{2q} \tag{2}$$

The latter expression implies that

$$q = \frac{L}{2d} \tag{3}$$

Through substituting (3) in (1), we obtain the following expression for the number of students in the university.

$$k = \frac{Lg}{2fd} \tag{4}$$

We presuppose now that there exists an upper bound $\overline{d}$ to the distance that a student accepts to commute to university (see SPIESS and WORLICH, 2010) and that this upper bound satisfies expression (4) as an equality.

$$k = \frac{Lg}{2f\overline{d}} \tag{5}$$

The meaning of (5) is straightforward. The critical mass of students which allows the university to break even, while it keeping spatially accessible to students, is k, as it is defined in the expression. Parameter k expresses the required level of coordination in this game (see HEINEMANN et al., 2009).

In general, we have $\overline{n} < k \leq n$. For simplicity, we will focus on the particular case where coordination among players implies their *unanimity*, so that we have

$$k = n = \frac{Lg}{2f\overline{d}} \tag{6}$$

In what follows, we specify the rewards x, y and the cost c. We bear in mind that, in this economy, a wage w_y is paid to skilled labor which is earned by adults with a university degree. By contrast, a wage $w_u < w_y$, is received by unskilled workers, who lack college attendance.

The cost of attending university by a student with a less favored background is $\frac{w_u}{2}$, i.e. one half of his household income, in addition to the college tuition fee f.

By assuming that the time discount rate, is r, we can write the payoffs and cost as

$$x = w_u \tag{7}$$

$$y - c = \frac{w_y}{1+r} - c \text{ where } c \text{ is given by} \tag{8}$$

$$c = f + \left(\frac{w_u}{2}\right) \tag{9}$$

While the reward of unskilled labor is received immediately, the payoff of skilled work is only received in the future *after* the completion of university in the future, hence it is discounted by rate r.

Then, we can simplify the game rules, while keeping invariant its best reply structure and set of Nash equilibria (see for this purpose WEIBULL, 1995). The steps of this transformation are:

1. We add c to the payoffs of strategies α and β, which thus become

$$\widetilde{x} = x + c = f + \frac{3w_u}{2}$$

$$\widetilde{y} = y + c = \frac{w_y}{1+r}$$

2. We multiply each resulting payoff by the positive factor $\frac{1}{\widetilde{y}}$ to obtain the

simplified payoffs

$$\widehat{x} = \frac{\widetilde{x}}{\widetilde{y}} = \left(f + \frac{3w_u}{2}\right)\left(\frac{1+r}{w_y}\right) \tag{10}$$

$$\widehat{y} = 1 \tag{11}$$

The rules of the game imply that $0 < \widehat{x} < \widehat{y}$, hence we have,

$$0 < \left(f + \frac{3w_u}{2}\right)\left(\frac{1+r}{w_y}\right) < 1 \tag{12}$$

By defining $w^* \equiv \dfrac{w_y}{w_u}$ as the wage premium associated with university attendance and $f^* \equiv \dfrac{f}{w_u}$ as the relative price of higher education, as compared with the income of a less favoured household, the payoffs (10) and (11) become

$$\widehat{x} = \left(f^* + \frac{3}{2}\right)\left(\frac{1+r}{w^*}\right) \tag{13}$$

$$\widehat{y} = 1 \tag{14}$$

From (13), the condition $\widehat{x} < 1$ can be written as

$$w^* > \left(f^* + \frac{3}{2}\right)(1+r) \tag{15}$$

Therefore, the rules of the coordination game imply that the wage premium of skill should be high in relation to the time discount rate and to the real price of higher education for a less favored household.

Then, we can express exactly the simplified rules of this n person *Stag Hunt* game. Each player can either select,

Strategy α , i.e., take a job immediately, and get a safe payoff $\widehat{x} < 1$.

Strategy β , i.e., enter the university and obtain

- Either a payoff $\widehat{y} = 1$, if **all** $n - \overline{n}$ less favored students decide to select β as well.
- Or a zero payoff otherwise.

2.2. Description of the set of Nash equilibria

A well known result is recalled here (see CARLSSON and VAN DAMME, 1993).

Proposition 1 *The education game has two strict Nash equilibria in pure strategies, namely an equilibrium where all* $n-\overline{n}$ *low background students choose strategy* α*, i.e. they opt to enter the labor market immediately, and an equilibrium where each of them selects strategy* β *and hence attends the university.*

Proof. It is clear that an outcome where *some* candidates select α and other potential students select β is **not** a Nash equilibrium. The reason is that a candidate of the latter type obtains a zero payoff, while he can get a positive payoff $\widehat{x}$ if he switches to α.

It is straightforward to prove that the situations where each less favored candidate selects α and all potential students with low background choose β are both strict Nash equilibria. ■

2.3. Selection of a Nash equilibrium through risk dominance

Since the education game has two strict Nash equilibria, namely "all candidates choose α" and "all candidates select β", we try now to select a unique equilibrium through the specification of the beliefs that each player holds on the behavior of the other participants. For that purpose, we will use the concept of *risk dominance*, which was put forward by HARSANYI and SELTEN (1988) and carefully explained again by VAN DAMME (2002). In the case of n person coordination game outline din this paper, the *risk dominance* procedure entails the following steps.

Firstly, we determine for each player i the prior expectation he holds about any other player selecting a given pure strategy, let us say, strategy β. We name this prior belief as probability $p_i(\beta)$. Player i assumes that every other player knows the true Nash equilibrium of the game. Hence, he has a subjective probability z_i that every other player selects β and a probability $1 - z_i$ that the other players choose α.

The players other than i, whom we label collectively as "$\_i$", do not observe the value of z_i. According to the principle of "insufficient reason", they assume that z_i is a random variable, that is uniformly distributed in $[0, 1]$.

From the viewpoint of players "$\_i$" (who behave here as some kind of external observer of i's actions), for each realization of random variable z_i player i will take the best reply to it. Let $b_i^{z_i}(\beta)$ label i's best reply to subjective probability z_i in terms of the probability that he assigns to playing pure strategy β.

$$b_i^{z_i}(\beta) = \begin{cases} 0 & \text{if} \quad z_i < \widehat{x} \\ 1 & \text{if} \quad z_i > \widehat{x} \end{cases} \tag{16}$$

Consequently, players "$\_i$" will compute the prior expectation that player i

holds about them selecting pure strategy β

$$
\begin{aligned}
p_i(\beta) &= \int_0^1 b_i^{z_i}(\beta)\, dz_i \\
p_i(\beta) &= \int_0^{\widehat{x}} b_i^{z_i}(\beta)\, dz_i + \int_{\widehat{x}}^1 b_i^{z_i}(\beta)\, dz_i \\
p_i(\beta) &= \int_0^{\widehat{x}} 0 \cdot dz_i + \int_{\widehat{x}}^1 1 \cdot dz_i \\
p_i(\beta) &= 1 - \widehat{x} \qquad (17)
\end{aligned}
$$

Secondly, each player computes the expected payoff of each one of his pure strategies and defines a plan which consists in selecting the best reply to his prior expectation $p_i(\beta)$. Since the game is symmetric, the players's plans of action will be identical. With the prior expectation derived in (17), the probability that every other "less favored" candidate chooses β is simply $(1-\widehat{x})^{(n-\overline{n}-1)}$. As the payoff of an unanimous choice of strategy β is 1, the associated expected payoff is also $(1-\widehat{x})^{(n-\overline{n}-1)}$. Recalling that $\widehat{x}$ is the certain payoff of strategy α, the optimal plan of action will be to play β iff

$$
(1-\widehat{x})^{(n-\overline{n}-1)} > \widehat{x} \qquad (18)
$$

To play α will be the optimal course of action if the reverse of (18) is satisfied.

Thirdly, since the game is symmetric, the plans of action selected by all players will be identical (either all players choose α or β). In any case, according to Proposition 1, each set of plans is coincident with one strict Nash equilibrium of the original game. Hence, following HARSANYI and SELTEN (1988), each one is robust to any further updating of expectations and can be viewed as the unique solution of the game, the *risk dominant* solution to be more precise.

We now assess the meaning of condition (18). By taking logs in both sides, it becomes

$$(n - \overline{n} - 1)\ln(1 - \widehat{x}) > \ln \widehat{x} \text{ or} \quad (19)$$

$$n - \widehat{n} < 1 + \gamma(\widehat{x}) \text{ where } \gamma(\widehat{x}) \equiv \frac{\ln \widehat{x}}{\ln(1 - \widehat{x})} \quad (20)$$

It can be checked through direct calculation that $\gamma(\widehat{x})$ is a strictly decreasing function.

We try now to rationalize the situation of coordination *failure*, where the solution of the game implies that no candidate enrolls in the university. The necessary and sufficient condition for this failure to take place is just the reverse of inequality (20),

$$n - \widehat{n} > 1 + \gamma(\widehat{x}) \quad (21)$$

2.4. Determinants of coordination failure in higher education

From (21), and taking into account that $\gamma'(\widehat{x}) < 0$, we can sort the factors that lead to failure in the development of higher education in the following way.

Factors explaining a high value of n: From (6), we realize that the value of n increases with g, the fixed cost of setting up a college, and decreases with $\overline{d}$, the maximal distance a student accepts to commute to the university. The former influence means that a more "laboratory oriented" university has higher fixed costs, so that it requires a higher critical mass of enrolled students in order to be able to break even. The latter factor may happen as a result of the deterioration of the transport system, in particular of public transportation. The worsening of transport systems may derive either from a rise in the prices or a decrease in the quality of transport services. In either case, a denser network of colleges is required to ensure proximity to students residences, thus requiring a higher critical mass of enrolled youngsters and leading to a more risky coordination.

A low value of $\overline{n}$: If, among the n families living in the region, only a few $\overline{n}$ are predetermined to send their children to college, then the achievement of unanimity in higher education attendance is harder to obtain than if many families had university attendance.

Factors behind a high value of $\widehat{x}$: The riskiness of enrolling in the university decreases obviously with the wage premium yielded by higher education w^*. It increases with the costs related with an investment in human capital, namely the relative price of education f^* and with the rate r through which the future benefits of this type of investment are discounted.

3. Development of post-compulsory education in Portugal

In Portugal, compulsory education rose fast since the institution of a democratic regime in 1974. At that time, it involved only six schooling years. In the aftermath, it was raised to nine years from 1985 on and it was eventually set at twelve years after 2008.

Schooling rates rose fast in Portugal in all education levels between 2004 and 2016, but they still lag behind the average in the EU28 countries (including the UK). The following table shows the population with at least secondary education (ISCED 3 or more) as a percentage of population with age between 25 and 64 years.

	2004	2016	Average annual growth rate
EU28	68.3	77.0	1.0
Portugal	24.9	46.9	5.3
Source: Eurostat			

(22)

The increase was dramatic for higher education, where the number of people endowed with this schooling level was raised about three times during the period between 1998 and 2018.

Nevertheless, as CARNEIRO (2014) remarked, the rate of premature abandonment of formal education and training remained high in comparison with European average values. The following table shows the evolution of the percentage of people who left education without even completing the secondary school in total population.

	2004	2016
EU28	16.0	10.7
Portugal	39.3	14.0
Source: Eurostat		

(23)

When interpreting Table (23), we should take into account that secondary education became compulsory in 2008 thereby constraining youngsters to remain in school up to 18 years old. In spite of this improvement, Portugal still ranks very low in what concerns school abandonment, indeed the $23th$ position within the EU.

By using the n persons coordination game that was outlined above, we can assess the factors of either development or backwardness is post-compulsory education.

Factors of educational development: we can enumerate a list of factors that decreased the level of risk associated with the investment in post-compulsory education. Firstly, the fast improvement in transport systems increased $\overline{d}$, the maximal distance a student accepts to commute to the nearest college, thereby reducing the "critical mass" of students as expressed in (6). Secondly, the orientation to an educational pattern with low fixed costs, i.e. low value of g, concentrated in areas such as such as law, management or educational training, explains the transition to a denser network of colleges and a reduction of the minimum number of students who must coordinate in order that the university breaks even. It is further evident that adverse demographic trends in Portugal have not led to a decrease in the density of the network of colleges. Thirdly, there is wide consensus that the wage premium associated with college attendance has remained ranked among the highest ones among developed countries (see TEIXEIRA et al., 2014), in spite of the fact that in Portugal the variation in such premium across students with heterogeneous socioeconomic background appears to be relatively high.

Factors of educational backwardness: We can also name a list of factors that explain that the development of education did not lead to a catch-up with other European countries. Firstly, the share of families where the parents (or the father) have attended higher education is comparatively very low in Portugal as compared with other European countries. A low value of $\overline{n}$ in (21) makes coordination among students more difficult and raises the level of risk of engaging in higher education. Secondly, wages of unskilled labor rank among the lowest within the EU. However, the high share of students who attend private schools and colleges in Portugal (by comparison with other European countries) shows that tuition fees and other direct financial costs are not a main deterrent of higher education.

4. Concluding remarks

In this paper, we try to assess the ability of educationally backward countries, such as Portugal, to catch-up with more developed nations within the EU. For that purpose, we use a framework composed by a symmetric coordination n person game. This *Stag Hunt* game is played by a set of candidates to attend a post-compulsory educational degree, such as university. There are two types of candidates, namely those whose parents have completed university, who are predetermined to attend a college, and those coming from families with a lower background. The latter can choose whether to enter immediately the labor market or to postpone it so as to attend the university. Higher education has a positive payoff only if a "critical mass" (indeed the *unanimity* in this paper) of low background students decide to attend the university.

This n person symmetric coordination game has two strict Nash equilibria, which are symmetric so that either all players decide to attend the university or none does it in equilibrium. By using the *risk dominance* approach to the selection of a unique Nash equilibrium that was suggested by HARSANYI and SELTEN (1988), we are able to recognize the factors that make either strict Nash equilibrium the likely.

In spite of the progress they have achieved in schooling at all levels, structurally lagging countries such as Portugal seem to be hindered in education development by the fact that, in a large majority of households, income is low and parents lack post-compulsory education. While tuition fees and other direct financial costs do not seem to have restrained the growth of higher education demand, the fact that only a small share of parents are highly educated makes the requirement of a "critical mass" of students who attend the university more difficult to be achieved and thus renders the benefits of college education riskier and less safe.

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